Dynamic Growth & Double Hashing

Concept: When the load factor ($\alpha$) reaches a fixed threshold, either (1) build a new double hash table whose size is fractionally larger ($n$) than the old table ($cn$) or (2) build a new double hash table whose size ($n$) is a fixed constant larger than the old table ($n - c$).

Analysis of expected insertion cost alone

Fractional Growth:

$$T(n) = T(cn) + dn$$

where $d$ is based on initializing new table and (total) expected insertion cost to reach $\alpha$.

$O(n)$ by substitution method:

Assume $T(k) \leq ek$ for $k < n$

$T(cn) \leq ecn$ for $c < 1$

$T(n) \leq ecn + dn$

$$= en + dn + (ec - e)n$$

$$\leq en \text{ if } e - ec \geq d$$

Fixed Growth:

$$T(n) = T(n - c) + dn$$

where $d$ is based on initializing new table and expected insertion cost to reach $\alpha$.

$O(n^2)$ by substitution method

Assume $T(k) \leq ek^2$ for $k < n$

$T(n - c) \leq e(n - c)^2 = en^2 - 2cen + ec^2$

$T(n) \leq en^2 - 2cen + ec^2 + dn$

$$\leq en^2 \text{ if } 2ce \geq d \text{ (ignoring } ec^2)$$