Pseudocode Conventions (p. 20-22)

Array Subscripts:

- Book: 1...n
- Notes/C/Java Code: 0...n – 1

1.A. QUADRATIC TIME SORTS:

Selection Sort (CLRS exercise 2.2-2)

```c
void selection(Item a[], int ell, int r)
{
    int i, j;
    for (i = ell; i < r; i++)
    {
        int min = i;
        for (j = i+1; j <= r; j++)
            if (less(a[j], a[min]))
                min = j;
        exch(a[i], a[min]);
    }
}
```

Always uses \[ \sum_{i=2}^{n}(i-1) = \sum_{i=1}^{n-1}i = \frac{n(n-1)}{2} \approx \frac{n^2}{2} \] comparisons and is not stable.

Insertion Sort (CLRS p.18, http://ranger.uta.edu/~weems/NOTES2320/insertionSort.c)

```c
void insertionSort(Item *a, int N) // Guaranteed stable
{
    int i,j;
    Item v;
    for (i=1; i<N; i++)
    {
        v=a[i];
        j=i;
        while (j>0 && less(v,a[j-1]))
        {
            a[j]=a[j-1];
            j--;
        }
        a[j]=v;
    }
}
```
Maximum ("worst case") number of times that body of j-loop executes for a particular value of i?

Maximum number of times that body of j-loop executes over entire sort?

\[ \sum_{i=1}^{k} i = \frac{k(k + 1)}{2} = ? \]

Expected ("average") number of times that body of j-loop executes for a particular value of i?

Expected number of times that body of j-loop executes over entire sort?

I.B. DIVIDE AND CONQUER (Decomposition)

1. Divide into subproblems (unless size allows a trivial solution).
2. Conquer the subproblems.
3. Combine solutions to subproblems.

(Binary) Mergesort – An “Optimal” Key-Comparison Sort (http://ranger.uta.edu/~weems/NOTES2320/mergesort.new.c)

1. Split (copy) array into two sub-arrays (unless n<2).
2. Call Mergesort recursively for each sub-array.
3. Merge together the two ordered sub-arrays.
int merge(int *in1, int *in2, int *out1, int in1Size, int in2Size)
{
    int i, j, k;
    i = j = k = 0;
    while (i < in1Size && j < in2Size)
    {
        if (in1[i] < in2[j])
            out1[k++] = in1[i++];
        else
            out1[k++] = in2[j++];
    }
    if (i < in1Size)
        for (; i < in1Size; i++)
            out1[k++] = in1[i];
    else
        for (; j < in2Size; j++)
            out1[k++] = in2[j];
    return k;
}

How are items with identical keys (“duplicates”) handled?

[Write body of while-loop with ?: expression. Code for linked lists, files, streams, etc.]

Fall 2009 Test Problem Applying Merge Concept

Two int arrays, A and B, contain m and n ints each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e., each table represents a set). Give Java code for a \( \Theta(m + n) \) algorithm to find the **symmetric difference** by producing a third array C (in ascending order) with the values that appear in *exactly* one of A and B and sets the variable p to the final number of elements copied to C. (Details of input/output, allocation, declarations, error checking, comments and style are unnecessary.)

```java
i = j = p = 0;
while (i < m && j < n)
    if (A[i] < B[j])
        C[p++] = A[i++];
    else if (A[i] > B[j])
        C[p++] = B[j++];
    else
        {
            i++;
            j++;
        }
for (; i < m; i++)
    C[p++] = A[i];
for (; j < n; j++)
    C[p++] = B[j];
```
How much work (time) in worse case? \( T(n) \) – a recurrence

1. Split: \( n \) steps. [Can reduce to constant time by computing pointer.]
2. Call recursively:

\[
T\left(\left\lfloor \frac{n}{2} \right\rfloor \right) + T\left(\left\lceil \frac{n}{2} \right\rceil \right)
\]

3. Merge together (\( n \) steps)

\[
T(n) = c_1 n + T\left(\left\lfloor \frac{n}{2} \right\rfloor \right) + T\left(\left\lceil \frac{n}{2} \right\rceil \right) + c_2 n = cn \log n
\]

Recursion Tree

\[
\begin{align*}
\pi(n=2^k) &= \\
\pi(n/2=2^{k-1}) &= \\
\pi(n/4=2^{k-2}) &= \\
\pi(n/8=2^{k-3}) &= \\
\pi(n/16=2^{k-4}) &= \\
&\vdots \\
\pi(n/2^k=2^{k-k}) &= 
\end{align*}
\]

[Don’t generalize from this example. More of these later.]
I.C. BINARY SEARCH - “Optimal” Search of an Ordered Table (or “Space”)

Concept – search *ordered* table in logarithmic time. Consider table with $2^k - 1$ slots.

Recursive binary search?

```c
int binSearch(int *a, int n, int key)
// Input: int array a[] with n elements in ascending order.
//        int key to find.
// Output: Returns some subscript of a where key is found.
//         Returns -1 if not found.
// Processing: Binary search.
{
    int low, high, mid;
    low = 0;
    high = n-1;
    // subscripts between low and high are in search range.
    // size of range halves in each iteration.
    while (low <= high)
    {
        mid = (low + high) / 2;
        if (a[mid] == key)
            return mid; // key found
        if (a[mid] < key)
            low = mid+1;
        else
            high = mid-1;
    }
    return (-1); // key does not appear
}
```

( [http://ranger.uta.edu/~weems/NOTES2320/binarySearch.c](http://ranger.uta.edu/~weems/NOTES2320/binarySearch.c) )
Find \( i \) such that \( a[i-1] < \text{key} \leq a[i] \)

```c
int binSearchFirst(int *a, int n, int key)
// Input: int array a[] with n elements in ascending order.
//        int key to find.
// Output: Returns subscript of the first a element \( \geq \text{key} \).
//         Returns n if key > a[n-1].
// Processing: Binary search.
{ int low, high, mid;
  low=0;
  high=n-1;
  // Subscripts between low and high are in search range.
  // Size of range halves in each iteration.
  // When low>high, low==high+1 and a[high]<key and a[low]\( \geq \text{key} \).
  while (low<=high)
  { mid=(low+high)/2;
    if (a[mid]<key)
      low=mid+1;
    else
      high=mid-1;
  }
  return low;
}
```

Relationship of low and high on return?

Find \( i \) such that \( a[i] \leq \text{key} < a[i+1] \)

```c
int binSearchLast(int *a, int n, int key)
{ // Input: int array a[] with n elements in ascending order.
  //        int key to find.
  // Output: Returns subscript of the last a element \( \leq \text{key} \).
  //         Returns \(-1\) if key < a[0].
  // Processing: Binary search.
  int low, high, mid;
  low=0;
  high=n-1;
  // Subscripts between low and high are in search range.
  // Size of range halves in each iteration.
  // When low>high, low==high+1 and a[high]\( \leq \text{key} \) and a[low]>key.
  while (low<=high)
  { mid=(low+high)/2;
    if (a[mid]\( \leq \text{key} \))
      low=mid+1;
    else
      high=mid-1;
  }
  return high;
}
```

Relationship of low and high on return?
Partial output from `binarySearchRange.c` (count is last-first+1)

<table>
<thead>
<tr>
<th>-- table --</th>
<th>key</th>
<th>first</th>
<th>last</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
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<td>1</td>
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<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
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<tr>
<td>3</td>
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<td>4</td>
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</tbody>
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