Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Suppose you are sorting millions of keys that consist of three decimal digits each. Which of the following sorts uses time beyond $O(n \lg n)$ in the worst case?
   A. counting
   B. heapsort
   C. merge
   D. quick

2. Which sort never compares two inputs (to each other) twice?
   A. heap
   B. merge
   C. quick
   D. shell

3. Suppose that a binary search is to be performed on a table with 30 elements. The maximum number of elements that could be examined (probes) is:
   A. 4
   B. 5
   C. 6
   D. 7

4. Which function is in neither $\Omega(2^n)$ nor $O(3^n)$?
   A. $2^n + n^2$
   B. $3^n - n^2$
   C. $2.5^n$
   D. $\ln n$

5. Which recurrence describes the worst-case time used by QUICKSORT?
   A. $T(n) = T\left(\frac{n}{2}\right) + n$
   B. $T(n) = 2T\left(\frac{n}{2}\right) + n$
   C. $T(n) = T(n-1) + n$
   D. $T(n) = T\left(\frac{n}{2}\right) + 1$

6. Which of the following sorts is not stable?
   A. insertion
   B. mergesort
   C. radix
   D. quick

7. What is the value of $H_4 - H_3$?
   A. $\lg 4$
   B. $\frac{1}{4}$
   C. $\frac{11}{6}$
   D. $3$

8. Which of the following will not be true regarding the decision tree for MERGE-SORT for sorting $n$ input values?
   A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
   B. The height of the tree is $\Omega(n \log n)$.
   C. There will be a path from the root to a leaf with $\Omega\left(n^2\right)$ decisions.
   D. There will be $n!$ leaves.

9. What is the value of $\sum_{k=0}^{\infty} \frac{1}{3^k}$?
10. Suppose the input to HEAPSORT is always a table of identical integers. The worst-case time will be
A. Θ(1)
B. Θ(n)
C. Θ(n log n)
D. Θ(n^2)

Long Answer
1. Prove that if \( g(n) \in \Omega(f(n)) \) then \( f(n) \in O(g(n)) \). 10 points

2. Suppose that a max-heap is used for the subfile production phase of external mergesort. If the available memory size is \( m \), what is the best case, worst case, and expected case for the size of the subfiles produced? 10 points

3. Use the substitution method to show that \( T(n) = 8T\left(\frac{n}{2}\right) + n^3 \) is in \( \Theta(n^3 \log n) \). 10 points

4. Use the recursion-tree method to show that \( T(n) = 8T\left(\frac{n}{2}\right) + n^3 \) is in \( \Theta(n^3 \log n) \). 10 points

5. Demonstrate how several executions of PARTITION may be used to determine the median of the following array. 10 points

6. Perform BUILD-MAX-HEAP and then EXTRACT-MAX (once). 10 points

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 4 points each

1. Why is it common for a circular queue implementation to waste one table element?
A. To avoid confusing an empty table with a full table
B. To have a place to store the tail and head values
C. To make sure the queue always has at least one element in use
D. To perform some loops faster

2. When evaluating a prefix expression, the stack contains
A. Both operands and operators
B. Both parentheses and operators
C. Operands only
D. Operators only

3. The worst-case time to find the maximum key in a circular, doubly-linked list with \( n \) nodes in ascending order is:
A. \( \Theta(1) \)
B. \( \Theta(\log n) \)
C. \( \Theta(n \log n) \)
D. \( \Theta(n) \)

4. How should the predecessor of a node with a left child in an unbalanced binary search tree be found?
A. Examine the ancestors of the node
B. Go left, then proceed to the right
C. Go right, then proceed to the left
D. Preorder traversal

5. If \text{Pop} is implemented as return stack[SP--], then \text{Push} of element X is implemented as:
   A. return stack[SP++]
   B. stack[SP++] = X
   C. stack[--SP] = X
   D. stack[++SP] = X

6. Given the load factor \( \alpha \), which expression is the most suitable upper bound on the expected number of probes for a successful search using double hashing?
   A. \( \frac{\alpha}{2} \)
   B. \( \alpha \)
   C. \( \frac{1}{1 - \alpha} \)
   D. \( \frac{1}{\alpha \ln \frac{1}{1 - \alpha}} \)

7. The stack for rat-in-a-maze stores
   A. all positions that have walls
   B. maze positions that must be in the final path
   C. the current path being explored
   D. the shortest known path that leads to the cheese

8. In the example of concatenating two lists in O(1) time, which situation holds?
   A. Both lists are not circular
   B. Both lists are circular
   C. The first list is circular, the second list is not
   D. The second list is circular, the first list is not

9. Which traversal will list the keys in a binary search tree in ascending order?
   A. Inorder
   B. Postorder
   C. Preorder
   D. All of the above

10. In which situation will a sentinel be inappropriate?
    A. Binary search for a key in an ordered table, to simplify and speed-up code
    B. Search for a key in an unordered table, to simplify and speed-up code
    C. Search for a key in an unordered linked list, to simplify and speed-up code
    D. Red-black tree, to simplify code

Long Answer
1. Describe the three situations that can occur for deletion from an unbalanced binary search tree. An example for each situation will be fine. (10 points)

2. Consider the following hash table whose keys were stored by linear probing using 
   \( h(key, i) = (key + i) \mod 13 \).

   a. Suppose 135 is to be stored (using linear probing). Which slot will be used? (5 points)
   b. Suppose 142 is to be stored (using linear probing) after 135 has been stored. Which slot will be used? (5 points)

3. Insert 75 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
4. Insert 95 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)

5. Consider the following hash table whose keys were stored by double hashing using
   \( h_1(key) = key \% 13 \) and \( h_2(key) = 1 + (key \% 12) \).

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</table>
   0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 |
   |   |   |   |   |   |   |   | 120| 186| 187| 162| 122|
   |   |   |   |   |   |   | 110|   |   |   |   | 194|

   a. Suppose 134 is to be inserted (using double hashing). Which slot will be used? (5 points)
   b. Suppose 500 is to be inserted (using double hashing) after 134 has been stored. Which slot will be used? (5 points)

6. How are deletions handled for open addressing? Be sure to address any issues regarding searches. (10 points)
C. there are no restarts  
D. there are no back edges

10. Before searching for a minimum cut in a network, it is useful to do the following:  
A. Determine the type of each edge using depth-first search.  
B. Find and record augmenting paths until none remains.  
C. Find one augmenting path.  
D. Perform a breadth-first search on the input network.

11. Suppose that a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, then its type will be:  
A. Back  B. Cross  C. Forward  D. Tree

12. The number of HEAP-EXTRACT-MINS to build a Huffman code tree for n symbols is:  
A. $\theta(\log n)$  B. n - 1  C. n  D. 2n - 2

13. Which of the following is not true for the activity scheduling problem?  
A. The activities may have various durations.  
B. The goal is to maximize the number of activities.  
C. The greedy solution is a heuristic.  
D. There may be several optimal solutions.

14. The goal of the optimal matrix multiplication problem is to:  
A. Minimize the number of $C(i,j)$ instances evaluated.  
B. Minimize the number of matrix multiplications.  
C. Minimize the number of scalar additions.  
D. Minimize the number of scalar multiplications.

15. The fractional knapsack problem may be solved optimally by a greedy method by taking less than the full available quantity of no more than this number of items.  
A. 0  B. 1  C. 2  D. 3

Long Answer  
1. What are the entries in the T-table (for Prim’s algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.

2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ordered. Write your answer in the tables below. 10 points
3. Demonstrate the Floyd-Warshall algorithm, with successors, for the following graph. You are not required to show the intermediate matrices. 15 points.

4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.

5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

$p[0]=8$
$p[1]=6$
$p[2]=1$
$p[3]=3$
$p[4]=3$
$p[5]=7$
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<tr>
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