Short Answer - 5 points each
1. Why is BUILD-MAX-HEAP better than using sorting?
2. Give the recurrence for the worst-case running time for merge sort.
3. Shellsort uses preprocessing to improve the performance of which sorting method?
4. Give the definition of $H_n$ and an approximation of its value.
5. Give a function that is in both $\Omega(2^n)$ and $O(3^n)$, but is not in $\Theta(2^n)$ or $\Theta(3^n)$.
6. What is the worst-case running time for PARTITION?
7. Explain how the elements of a heap are mapped to an array.
8. Use integrals to bound, both above and below, the value of

$$\sum_{i=1}^{n} i$$

9. In the study of sorting, why is the fact that $\lg(n!) = \Omega(n \lg n)$ significant?
10. List the sorts that require only $O(1)$ additional space besides the array to be sorted.

Long Answer.
1. Use the definitions of $O$ and $\Omega$ to prove that if $f(n) \in O(g(n))$, then $g(n) \in \Omega(f(n))$. 10 points
2. Use the recursion tree method to show that $T(n) = T(n/2) + \lg n$ is in $\Theta(\lg^2 n)$. (Recall that $\lg^2 n = (\lg n)^2$.) 10 points.
3. Use the substitution method to show that $T(n) = T(n/2) + \lg n$ is in $\Theta(\lg^2 n)$. (Recall that $\lg^2 n = (\lg n)^2$.) 10 points.
4. Suppose an array stores 1000 distinct integers in descending order. Give code for a binary search to determine the subscript (if it exists) where a supplied number is stored in the table. 10 points
5. Demonstrate PARTITION on the following array. 10 points

8 0 4 9 1 2 3 6 7 5
2. Use linear probing with remaindering as the hash function to place the keys in the hash table. The insertions must be done in the order given:

<table>
<thead>
<tr>
<th>86</th>
<th>94</th>
<th>87</th>
<th>62</th>
<th>122</th>
<th>110</th>
<th>20</th>
<th>30</th>
<th>88</th>
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</thead>
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</tr>
</tbody>
</table>

3. Describe each of the linked list variations that were studied and indicate why each is useful.

4. Insert 35 into the given red-black tree. Be sure to indicate the cases that you used. 10 points

```
80
 /   \
20 B   100 B
 /     /   \
10 B   60 R   90 B
 /   /   /   \
30 R 40 B 70 B
 /   /   \
30 R 50 R
```

5. Insert 35 into the given red-black tree. Be sure to indicate the cases that you used. 10 points

```
60 B
 /   \
20 R   80 R
 /     /   \
10 B   40 B   70 B
 /   /     /   \
30 R 40 B 70 B
 /   /   \
30 R 50 R
```

6. Delete 60 from the given red-black tree. Be sure to indicate the cases that you used. 10 points
7. Delete 150 from the given red-black tree. Be sure to indicate the cases that you used. 10 points

```
  60
  /   \\
 20     120
 /   /   /   \
10  40 100 140
    /   /         \
   50  80 110   150
  /   /   /   \
70  90  130   \
```

CSE 2320
Test 3
100 points

Short Answer - 5 points each

1. Give an instance of the network flow problem that could have an exponential number of augmenting paths if breadth-first search is not used.

2. What is the worst-case time for the algorithm that determines the strongly-connected components of a directed graph?

3. State the cost function for the longest common subsequence problem.

4. Explain the meaning of each of the “colors” used during breadth-first search.

Long Answer.

1. Complete the following instance of KMP failure link table construction. 15 points

```
  0 a
  1 c
  2 a
  3 b
  4 a
  5 c
```
2. Perform depth-first search on the following graph, including edge types. Assume that the adjacency lists are ordered. 10 points

3. Demonstrate Warshall’s algorithm on the following graph by giving either the final successor matrix or the final predecessor matrix. 10 points

4. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
p[0]=5
p[1]=6
p[2]=4
p[3]=7
p[4]=8
p[5]=4
```

```
1  0  0  120  1  260  2  504  2  ???  5
2  -------  0  0  168  2  416  2  432  2
3  -------  -------  0  0  224  3  336  3
4  -------  -------  -------  0  0  224  4
5  -------  -------  -------  -------  0  0
```
5. For the given network, determine a maximum flow and the minimum cut. Be sure to give each augmenting path, the amount of additional flow that it provides, and the residual graph after each augmenting path is recorded. You may choose the augmenting paths in any manner you wish. 15 points

Augmenting Path/Flow:
.
.
.
Minimum Cut:

6. Give an optimal Huffman code tree for the provided symbols and probabilities. In addition, compute the expected number of bits per symbol. 10 points

A .15
B .2
C .15
D .13
E .2
F .07
G .1

7. What are the entries in the heap (for Prim’s algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points