Asymptotic Bounds

**Review:**

Use ratio and limit to infinity to compare function growth. Examples:



Examples of functions that have N2 growth:

Speed of Growth:

\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_<\_\_\_\_

Fill in each column functions that have the same growth as the column label.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | lgN | N | NlgN | N2 | N3 | 2N |
|  |  |  |  |  |  |  |

The functions listed in a column are \_\_\_\_ of the function that labels the column. E.g. \_\_\_\_\_\_\_\_\_ = \_\_\_( N2)

From ordering by growth to asymptotic notation

…………………………………………………………………………………………………………………………………………………………………………………

…………………………………………………………………………………………………………………………………………………………………………………

Symbol: Θ O o Ω ⍵

Meaning:

Examples: N3/10 – 500N2 -1000 = O(N3) True/False Solution: find dominant term(s) and compare their growth

Notation abuse: = instead of

Finding Θ , O , Ω for an algorithm.

|  |  |
| --- | --- |
| Worst case is Θ(\_\_\_) then the algorithm is \_\_\_ ( \_\_\_\_\_\_)Best case is Θ(\_\_\_) then the algorithm is \_\_\_ ( \_\_\_\_\_\_) | Insertion sort:Worst case is Θ(\_\_\_) then insertion sort is \_\_\_ ( \_\_\_\_\_\_)Best case is Θ(\_\_\_) then insertion sort is \_\_\_ ( \_\_\_\_\_\_) |

An algorithm is Θ(\_\_\_\_\_) iff best case and worst case have \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ TC.

Exercise: Assume:

* Alg 1 is O(N2) ……can be: ……………………………………………………….. cannot be: ………………………………………………………..
* Alg 2 is Θ(NlgN) can be: ……………………………………………………….. cannot be: ………………………………………………………..
* Alg 3 is Ω(N) …… can be: ……………………………………………………….. cannot be: ………………………………………………………..
* what could be the TC function in each case?
* which one is “better”? (define better)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Symbol | Name | Meaning | Notation | Examples | Limit theorem |
| ⍵ |  |  |  |  |  |
| Ω |  |  |  |  |  |
| Θ |  |  |  |  |  |
| O |  |  |  |  |  |
| o |  |  |  |  |  |

Ex: Fill in s.t the statements are correct

TC = Θ(log3(N)) => TC = o(\_\_\_\_\_\_) , TC = O(\_\_\_\_\_\_), TC = Ω(\_\_\_\_\_\_), TC = ⍵(\_\_\_\_\_\_)

⍵: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ω: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

TC = Θ(N2) => TC = o(\_\_\_\_\_\_) , TC = O(\_\_\_\_\_\_), TC = Ω(\_\_\_\_\_\_), TC = ⍵(\_\_\_\_\_\_)

⍵: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ω: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

TC = O(N2)

⍵: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ω: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ο: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Asymptotic Bounds as Limits:

f(n) = ω(g(n)) ⇔

English Translation:

 f(n) = Ω(g(n)) ⇔

English Translation:

f(n) = Θ(g(n)) ⇔   (limit is a non-zero constant)

English Translation:

 f(n) = O(g(n)) ⇔

English Translation:

 f(n) = o(g(n)) ⇔

English Translation:

Properties:

1.

2.

3.

4.

5.

\*\*\* Transitivity (From Discrete Structures): If a > b and b > c, then a > c. This concept can be applied to TC bounds as well.

6.

7.