## **Asymptotic Bounds**

## **Review:**

Use ratio and limit to infinity to compare function growth. Examples:

a) 
$$N^2 < N^3$$
  $\frac{N^2}{N^3} = \frac{1}{N}$  use **1** grows slower than **N** or  $\lim_{N \to \infty} \frac{1}{N} = \mathbf{0}$   $\Rightarrow$  top grows slower

b) 
$$N\sqrt{N} > N$$
  $\frac{N\sqrt{N}}{N} = \frac{\sqrt{N}}{1}$  use  $\sqrt{N}$  grows faster than 1 or  $\lim_{N\to\infty} \frac{\sqrt{N}}{1} = \infty$   $\Rightarrow$  top grows faster

c) 
$$N\sqrt{N} < N^2$$
  $\frac{N\sqrt{N}}{N^2} = \frac{1}{\sqrt{N}}$  use **1** grows slower than  $\sqrt{N}$  or  $\lim_{N\to\infty} \frac{1}{\sqrt{N}} = 0$   $\Rightarrow$  top grows slower

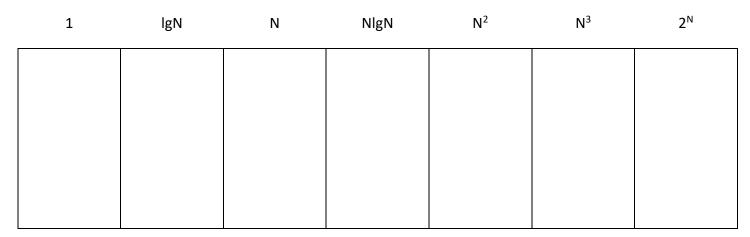
d) 
$$lgN \approx log_3N$$
  $\frac{lgN}{log_3N} = \frac{lgN}{\frac{lgN}{lg3}} = \frac{lgN}{1} * \frac{lg3}{lgN} = lg3$  use  $lg3$  is a constant (no N) or  $\lim_{N\to\infty} \frac{lgN}{log_3N} = lg3 \Rightarrow same\ growth$ 

Examples of functions that have N<sup>2</sup> growth:

Speed of Growth:

\_\_<\_\_<\_\_<\_\_<\_\_<\_\_<\_\_<\_\_

Fill in each column functions that have the same growth as the column label.



The functions listed in a column are \_\_\_\_\_ of the function that labels the column. E.g. \_\_\_\_ = \_\_\_( N^2)

From ordering by growth to asymptotic notation

Symbol:  $\Theta$  O o  $\Omega$   $\omega$ 

Meaning:

Examples:  $N^3/10 - 500N2 - 1000 = O(N^3)$  True/False Solution: find dominant term(s) and compare their growth

Notation abuse: = instead of ∈

		r an algorithm.		tion sort:	an insertion sort is	
orst case is $\Theta(\underline{\hspace{0.1cm}})$ then the algorithm is $\underline{\hspace{0.1cm}}$ (st case $\hspace{0.1cm}$ is $\Theta(\underline{\hspace{0.1cm}})$ then the algorithm is $\underline{\hspace{0.1cm}}$ (				() Worst case is Θ() then insertion sort is ( ( ) Best case is Θ( ) then insertion sort is (		
		,		(	(	
gorithr	n is Θ(	) iff best case and wo	orst case have	TC.		
.: <b>^</b> -						
cise: As		can he	,	cannot he		
				cannot be:cannot be:		
				cannot be:		
		e TC function in each ca	se?			
vhich o	ne is "bet	ter"? (define better)				
Symbol	Name	Meaning	Notation	Examples	Limit theorem	
ω						
Ω						
Θ						
0						
0						
U						
		ments are correct	TC = O/	- O/ \ TC - 4	· /	
IC =	= O(log;(lv	)) => TC = o()	, 10 = 0(), 10	= 12(), TC = (	ω()	

2

ω: Ω: Ω: Ω: ω:

 $\omega : \underline{\hspace{1cm}} O : \underline{\hspace{1cm}} o : \underline{\hspace{1cm}}$ 

 $TC = O(N^2)$ 

**Asymptotic Bounds as Limits:** 

$$f(n) = \omega(g(n)) \Leftrightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$$
 English Translation:

$$f(n) = \Omega(g(n)) \Leftrightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \ or \ c \neq 0$$
 English Translation:

$$f(n) = \Theta(g(n)) \Leftrightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = c \neq 0 \quad \text{(limit is a non-zero constant)}$$
 English Translation:

$$\mathsf{f}(\mathsf{n}) = \mathsf{O}(\mathsf{g}(\mathsf{n})) \Leftrightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \ or \ c$$

**English Translation:** 

$$\mathsf{f}(\mathsf{n}) = \mathsf{o}(\mathsf{g}(\mathsf{n})) \Leftrightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

**English Translation:** 

**Properties:** 

2.

3.

4.

5.

\*\*\* Transitivity (From Discrete Structures): If a > b and b > c, then a > c. This concept can be applied to TC bounds as well.

6.

7.