Asymptotic Bounds

Review:

Use ratio and limit to infinity to compare function growth. Examples:

a) $N^2 < N^3$ $\quad \frac{N^2}{N^3} = \frac{1}{N}$ $\quad$ *use* $\mathbf{1}$ ***grows slower than*** $\boldsymbol{N}$ $\quad$ *or* $\quad \lim_{N\to\infty} \frac{\mathbf{1}}{\boldsymbol{N}} = \mathbf{0}$ $\quad \Rightarrow$ *top grows slower*

b) $N\sqrt{N} > N$ $\quad \frac{N\sqrt{N}}{N} = \frac{\sqrt{N}}{1}$ $\quad$ *use* $\sqrt{\boldsymbol{N}}$ ***grows faster than*** $\mathbf{1}$ $\quad$ *or* $\quad \lim_{N\to\infty} \frac{\sqrt{N}}{\mathbf{1}} = \infty$ $\quad \Rightarrow$ *top grows faster*

c) $N\sqrt{N} < N^2$ $\quad \frac{N\sqrt{N}}{N^2} = \frac{1}{\sqrt{N}}$ $\quad$ *use* $\mathbf{1}$ ***grows slower than*** $\sqrt{\boldsymbol{N}}$ $\quad$ *or* $\quad \lim_{N\to\infty} \frac{\mathbf{1}}{\sqrt{N}} = \mathbf{0}$ $\quad \Rightarrow$ *top grows slower*

d) $lgN \approx log_3N$ $\quad \frac{lgN}{\log_3 N} = \frac{lgN}{\frac{lgN}{lg3}} = \frac{lgN}{1} * \frac{lg3}{lgN} = lg3$ $\quad$ *use* ***lg3 is a constant (no N)*** $\quad$ *or* $\quad \lim_{N\to\infty} \frac{lgN}{\log_3 N} = lg3$ $\quad \Rightarrow$ *same growth*

Examples of functions that have N^2 growth:

Speed of Growth:

____<____<____<____<____<____<____<____<____<____<____<____<____<____

Fill in each column functions that have the same growth as the column label.

1	lgN	N	NlgN	N^2	N^3	2^N

The functions listed in a column are ____ of the function that labels the column. E.g. _________ = ___(N^2)

From ordering by growth to asymptotic notation

……

……

Symbol: Θ O o Ω ω

Meaning:

Examples: $N^3/10 - 500N2 - 1000 = O(N^3)$ True/False Solution: find dominant term(s) and compare their growth

Notation abuse: = instead of ∈

Finding Θ , O , Ω for an algorithm.

Worst case is Θ(___) then the algorithm is ___ (______) Best case is Θ(___) then the algorithm is ___ (______)	Insertion sort: Worst case is Θ(___) then insertion sort is ___ (______) Best case is Θ(___) then insertion sort is ___ (______)

An algorithm is Θ(_____) iff best case and worst case have __________________ TC.

Exercise: Assume:

- Alg 1 is $O(N^2)$ ……can be: ……………………………………………………. cannot be: …………………………………………………….
- Alg 2 is Θ(NlgN) can be: ……………………………………………………. cannot be: …………………………………………………….
- Alg 3 is Ω(N) …… can be: ……………………………………………………. cannot be: …………………………………………………….
- what could be the TC function in each case?
- which one is "better"? (define better)

Symbol	Name	Meaning	Notation	Examples	Limit theorem
ω					
Ω					
Θ					
O					
o					

Ex: Fill in s.t the statements are correct

TC = $\Theta(\log_3(N))$ => TC = o(______) , TC = O(______), TC = Ω(______), TC = ω(______)

ω: ____________________ O: _____________________ Ω: ___________________ o: ______________________

TC = $\Theta(N^2)$ => TC = o(______) , TC = O(______), TC = Ω(______), TC = ω(______)

ω: ____________________ O: _____________________ Ω: ___________________ o: ______________________

TC = $O(N^2)$

ω: ____________________ O: _____________________ Ω: ___________________ o: ______________________

Asymptotic Bounds as Limits:

f(n) = ω(g(n)) ⇔ $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \infty$
English Translation:

f(n) = Ω(g(n)) ⇔ $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \infty\ or\ c \neq 0$
English Translation:

f(n) = Θ(g(n)) ⇔ $\lim_{n\to\infty}\frac{f(n)}{g(n)} = c \neq 0$ (limit is a non-zero constant)
English Translation:

f(n) = O(g(n)) ⇔ $\lim_{n\to\infty}\frac{f(n)}{g(n)} = 0\ or\ c$
English Translation:

f(n) = o(g(n)) ⇔ $\lim_{n\to\infty}\frac{f(n)}{g(n)} = 0$

English Translation:

Properties:
1.

2.

3.

4.

5.

*** Transitivity (From Discrete Structures): If a > b and b > c, then a > c. This concept can be applied to TC bounds as well.

6.

7.