

Tree review

complete tree

- if it has height h the number of nodes is _____
- if it has N nodes, the number of leaves (nodes on the last level) is _____
- if it has N nodes, if each leaf keeps swapping with its parent, until it gets to the root, then one leaf will do _____ swaps and all leaves together will do _____ swaps.

nearly complete tree

- if it has height h the number of nodes N is: _____ $\leq N \leq$ _____
- if it has N nodes, the height is _____
- if it has N nodes, the longest path has length _____
- if it has N nodes, the number of leaves is _____

Array traversal using

- $idx = idx/2$

`for(idx = N; idx >= 1; idx = idx/2) // repeats _____`

- $left = idx*2, right = idx*2+1$

`for(idx = 1; idx <= N; idx = idx*2) // repeats _____`

`for(idx = 1; idx <= N; idx = idx*2+1) // repeats _____`

Priority Queues/Heaps:

What is a priority queue or a max-heap?

Uses:

Necessary operations:

Visualizing Heaps in Tree Format:

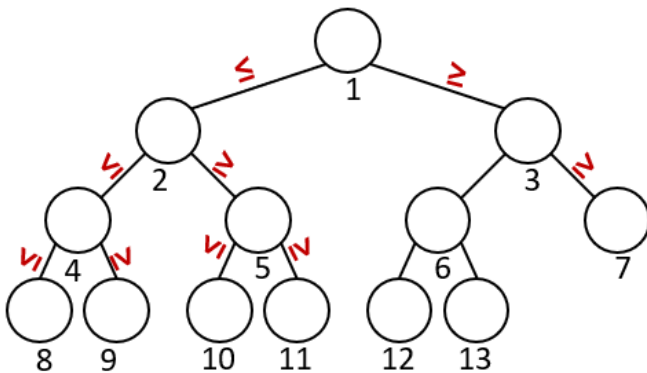
Drawing out heaps as a tree can make it much easier to understand the relationship certain elements have with each other based on the indexes they are stored at. However, note that a heap is NOT a _____. It is an _____.

Required Tree Properties for a Max-Heap:

- Order of the Elements
 - Values at children indexes/nodes are _____ than values at their parent indexes/nodes
 - Items must respect order only along _____. Nodes in 2 subtrees have _____
- Shape of the Tree
 - The tree is a _____
 - There are no _____
 - All levels are complete but possibly the _____
 - If not complete, all nodes are to the _____ on the last level

Ex: Heap array

value	-	9	7	5	3	5	4	3	2	1	1	3	4	1
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13



Index computation when 1st item is at index 1 (root is at index 1)

```
int left(int idx)    {return  idx*2    ;}
int right(int idx)   {return (idx*2)+1 ;}
int parent(int idx)  {return  idx/2    ;}
```

E.g.:

left(4) -> _____
 right(4) -> _____
 parent(4) -> _____
 left(5) -> _____
 right(5) -> _____
 parent(5) -> _____

Inserting New Item:

- 1.) Increase size of array and add a new element to end
- 2.) Continually compare and swap new element with elements at the parent indexes until heap order is correct again (Perform swimUp())

```
swimUp(int* A, int idx){
```

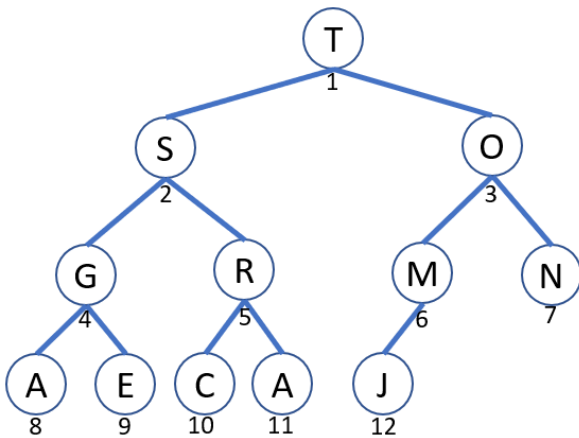
```
}
```

TC:

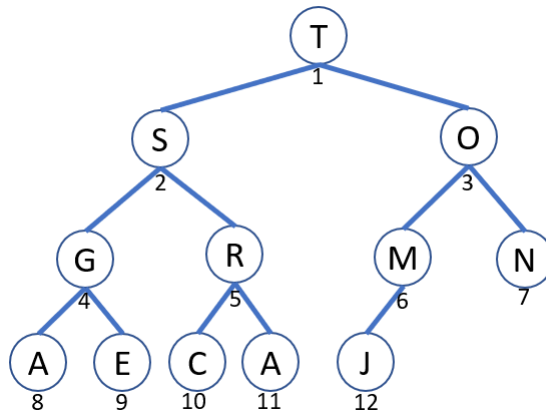
SC:

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z

Original Heap:



Add _____. Show the new heap below and affected/swapped elements on the original heap to the left.



Batch Initialization of Heap:

- Bottom Up Batch: Turns an array into a heap by reordering its elements to fit requirements for heap order

```
buildMaxHeap(int* A, int N) {
```

```
}
```

TC:

SC:

- Top Down Batch: Builds a heap by **repeated insertions** in an originally empty heap

TC:

SC:

Heapsort:

Implementation:

Code:

```
Heapsort(int* A,int N){
```

```
}
```

TC:

SC:

Stable? _____ Adaptive? _____

Index	0	1	2	3	4	5
Original Array						
Heap						
Removal 1						
Removal 2						
Removal 3						
Removal 4						
Removal 5						

Variations:

What are the required adjustments to create a min heap instead of a max heap?

When would it be better to use a min-heap?

Index calculation

Index computation when 1st item is at index 1 (root is at index 1)

```
int left(int idx) {return  idx*2 ;}
int right(int idx) {return ( idx*2)+1 ;}
int parent(int idx) {return  idx/2 ;}
```

E.g.:

left(4) -> ____

right(4) -> ____

parent(4)-> ____

left(5) -> ____

right(5) -> ____

parent(5)-> ____

index of 1st item: ____

index of last item (based on size N): ____

Index computation when 1st item is at index 0 (root is at index 0)

```
int left(int idx) {return ( idx*2)+1 ;}
int right(int idx) {return ( idx*2)+2 ;}
int parent(int idx) {return (idx -1)/2 ;}
```

E.g.:

left(4) -> ____

right(4) -> ____

parent(4)-> ____

left(5) -> ____

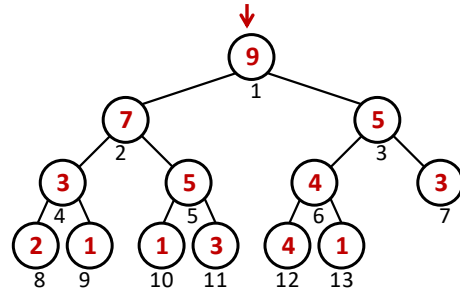
right(5) -> ____

parent(5)-> ____

index of 1st item: ____

index of last item (based on size N): ____

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