

Summary

- Simple runtime problems
- ‘Counting’ instructions
 - Detailed
- Terminology and notation:
 - $\log_2 N = \lg N$

Estimate runtime

- Problem:

The total number of instructions in a program (or a piece of code) is 10^{12} and it runs on a computer that executes 10^9 instructions per second. How long will it take to run this program? Give the answer in seconds. If it is very large, transform it in larger units (hours, days, years).

- Summary:

- Total instructions: 10^{12}
- Speed: 10^9 instructions/second

- Answer:

- Time = (total instructions)/speed =
 $(10^{12} \text{ instructions}) / (10^9 \text{ instr/sec}) = 10^3 \text{ seconds} \sim 15 \text{ minutes}$

- Note that this computation is similar to computing the time it takes to travel a certain distance (e.g. 120miles) given the speed (e.g. 60 miles/hour).

Estimate runtime

- A slightly different way to formulate the same problem:
 - total number of instructions in a program (or a piece of code) is 10^{12} and
 - it runs on a computer that executes one instruction in one nanosecond (10^{-9} seconds)
 - How long will it take to run this program? Give the answer in seconds. If it is very large, transform it in larger units (hours, days, years)
- Summary:
 - 10^{12} total instructions
 - **10^{-9} seconds per instruction**
- Answer:
 - Time = (total instructions) * (seconds per instruction) = $(10^{12} \text{ instructions}) * (\mathbf{10^{-9} \text{ sec/instr}}) = 10^3 \text{ seconds} \sim 15 \text{ minutes}$

C conventions

- Body of loops :
 - Several instructions with curly braces
 - One instruction indented (with or without curly braces)
 - No instruction
 - With semicolon, or without

Counting instructions: detailed

Worksheet

Count in detail the total number of instructions executed by each of the following pieces of code:

// Example A. Notice the ; at the end of the for loop.

```
temp = 5; x = temp * 2;  
for (i = 0; i < n; i++) ;
```

// Example B (source: Dr. Bob Weems)

```
for (i=0; i < n; i++)  
    for (t=0; t < p; t++)  
    {  
        c[i][t]=0;  
        for (k=0; k < r; k++)  
            c[i][t] += a[i][k] * b[k][t];  
    }
```

Counting instructions: detailed

Answers

$$1 + 1 + n * (1 + 1 + \text{body_instr_count})$$

Diagram showing arrows pointing from the terms in the formula to the corresponding parts of the for loop structure below.

```
for (init; cond; update) // assume the condition is TRUE n times
    body
```

// Example A. Notice the ; at the end of the for loop.

```
temp = 5; x = temp * 2;
for (i = 0; i < n; i++) ;
```

$$2 + 1 + 1 + n * (1 + 1 + 0) = 4 + 2n$$

// Example B (source: Dr. Bob Weems)

```
for (i=0; i<n; i++) → 1 + 1 + n * (1 + 1 + ___ ) = 2 + n * (2 + 2 + 5*p + 3*p*r) = 2 + 4*n + 5*n*p + 3*n*p*r
for (t=0; t<p; t++) → 1 + 1 + p * (1 + 1 + 1 + ___ ) = 2 + p * (3 + 2 + 3*r) = 2 + 5*p + 3*p*r
{
    c[i][t]=0;
    for (k=0; k<r; k++) → 1 + 1 + r * (1 + 1 + 1) = 2 + 3 * r
        c[i][t] += a[i][k] * b[k][t];
}
```

Counting instructions: sequential vs nested loops

Worksheet

```
// Example sequential vs nested
for (t=0; t<n; t++)
    printf("A");
for (i=0; i<p; i++) {
    for (k=0; k<r; k++)
        printf("B");
}
```

Counting instructions: sequential vs nested loops

Answers

```
// Example sequential vs nested
```

```
for (t=0; t<n; t++)
```

```
    printf("A");
```

```
for (i=0; i<p; i++){
```

```
    for (k=0; k<r; k++)
```

```
        printf("B");
```

```
}
```

$$\underline{\quad} + \underline{\quad} = (2 + 3*n) + (2 + 4*p + 3*p*r)$$

$$1 + 1 + n * (1 + 1 + 1) = 2 + 3*n$$

$$1 + 1 + p * (1 + 1 + \underline{\quad}) = 2 + p * (2 + 2 + 3*r) = 2 + 4*p + 3*p*r$$

$$1 + 1 + r * (1 + 1 + 1) = 2 + 3*r$$

CLRS

- The textbook, CLRS, provides a more detailed analysis that uses different costs (time) for instructions (based on their type).
 - Easy to adapt the above method to do that: just reapply the method, but count only specific instructions (assignments, additions, comparisons,...)
 - More details that we will want to skip over in the end (=> use Big-Oh).

Detailed instruction count

Worksheet

- What does this code do?
- Give a detailed count of instructions for the case when the while loop stops because the condition (**left <= right**) is false.

```
/* code adapted from Sedgewick
   Assume A is sorted in increasing order and that left and
   right are in range indexes for A.*/

1. int mystery(int A[], int v, int left, int
right){
3.     while (left <= right) {
4.         int m = (left+right)/2;
5.         if (v == A[m]) return m;
6.         if (v < A[m])
7.             right = m-1;
8.         else
9.             left = m+1;
10.    }
11.    return -1;
12. }
```