Extra topic , NOT required

Calculate exact number of iterations of a loop as a function of the variable(s) that control the loop

Formula for values of i and exact calculation of number of loop iterations – Example 1

```
for (i=0; i<=N; i=i+3)
    printf("A");
```

i takes values: **0,3,6,9,12,….** $\leq$ **N**

We notice that these are consecutive multiples of 3 so we will explicitly show that by writing i as a function of another variable:

i = 3*e

Where **e takes values: 0,1,2,3,….,p**

Here we use p to refer to that last multiple of 3 that is $\leq$N.

The loop executes (the condition is true) for all i = 3e where e takes values: 0,1,2,3,…,p => 1+p total values (because of the 0) => the loop iterates 1+p times. (A)

Next we will compute the exact formula for p:

Because of how we chose p we have: **3p $\leq$ N** but p is an integer and largest with this property => $\mathbf{p = \left\lfloor \dfrac{N}{3} \right\rfloor}$ (B)

From (A) and (B) it follows that the loop executes $1 + p = 1 + \left\lfloor \dfrac{N}{3} \right\rfloor$ times =>

The loop executes exactly: $1 + \left\lfloor \dfrac{N}{3} \right\rfloor$ times.

As a verification step you should check that the formula does give the exact number of loop iterations for a few values of N: 0,1,2,3,4,15,17

Practice: how would you solve: `for (i=3; i<=N; i=i+3) printf("A");`

e	i=3e
0	0
1	3
2	6
3	9
...	...
e	3e
...	...
p	3p (i_{last} = **3p**, i_{last} $\leq$N 3p$\leq$N => $p = \lfloor N/3 \rfloor$

Formula for values of i and exact calculation of number of loop iterations – Example 1

```
for (i=2; i<=N; i=i+3)
    printf("A");
```

i takes values: **2,5,8,11,14**,…. $\leq$ N

We notice that these are consecutive multiples of 3 with an offset of 2. We will explicitly show that by writing i as a function of another variable:

i = 2+(3*e)

Where e takes values: 0,1,2,3,….,p

Here we use p to refer to that last value **2+3p that is $\leq$ N.**

The loop executes (the condition is true) for all i = 2+3e where e takes values: 0,1,2,3,…,p => 1+p total values (because of the 0) => the loop iterates 1+p times. (A)

Next we will compute the exact value of p: 2+3p $\leq$ N => 3p $\leq$ (N-2) => p $\leq$ (N-2)/3, but p is an integer and largest with this property => $\mathbf{p} = \left\lfloor \dfrac{N-2}{3} \right\rfloor$ (B)

FINAL ANSWER form (A) and (B): The loop executes exactly: $\mathbf{1} + \left\lfloor \dfrac{N-2}{3} \right\rfloor$ times.

As a verification step you should check that the formula does give the exact number of loop iterations for a few values of N: **2,3,4,5,6** (Note that you start with the smallest value of N for which the loop iterates at least one time: 2. You do not use 0 or 1 for N in this verification.)

e	i=2+3e
0	2
1	5
2	8
3	11
…	…
e	i = 2+3e
…	…
p	i_{last} <=N (i_{last} =2+3p)

Formula for values of i and exact calculation of number of loop iterations – Example 3

```
for (i=1; i<=N; i=i*5)
    printf("A");
```

i takes values: **1,5,25,125,,….<=N**
We notice that these are consecutive multiples powers of 5 so we will explicitly show that by writing i as a function of another variable:
i = 5^e
Where e takes values: 0,1,2,3,….,p
Here we use p to refer to that largest value **5^p that is ≤N.**

The loop executes (the condition is true) for all i = **5^e** where e takes values: 0,1,2,3,…,p => 1+p total values (because of the 0) => **the loop iterates 1+p times**. (A)

Next we will compute the exact formula for p.
Because of how we picked p we have: **5$^p \leq N$**, where p is an integer and largest with this property.
Take log$_5$ on both sides $\Rightarrow p \leq log_5 N \Rightarrow p = \lfloor \log_5 N \rfloor$ (B)

From (A) and (B) it follows that the loop executes **$1 + p$ = $1 + \lfloor \log_5 N \rfloor$** times =>

ANSWER: The loop executes exactly: **$1 + \lfloor \log_5 N \rfloor$** times.

As a verification step you should check that the formula does give the exact number of loop iterations for a few values of N: **1,5,25,26,29,30**

e	i=5^e
0	1
1	5
2	25
3	125
…	…
e	i=5^e
…	…
p	i$_{last}$ <=N (i$_{last}$ =5^p) => $p = \lfloor \log_5 N \rfloor$