Stacks – Calculator Application

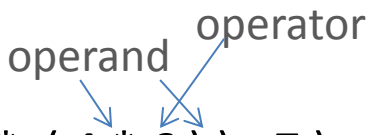
Alexandra Stefan

Infix and Postfix Notation

- The standard notation we use for writing mathematical expressions is called **infix notation**.
  - The operators are between the operands.

- There are two alternative notations:
  - **prefix notation**: the operator comes before the operands.
  - **postfix notation**: the operator comes after the operands.

- Example:



  - **infix:** 5 * (((9 + 8) * (4 * 6)) - 7)
  - **prefix:** (* 5 (- (* (+ 9 8) (* 4 6)) 7))
  - **postfix:** 5 9 8 + 4 6 * * 7 - * (use , if needed: 5, 9, 8, +, 4, 6, *, *, 7, -, *
    - No parentheses needed.
    - Can be easily evaluated using a stack.

Processing a Symbolic Expression

- How do we process an expression such as:
  - 5 * (((9 + 8) * (4 * 6)) - 7)
  - postfix: 5, 9, 8, +, 4, 6, *, *, 7, -, *

- Think of the input as a **list of tokens**.
  - Assume it is already tokenized

- A **token** is a **logical unit** of input, such as:
  - A number
  - An operator
  - A parenthesis.

Tokens

- A **token** is a **logical unit** of input, such as:
  - A number
  - An operator
  - A parenthesis.

- What are the tokens in:
  - 51 * (((195 + 8) * (4 - 6)) + 7)

- Answer: 51, *, (, (, (, 195, +, 8,), *, (, 4, -, 6,),), +, 7,)
  - 19 tokens.
  - Note that a token is NOT a character. For example 195 is one token, but it contains 3 charatcters.
  - We will not discuss how to build tokens from characters.
    - The numbers are the difficult part.

Converting Infix to Postfix

Input: a list/stream of tokens in infix order.

Output: a list of tokens in postfix order.

Assumptions:

1. Each operator has two operands.
2. The input is fully parenthesized.

   Every operation (that contains an operator and its two operands) is enclosed in parentheses.

Fully parenthesized	**Not fully parenthesized** (not allowed as input)
(3+5)	3+5
(2+(5-4))	(2+5-4) 2+(5-4) 2+((5-4))
((2 + 9) - (4 + 5))	(2 + 9) - (4 + 5)

T	op_stack	result list
5		5
*	*	
2		5 2
+	* +	
8		5 2 8
)	*	5 2 8 +
/	* /	
6		5 2 8 + 6
-	* / -	
4		5 2 8 + 6 4
)	* /	5 2 8 + 6 4 -
)	*	5 2 8 + 6 4 - /
-	* -	
7		5 2 8 + 6 4 - / 7
)	*	5 2 8 + 6 4 - / 7 -
)		5 2 8 + 6 4 - / 7 - *

input: a stream of tokens in infix order.

output: a list, `result`, of tokens in postfix order.

(Uses a stack: `op_stack`)

```
result = empty list
op_stack = empty stack
while(the input stream is not empty)
    T = next token
    If T is left parenthesis, ignore.
    If T is a number, insertAtEnd(result, T)
    If T is an operator, push(op_stack, T).
    If T is right parenthesis:
        op = pop(op_stack)
        insertAtEnd(result, op)
```

Infix → Postfix

(5 * (((2 + 8) / (6 - 4)) - 7))

- Numbers go in the `result` list
- Operators go on the `op_stack`

(the stack shown grows to the right)

- Left parenthesis, (, are ignored.
- At right parenthesis,), pop operator from `op_stack` and add it to the `result` list.

Evaluating Expressions in Postfix Notation

Input: a list tokens in infix order.

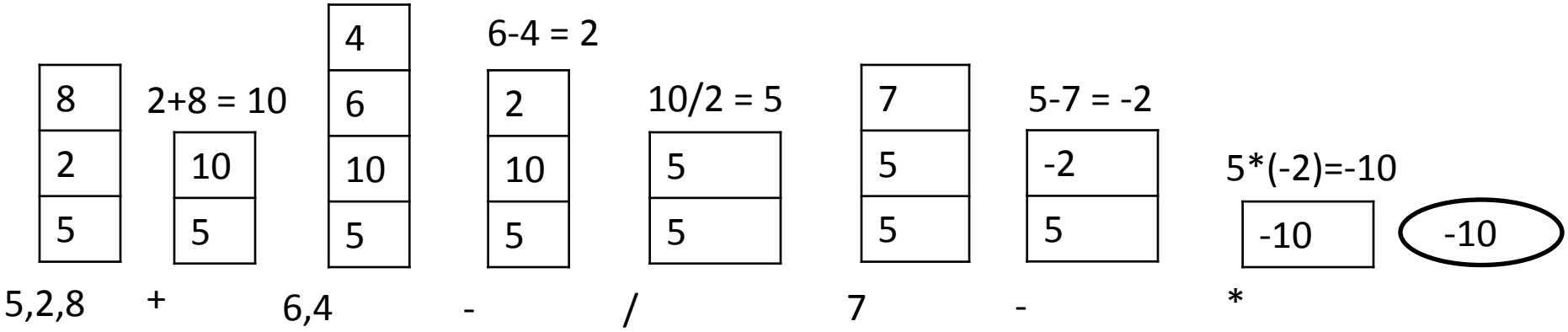
Output: the result of the calculation (a number).

Assumption: the list of tokens is be provided as input.

```
while(token list is not empty)
    T = remove next token (number or operator) from list.
    If T is a number, push(stack, T).
    If T is an operator:
        A = pop(stack)
        B = pop(stack)
        C = apply operator T on A and B
                (order: B T A, e.g.: B-A)
        push(stack, C)
final_result = pop(stack)
```

Postfix: 5 2 8 + 6 4 - / 7 - *

Token list: 5, 2, 8, +, 6, 4, -, /, 7, -, *



Here the * indicates the multiplication operator, not a pop() operation on the stack.

We do not explicitly show the pop operations. Instead, for each operator we pop, pop, calculate, push.

Another example

T	op_stack	result list
5		5
*	*	
9		5, 9
+	* +	
8		5, 9, 8
)	*	5 , 9, 8, +
/	* /	
4		5, 9, 8, +, 4
*	* / *	
6		5, 9, 8, +, 4, 6
)	* /	5, 9, 8, +, 4, 6, *
)	*	5, 9, 8, +, 4, 6, *, /
-	* -	
7		5, 9, 8, +, 4, 6, *, /, 7
)	*	5, 9, 8, +, 4, 6, *, /, 7, -
)		5, 9, 8, +, 4, 6, *, /, 7, -, *

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Postfix: 5 9 8 + 4 6 * / 7 - *

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