# Hashing

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# Hash tables

- Tables
  - **Direct access** table (or key-index table): key => index
  - **Hash** table: key => hash value => index
- Main components
  - Hash function
  - Collision resolution
    - Different keys mapped to the same index
- Dynamic hashing/rehashing – reallocate the table as needed
  - If an Insert operation brings the load factor past a threshold, e.g. 0.75, double the table capacity.
    - Other load factors may be used
  - If a Delete operation brings the load factor to 1/8 , half the table capacity.
- Properties:
  - Good time-space trade-off
  - Good for:
    - Search, insert, delete – O(1) – average time
  - Not good for:
    - Select, sort – not supported, must use a different method
- Reading: chapter 11, CLRS (chapter 14, Sedgewick – has more complexity analysis)
- [Hash Tables and Hash Functions](Hash Tables and Hash Functions) - youtube video

# Example

- M – table capacity.
- h – hash function that maps a key to an index

- Let M = 10, h(k) = k%10
  - Note: 10 is a bad table size, but is used here for ease of calculation

Insert keys:
46 -> 6
15 -> 5
20 -> 0
37 -> 7
23 -> 3
25 -> 5 collision
35 ->
9 ->

Collision resolution:
- Separate chaining
- Open addressing
  - Linear probing
  - Quadratic probing
  - Double hashing

| index | k |
|---|---|
| 0 | 20 |
| 1 | |
| 2 | |
| 3 | 23 |
| 4 | |
| **5** | **15** |
| 6 | 46 |
| 7 | 37 |
| 8 | |
| 9 | |

# Hash functions

- M – table size.
- h – hash function that maps a key to an index
  - We want random-like behavior:
    - any key can be mapped to any index with equal probability.

  - Typical functions for numeric keys:
    - h(k,M) = k % M
      - Best M is a prime number. (Avoid M that has a power of 2 factor, will generate more collisions).
      - Choose M a prime number that is closest to the desired table capacity.
      - If $M = 2^p$, it uses only the lower order p bits => bad, ideally use all bits.
    - h(k,M) = floor( ((k-A)/(B-A))* M )
      - Here $A \le k < B$.
      - Simple, good if keys are random, not so good otherwise.
    - h(k,M) = floor(M*(k*A mod 1)) ,   0<A<1  (in CLRS)
      - Good A = 0.6180339887 (the golden ratio)
      - Useful when M is not prime (can pick M to be a power of 2)
      - Alternative: h(k,M) = (16161 * (unsigned)k ) % M     (from Sedgewick)

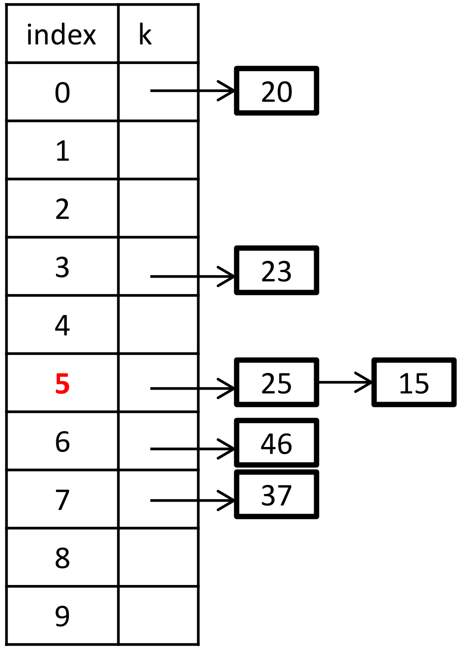
# Collision Resolution: Separate Chaining

- $\alpha = N/M$ (N – items in the table, M – table size)
  - load factor
- Separate chaining
  - Each table entry points to a list of all items whose keys were mapped to that index.
  - Requires extra space for links in lists
  - Lists will be short. On average, size $\alpha$.
  - Preferred when the table must support deletions.
- Operations:
  - Chain_Insert(T, x) - O(1)
    - insert x in list T[h(x.key)] at beginning. No search for duplicates
  - Chain_Delete(T, x) – O(1)
    - delete x from list T[h(x.key)] (Here x is the record so we do not need to search for it)
  - Chain_Search(T, k) – $\Theta(1+\alpha)$ (both successful and unsuccessful)
    - search in list T[h(k)] for an item x with x.key == k.

# Separate Chaining
## Example: insert 25

- Let M = 10, h(k) = k%10

Insert keys:
46 -> 6
15 -> 5
20 -> 0
37 -> 7
23 -> 3
25 -> 5 collision
35
9 ->

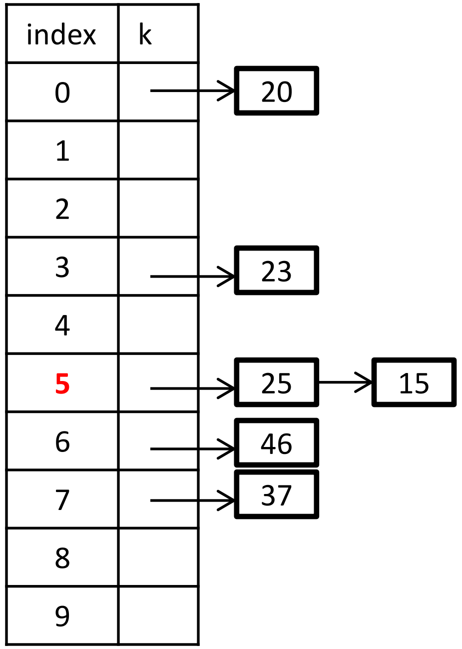



Inserting at the beginning of the list is advantageous in cases where the more recent data is more likely to be accessed again (e.g. new students will have to visit several offices at UTA and so they will be looked-up more frequently than continuing students.

α =

# resize table

- double space



# hashing strings

- review base conversion 4 to 10:
    - 312 (base 4) = > to base 10: 2*1 + 1*4 + 3*16

- sum of ascii codes
    - dog => (100+111+103)%10 = 314%10 = 4
    - problem: does not consider location of letters:
        - cat
        - act
        - tac

# Hashing Strings – Polynomial hash code

## (Implementation - avoid number overflow)

$Let\ s = s_0 s_1 s_2 \dots s_t,$ $be\ a\ string\ (here\ s_0\ is\ the\ first\ character\ and\ s_t\ is\ the\ last\ character\ in\ the\ string).$
$Polynomial\ hash\ code\ for\ s\ (with\ result\ in\ range\ [0, M-1]):$

$$h_a(s, M) = (s_0 * a^t + s_1 * a^{t-1} + \dots + s_{t-1} * a + s_t * 1)\%M \quad (1)$$

$in\ order\ to\ avoid\ overflow\ compute\ it\ iteratively\ and\ apply\ mod\ M\ at\ each\ step:$

$$h_a(s, M) = (((\dots((s_0\%M) * a + s_1)\%M) * a + s_2)\%M) * a + \dots + s_{t-1})\%M) * a + s_t)\%M \quad (2)$$

*(Both expressions give the same value for* $h_a(s,M)$ *)*

Recommendations:

- Choose *a* and *M* to be prime numbers.
- Recommended values for *a*: 33, 37,39,41 (citation: "Data Structures and Algorithms in Java" by Goodrich and Tamassia)

E.g. $h_{33}$("dog",101) =( ASCII(d)*$33^2$ + ASCII(o)*33 + ASCII(g) ) %101 = (100*1089 + 111*33+ 103)%101 = 112666 % 101 = 51

The other method:

Discussion:

1) *a* should have some non-zero lower order bits. (Reason: If all the lower order bits of *a* are 0, in expression (1), when multiplying by powers of a and then truncating due to overflow, the influence of first letters of s is removed. Since expressions (1) and (2) give the same result, it means that this happens even when explicitly avoiding the overflow.)

2) If a and M have a common divisor, other issues may appear and thus a better choice is to pick *a* and *M* to be relatively prime with each other. (See a discussion here: [Magic numbers in polynomial hash functions](#).)

=> Note that if both *a* and *M* are prime, then they are also prime with each other and because a is not a multiple of any power of 2 it will also have non-zero lower order bits.

To understand overflow, pick a = 33 > 32 = $2^5$ the term $33^t > 32^t = (2^5)^t = 2^{5t} > 2^{32}$ for t≥7 => for strings of length 8 or more, the $a^t$ term alone overflows a 4B integer (even more so after multiplying with $s_0$ and adding the other terms).

```
int hash(char *s, int M)  {
  int i, h = 0, a = 33;
  for (i = 0; i<strlen(s); i++) {
    h = (a*h + s[i]) % M;
    // here s[i] will use the ASCII code
  }
  return h;
}
```

Worked-out code for s = "dog"
Function call: `hash("dog",101)`
M=101, s = "dog" (ASCII(d)=100, ASCII(o)=111, ASCII(g)=103 )
h=0, a=33
i = 0, s[0]->d : h = (33*0+100)%101 = 100
i = 1, s[1]->o : h = (33*100+111)%101=78
i = 2, s[2]->g : h = (33*78+103)%101 = 51
return 51

# Hashing strings

- Note that the hash function for strings given in the previous slide can be used as the initial hash function. Based on what type of hash table you have, you will need to do additional work
  - If you are using separate chaining, you will create a node with this word and insert it in the linked list (or if you were doing a search, you would search in the linked list)
  - If you are using open addressing:
    - For linear, you can use the next cells as needed
    - For quadratic, you would use this function in the expression with the quadratic terms

      E.g. index = $[h_{33}(\text{"dog"},101) + 2i+i^2]\%101$
    - For double hashing you will need to choose another hash function (e.g. `1+h`$_{41}$`("dog",100)` ) to get the jump size

      E.g. index = $\{h_{33}(dog, 101) + i*[1+h_{41}(dog,100)]\}\%101$

# Collision Resolution: Open Addressing

- $\alpha = N/M$ (N – items in the table, M – table size)
  - load factor
- Open addressing:
  - Use empty cells in the table to store colliding elements.
  - M > N
  - α – ratio of used cells from the table (<1).
  - Probing – examining slots in the table. Number of probes = number of slots examined.
  - $h(k,f,M)$ where f gives the number of failed probes/attempts: f=0,1,2,… until successful hash.

  - Linear probing: $h(k,f,M) = (h_1(k) + f) \% M$,
    - If the slot where the key is hashed is taken, use the next available slot and wrap around the table.
    - Very bad: primary clustering - long chains; snowball effect: the longer the chain, the higher the chance to grow
  - 3 reshash (linear): $h(k,f,M) = (h_1(k,M) + 3f) \%M$
    - Bad: secondary clustering - If two keys hash to the same value, they follow the same set of probes. But better than linear.
  - Quadratic probing: $h(k,f,M) = (h_1(k) + c_1 f + c_2 f^2) \% M$,
    - Bad: secondary clustering - If two keys hash to the same value, they follow the same set of probes. But better than linear.
  - Double hashing: $h(k,f,M) = (h_1(k,M) + f * h_2(k,M)) \% M$,
    - $h_2(k,M)$ <u>should NEVER be 0</u>. (E.g. use: $h_2(k,M) = 1 + k\%(M-1)$ )
    - Use a second hash value as the jump size (as opposed to size 1 in linear probing).
    - Want: $h_2(k)$ relatively prime with $M$. (relatively prime: they have no common divisor)
      - M prime and $h_2(k,M) = 1 + k\%(M-1)$
      - $M = 2^p$ and $h_2(k,M) = odd$ (M and $h_2(k)$ will be relatively prime since all the divisors of M are powers of 2, thus even).
  - See figure 14.10, page 596 (Sedgewick) for clustering produced by linear probing and double hashing.

# Open Addressing: quadratic Worksheet

M = 10, $h_1(k)$ = k%10.

Table already contains keys: 46, 15, 20, 37, 23

Next want to insert 25:

$h_1(25)$ = 5 (collision: 25 with 15)

## Linear probing

- $h(k,f,M) = (h_1(k) + f)\% M$
  (try slots: 5,6,7,8)

## Quadratic probing example:

- $h(k,f,M) = (h_1(k) + 2f+f^2)\% M$
  (try slots: 5, 8)
- Inserting 35(not shown in table):
  (try slots: 5, 8, 3,0)

| f (probe) | $h_1(k) + 2f+f^2$ | %10 |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

| Index | Linear | Quadratic | Double hashing $h_2(k)$ = 1+(k%7) | Double hashing $h_2(k)$ = 1+(k%9) |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 | 20 |
| 1 | | | | |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 |
| 4 | | | | |
| 5 | 15 | 15 | 15 | 15 |
| 6 | 46 | 46 | 46 | 46 |
| 7 | 37 | 37 | 37 | 37 |
| 8 | | | | |
| 9 | | | | |

Where will 9 be inserted now (after 35)?

# Open Addressing: quadratic
# Answers

$M = 10$, $h_1(k) = k\%10$.

Table already contains keys: 46, 15, 20, 37, 23

Next want to insert 25:

$h_1(25) = 5$ (collision: 25 with 15)

## Linear probing

- $h(k,f,M) = (h_1(k) + f)\% M$
  (try slots: 5,6,7,8)

## Quadratic probing example:

- $h(k,f,M) = (h_1(k) + 2f+f^2)\% M$
  (try slots: 5, 8)
- Inserting 35(not shown in table):
  (try slots: 5, 8, 3,0)

| f (probe) | $h_1(k) + 2f+f^2$ | %10 |
|---|---|---|
| 0 | 5+0=5 | 5 |
| 1 | 5+3=8 | 8 |
| 2 | 5+8=13 | 3 |
| 3 | $5+2*3+3^2 = 5+15=20$ | 0 |
| 4 | | |

| Index | Linear | Quadratic | Double hashing $h_2(k) = 1+(k\%7)$ | Double hashing $h_2(k) = 1+(k\%9)$ |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 ● | 20 |
| 1 | | | | **25** ● |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 ● |
| 4 | | | | |
| 5 | 15 ● | 15 ● | 15 ● | 15 ● |
| 6 | 46 ● | 46 | 46 | 46 |
| 7 | 37 ● | 37 | 37 | 37 |
| 8 | **25** ● | **25** ● | | |
| 9 | | | | |

Where will 9 be inserted now (after 35)?

# Open Addressing : double hashing - Worksheet

M = 10, $h_1(k)$ = k%10.

Table already contains keys: 46, 15, 20, 37, 23

Try to insert 25:

$h_1(25)$ = 5 (collision: 25 with 15)

## Double hashing example

- $h(k,f,M) = (h_1(k) + f^* h_2(k)) \% M$

Choice of $h_2$ matters:

- $h_{2a}(k) = 1+(k\%7)$: try slots: 5, 9,
  - $h_2(25) = 1+ (25\%7) = 1+ 4 = 5$ =>
    h(k,f,M) = (5 + f*5)%M => slots: 5,0,5,0,...
    Cannot insert 25.
- $h_{2b}(k) = 1+(k\%9)$:
  - $h_2(25) = 1 + (25\%9) = 1 + 7 = 8$ =>
    h(k,f,M) = (5 + f*8)%M => slots: 5,3,1,9,7,5,...

### Quadratic probing

| f (probe) | $h_1(k) + 2f+f^2$ | %10 |
|---|---|---|
| 0 | 5+0=5 | 5 |
| 1 | 5+3=8 | 8 |
| 2 | 5+8=13 | 3 |
| 3 | $5+2*3+3^2$ = 5+15=20 | 0 |
| 4 | | |

| Index | Linear | Quadratic | Double hashing $h_2(k) = 1+(k\%7)$ | Double hashing $h_2(k) = 1+(k\%9)$ |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 ● | 20 |
| 1 | | | | 25 ● |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 ● |
| 4 | | | | |
| 5 | 15 ● | 15 ● | 15 ● | 15 ● |
| 6 | 46 ● | 46 | 46 | 46 |
| 7 | 37 ● | 37 | 37 | 37 |
| 8 | 25 ● | 25 ● | | |
| 9 | | | | |

Where will 9 be inserted now?

### Double hashing

| f (probe) | Index | $(h_1(k) + f^*h_{2a}(k))\%M$ (5+f*5)%10 |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |

| f (probe) | Index | $(h_1(k) + f^*h_{2b}(k))\%M$ (5+f*8)%10 |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

# Open Addressing : double hashing - Answers

M = 10,   $h_1(k)$ = k%10.

Table already contains keys:  46, 15, 20, 37, 23

Try to insert 25:

$h_1(25)$  = 5  (collision: 25 with 15)

## Double hashing example

- $h(k,f,M) = (h_1(k) + f{*}\ h_2(k))\ \%\ M$

Choice of $h_2$ matters:

- $h_{2a}(k) = 1+(k\%7)$: try slots: 5, 9,
  - $h_2(25) = 1+ (25\%7) = 1+ 4 = 5$ =>
    $h(k,f,M) = (5 + f{*}5)\%M$ => slots: 5,0,5,0,...
    Cannot insert 25.
- $h_{2b}(k) = 1+(k\%9)$:
  - $h_2(25) = 1 + (25\%9) = 1 + 7 = 8$  =>
    $h(k,f,M) = (5 + f{*}8)\%M$ => slots: 5,3,1,9,7,5,...

## Quadratic probing

| f (probe) | $h_1(k) + 2f+f^2$ | %10 |
|---|---|---|
| 0 | 5+0=5 | 5 |
| 1 | 5+3=8 | 8 |
| 2 | 5+8=13 | 3 |
| 3 | $5+2{*}3+3^2$ = 5+15=20 | 0 |
| 4 | | |

## Double hashing

| f (probe) | Index | $(h_1(k) + f{*}h_{2a}(k))\%M$ (5+f*5)%10 |
|---|---|---|
| 0 | **5** | (5+0)%10=5 |
| 1 | 0 | (5+5)%10=0 |
| 2 | **5** | (5+2*5)%10 =15%10= 5<br>Cycles back to 5 =><br>Cannot insert 25 |
| 3 | 0 | (5+3*5)%10=20%10= 0 |

| f (probe) | Index | $(h_1(k) + f{*}h_{2b}(k))\%M$ (5+f*8)%10 |
|---|---|---|
| 0 | **5** | (5+0)%10=5 |
| 1 | 3 | (5+8)%10=3 |
| 2 | 1 | (5+2*8)%10 =21%10= 1 |
| 3 | 9 | (5+3*8)%10=29%10= 9 |
| 4 | 7 | (5+4*8)%10=37%10= 7 |
| 5 | **5** | (5+5*8)%10=45%10= 5<br>Cycles back to 5 |

| Index | Linear | Quadratic | Double hashing $h_2(k) = 1+(k\%7)$ | Double hashing $h_2(k) = 1+(k\%9)$ |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 | 20 |
| 1 | | | | **25** |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 |
| 4 | | | | |
| **5** | 15 | 15 | 15 | 15 |
| 6 | 46 | 46 | 46 | 46 |
| 7 | 37 | 37 | 37 | 37 |
| 8 | **25** | **25** | | |
| 9 | | | | |

Where will  9 be inserted now?

# Open Addressing:
# Quadratic vs double hashing

$M = 10$, $h_1(k) = k\%10$.

Table already contains keys: 46, 15, 20, 37, 23

Try to insert 25:

$h_1(25) = 5$ (collision: 25 with 15)

Quadratic hashing with: $2i+i^2$
$h(k)= (h_1(k) + 2f+f^2)\%M$ where
$h_1(k) = k\%M$

| f (probe) | Index | $h(k)=(h_1(k) + 2f+f^2)\%M$ $=(5 + 2f+f^2)\%10$ (k=25) |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

Double hashing:
$h(k)=(h_1(k)+f*h_2(k))\%M$ where:
$h_1(k) = k\%M$
$h_2(k) = 1+(k\%(M-1)) = 1+(k\%9)$
$h_2(25) = 1+(25\%9) = 1+7 = 8$

| f (probe) | Index | $h(k)=(h_1(k)+f*h_2(k))\%M$ $= (5+f*8)\%10$ (for k=25) |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

| Index | Linear | Quadratic | Double hashing $h_2(k) = 1+(k\%7)$ | Double hashing $h_2(k) = 1+(k\%9)$ |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 ● | 20 |
| 1 | | | | **25** ● |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 ● |
| 4 | | | | |
| 5 | 15 ● | 15 ● | 15 ● | 15 ● |
| 6 | 46 ● | 46 | 46 | 46 |
| 7 | 37 ● | 37 | 37 | 37 |
| 8 | **25** ● | **25** ● | | |
| 9 | | | | |

Choice of $h_2$ matters. See $h_2(k) = 1+(k\%7)$
$h_2(25) = 1+4 = 5$ => h(25) cycles: 5,0,5,0
=> Could not insert 25.

Where will 9 be inserted now?

# Open Addressing:
# Quadratic vs double hashing

M = 10, $h_1(k)$ = k%10.

Table already contains keys: 46, 15, 20, 37, 23

Try to insert 25:

$h_1(25)$ = 5 (collision: 25 with 15)

Quadratic hashing with: $2f+f^2$
$h(k) = (h_1(k) + 2f+f^2)\%M$ where
$h_1(k) = k\%M$

| f (probe) | Index | $h(k)=(h_1(k) + 2f+f^2)\%M$ $=(5 + 2f+f^2)\%10$ (k=25) |
|---|---|---|
| 0 | 5 | 5+0=5 |
| 1 | 8 | 5+3=8 |
| 2 | 3 | 5+8=13 |
| 3 | 0 | $5+2*3+3^2 = 5+15=20$ |
| 4 | | |

Double hashing:
$h(k)=(h_1(k)+f*h_2(k))\%M$ where:
$h_1(k) = k\%M$
$h_2(k) = 1+(k\%(M-1)) = 1+(k\%9)$
$h_2(25) = 1+(25\%9) = 1+7 = 8$

| f (probe) | Index | $h(k)=(h_1(k)+f*h_2(k))\%M$ = (5+f*8)%10 (for k=25) |
|---|---|---|
| 0 | 5 | (5+0)%10=5 |
| 1 | 3 | (5+8)%10=3 |
| 2 | 1 | (5+2*8)%10 =31%10= 1 |
| 3 | 9 | (5+3*8)%10=29%10= 9 |
| 4 | 7 | (5+4*8)%10=37%10= 7 |
| 5 | 5 | (5+5*8)%10=45%10= 5<br>Cycles back to 5 |

Choice of $h_2$ matters. See $h_2(k) = 1+(k\%7)$
$h_2(25) = 1+4 = 5$ => h(25) cycles: 5,0,5,0
=> Could not insert 25.

| Index | Linear | Quadratic | Double hashing $h_2(k) = 1+(k\%7)$ | Double hashing $h_2(k) = 1+(k\%9)$ |
|---|---|---|---|---|
| 0 | 20 | 20 | 20 ● | 20 |
| 1 | | | | 25 ● |
| 2 | | | | |
| 3 | 23 | 23 | 23 | 23 ● |
| 4 | | | | |
| 5 | 15 ● | 15 ● | 15 ● | 15 ● |
| 6 | 46 ● | 46 | 46 | 46 |
| 7 | 37 ● | 37 | 37 | 37 |
| 8 | 25 ● | 25 ● | | |
| 9 | | | | |

Where will 9 be inserted now?

# Search and Deletion in Open Addressing

- Searching:
  - Report as not found when land on an EMPTY cell
- Deletion:
  - Mark the cell as ‘DELETED’, not as an EMPTY cell
    - Otherwise you will break the chain and not be able to find elements following in that chain.
    - E.g., with linear probing, and hash function h(k,i,10) = (k + i) %10, insert 15,25,35,5, search for 5, then delete 25 and search for 5 or 35.

# Open Addressing: clustering

- Linear probing
  - primary clustering: the longer the chain, the higher the probability that it will increase.
  - Given a chain of size T in a table of size M, what is the probability that this chain will increase after a new insertion?

- Quadratic probing
  - Secondary clustering

# Expected Time Complexity for Hash Operations
## (under ‘perfect’ conditions)

| Operation \Methods | Separate chaining | Open Addressing |
|---|---|---|
| Successful search | $\Theta(1+\alpha)$ | $(1/\alpha)\ln(1/(1-\alpha))$ |
| Unsuccessful search | $\Theta(1+\alpha)$ | $1/(1-\alpha)$ |
| Insert | $\Theta(1)$<br>When: insert at beginning and no search for duplicates | $1/(1-\alpha)$ |
| Delete | $\Theta(1)$<br>Assumes: doubly-linked list and node with item to be deleted is given. | The time complexity does not depend only on α, (but also on the deleted cells). In such cases separate chaining may be preferred to open addressing as its behavior has better guarantees. |
| Perfect conditions: | simple uniform hashing | uniform hashing |
| Reference | Theorem 11.1 and 11.2 | Theorem 11.6 and 11.8 and corollary 11.7  in CLRS |

$\alpha = N/M$ is the load factor

# Perfect Hashing

Like separate chaining, but **use another hash table instead of a linked list**.

– Can be done for ***static*** keys (once the keys are stored in the table, the set of keys never changes).
– Corollary 11.11:Suppose that we store N keys in a hash table using perfect hashing. Then the **expected storage used for the secondary hash tables is less than 2N**.

# Hash Tables in Popular Languages

- "An Analysis of Hash Map Implementations in Popular Languages"
  - Reference: https://rcoh.me/posts/hash-map-analysis/
  - Chaining: Java, C++, C#, Scala, Go
  - Open addressing: Ruby, Python

- HashMap in Java 8
  - Reference: https://www.geeksforgeeks.org/internal-working-of-hashmap-java/
  - In Java 8, a HashMap is implemented with separate chaining.
  - Default load factor is 0.75
  - For efficiency, the bucket (chain) starts as a linked list but after a certain threshold it is replaced with a balanced tree.
- C++
  - Reference: http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2013/n3690.pdf
  - "The elements of an unordered associative container are ***organized into buckets***. Keys with the same hash code appear in the same bucket. The number of buckets is automatically increased as elements are added to an unordered associative container, so that the average number of elements per bucket is kept below a bound. Rehashing invalidates iterators, changes ordering between elements, and changes which buckets elements appear in, but does not invalidate pointers or references to elements. For unordered_multiset and unordered_multimap, rehashing preserves the relative ordering of equivalent elements."

# %

- % – is the remainder operator
- Note that if the number on the left is smaller than the number on the right, the result is the number on the left (and NOT 0; 0 is the result of the integer division in that case, but not the remainder)
- 7 % 10 is 7
- 4 % 9 is 4

- 48 % 13 is 9

# Extra