Dynamic Programming, Greedy - Practice

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Book (CLRS) problems:

1. Dynamic programming: 15.4-1, 15.4-2, 15.4-3 and 15.4-5 (here justify the complexity of all steps you take). (page 396)
2. Greedy: 16.1-3 (page 422)

Other sources: Problems/issues discussed in class, Homework problems, Blackboard quizzes.

Types of problems:

1) Given solution table partially filled out, finish filling it out.
2) Given the gain/cost solution, recover the **solution choices** that gave this optimal value.
3) Time and space complexity for all covered algorithms.

Edit Distance (ED), Longest Common Subsequence (LCS), Longest Increasing Subsequence (LIS)

P1-MIX. Short answer:

a) True/False for any DP problem, we can only recover the choices that give the optimal solution AFTER we computed the actual optimal value. **True**

b) (5 pts) Give the function for computing the longest common subsequence (LCS): D[i][j] = …. You can use max/min functions on however many arguments. You can write the function as a math function or as code. Assume i is the index for string X = $x_1\ x_2\ x_{3\ldots}\ x_M$ and j is the index for string Y = $y_1\ y_2\ y_{3\ldots}\ y_N$.

$$D[i][0] = D[0][j] = 0$$

$$D[i][j] = \begin{cases} 1 + D[i-1][j-1], & x_i == y_j \\ \max\{\, D[i-1][j],\ D[i][j-1]\,\}, & x_i \neq y_j \end{cases}$$

c) (6 pts) Give the math function (or code piece) for computing the cell Dist[i][j] for the Edit Distance between two strings X and Y. Assume that in the table, X will be vertical and Y will be horizontal. Assume the index of the first letter in a string is 0 (not 1). (Same question applies to **all** DP problems covered in class.)

$$D[i][0] = i$$

$$D[0][j] = j$$

$$D[i][j] = \begin{cases} \min\{\, D[i-1][j]+1, D[i][j-1]+1, D[i-1][j-1] \,\}, & x_i == y_j \\ \min\{\, D[i-1][j]+1, D[i][j-1]+1, D[i-1][j-1]+1 \,\}, & x_i \neq y_j \end{cases}$$

d) (5 pts) This is meant to be an edit distance table, but something is wrong. What is the problem?

0	**1**	**2**	**3**
1	**1**	**3**	**4**
2	**2**	**2**	**4**
3	**3**	**3**	**3**

The highlighted cell cannot be 3. It should be at most 2 (diag+1= left +1 =2).

e) (4 pts) In the table below, the highlighted value should be 2 instead of 3, because it calculates the minimum of 2,3,4 when the letter is the same: o. True/False. EXPLAIN.

		s	o	m	e	t	h	i	n	g
	0	1	2	3	4	5	6	7	8	9
s	1	0	1	2	3	4	5	6	7	8
o	2	1	0	1	2	3	4	5	6	7
m	3	2	1	0	1	2	3	4	5	6
e	4	3	2	1	0	1	2	3	4	5
o	5	4	3	2	1	1	2	3	4	5
n	6	5	4	3	2	2	2	3	3	4

False.
The letter is used ONLY for diagonal:
3+0 = 3
When coming from the top: 2+1 = 3

P2-LIS. (8pts) In class we discussed how we can solve the Longest Increasing Subsequence (LIS) problem by reducing it to another problem.

a) (3pts) What other type of problem did we reduce it to? Longest Common Subsequence (LCS)

b) (5pts) Assume the specific instance you need to solve is 7, 1, 3, 6, 4, 1, 2, 3, 0, 6, and you allow repetitions in the increasing sequence (it is NOT strictly increasing: 3,3,6 is allowed). Give the reduction: produce a problem **instance** of the other problem type. You do NOT need to solve, it, just give the new instance problem corresponding to the given LIS problem.

Make a copy, Y, of the sequence X.

Sort Y in increasing order.

Find the LCS between X and Y. That is the answer: LIS(X) = LCS(X,Y)

P3-ED1. (6pts) Fill out the last two rows in the table below:

		N	O	N	S	T	O	P
	0	1	2	3	4	5	6	7
R	1	1	2	3	4	5	6	7
O	2	2	1	2	3	4	5	6
U	3	3	2	2	3	4	5	6
N	4	3	3	2	3	4	5	6
D	5	4	4	3	3	4	5	6

P4-ED2. (6pts) Fill-out the last two rows in the edit distance table below. (Here STEM is the complete second word. ME is the end of the first word. For example the first word could be NAME, TESTME).

		S	T	E	M
…	…	…	…	…	…
…	3	3	2	2	3
M	4	4	3	3	2
E	5	5	4	3	3

0/1 Knapsack normal and fractional. Dynamic Programing & Greedy

NOTE: *Unless otherwise specified, any Knapsack problem is assumed to NOT be fractional: items can be picked up as the whole item or not at all.*

KP1. Short answer:

a) (3pts) The time complexity for the ITERATIVE solution for the 0/1 Knapsack of max capacity N and d items is: is Θ(__**N*d**____)

b) (3pts) What information is used to recover the items that give the optimal solution for the 0/1 Knapsack:

A) a star B) an arrow C) the Picked array D) None of these.

c) (5 pts) Give the function for computing the cell **sol[i][j]** for the 0/1 Knapsack. Assume that i is the index of the item. You can use value[i] and weight[i] to refer to the value and the weight of item i.

sol[i][j] = max{ sol[i-1][j], sol[i-1][j-weight[i]]+value[i] }

d) (5 pts) Is there any Knapsack version for which Greedy (with the ratio value to weight) gives an optimal solution for? Yes: The **fractional** Knapsack (both 0/1 and Unbounded versions)

KP2. (14 pts) Use a Greedy method based on the **largest value** to solve a **0/1** Knapsack problem.

a) (6pts) Give the pseudocode for the algorithm (what it computes, what actions it takes).

```
// A sufficient version for the exam:
L = list of items sorted in increasing order of value.
currentWeight = original weight
totalValue = 0
Repeat {
     x = first item in L s.t. x->weight<=currentWeight
     // if no item in L has weight <= currentWeight, x is NULL
     If (x == NULL)
          Break
     Else  {
          x = curr // x is the first item that fits
          currentWeight = currentWeight - x->weight
          totalValue = totalValue + x->value
          remove x from L      // because 0/1 Knapsack
     }
}
```

```
// A more detailed version:
L = list of items sorted in increasing order of value.
currentWeight = original weight
totalValue = 0
x = L // just to start the loop. It gets overwritten
while (x!=null):
     x = null
     curr = L->next // assume list has dummy node
     while (curr != null && curr->weight >currentWeight)
          curr = curr->next
     if (curr != null)
          x = curr // x is the first item that fits
          currentWeight = currentWeight - x->weight
          totalValue = totalValue + x->value
          remove x from L      // because 0/1 Knapsack
     // else: L is empty or no item fit in
```

b) (8pts) Does it give an optimal solution or not? If Yes, justify, if No, prove it with a counter example.

No. Idea: if pick the item with most value, no other fits in. Greedy picks this. Optimal: have 2 other items of smaller individual value, but larger when added and they both fit.

Item	A	B	C
Weight	3	2	2
Value	5	4	4

Use Knapsack capacity: 4.
Greedy can only pick A (remaining weight 1) total value: 5
Optimal solution: pick B and C. Total value: 8

Job Scheduling (or Weighted Interval Scheduling): DP & Greedy

JS1. Short answer

a) Assume that the data has been preprocessed to the point where you have all the pi values for a Job Scheduling problem with N jobs. What is the time complexity to solve this problem? Circle the tightest asymptotic bound (O, Θ or Ω) that applies and fill in the function as well. O / Θ / Ω (…N……..)

b)

JS2. (12 pts) a) (10 pts) Use Dynamic Programming to solve the Weighted Job Scheduling below for jobs 1-6 with job values given by v_i. Here p_i gives the last job that does not overlap with job j. When filling in the answer for m(i) show your work as done in class.

b) (2 pts) Recover the solution (fill in in the rightmost column). **Jobs: ~~2,~~1,4,6 – corrected 4/5/19**

i	v_i	p_i	m(i)	m(i) used i: Y/N	In opt Solution: Y/N
0			0		
1	4	0	4 (max of: 0,4)	Y	Y
2	3	0	4 (max of: 4,3)	N	~~Y~~
3	4	1	8 (max of: 4,4+4)	Y	
4	5	2	9 (max of: 8,4+5)	Y	Y
5	3	2	9 (max of: 9,4+3)	N	
6	2	4	11 (max of: 9,9+2)	Y	Y

JS3. (11pts) For a Job-Scheduling problem where each job has the same value (e.g. $10), your goal is to choose the maximum number of non-overlapping jobs. You take the following approach:

- Sort the jobs in increasing order of START TIME
- Repeat until no job left:
  - Pick the job,J, that starts first. Assume no two jobs have the same start time.
  - From the remaining jobs, remove all the jobs that overlap with job J.

a) (3 pts) Is this Greedy, Dynamic Programming or Brute Force Search? **Greedy**

b)(8 pts) Does it give an optimal solution or not? If Yes, justify, if No, prove it with a counter example.

No. Idea: have two overlapping jobs, greedy picks first one to start but the other one gives more value.

Job	Start	End	Value
1	1	3	2

2	2	5	5

Greedy picks job, total value: 2 .

Optimal solution job 2, total value: 5 .

JS4. () When we solved the Job Scheduling problem, the problem was already 'preprocessed' to lend itself to a Dynamic Programing (DP) solution.

a) (5 pts) Fill in the 2 sentences below to say what the 'preprocessing" does.

Step 1:_Reorder jobs in increasing order of end time (Job 1 is the first one to end, job N is the last one to end)

p_i is _ the last job that ends before job i starts _____

b) (6 pts) Given the jobs in the table to the right, do the preprocessing so that the problem can be solved with DP. Solve up to and including the p_i.

Start time	End time	Value
5	7	5
1	2	4
2	6	4
4	9	3
3	4	3
8	10	2

Draw the picture and fill in the table below:

Job	Start	End	Value	p_i
0	-	-	-	-
1	1	2	4	0
2	3	4	3	1
3	2	6	4	1
4	5	7	5	2
5	4	9	3	2
6	8	10	2	4