# Solved problems for time complexity of loops

Last updated 9/17/2024

General comments/hints

1. If a loop variable takes consecutive values, you do not have write it as a function of the iteration number, r.
2. r can start from 0 or 1, whatever is more convenient in writing the loop variable as a function of r
3. lg = log2

A) 6.

// int mistery(int len, int v); has Θ(len2)

for(i=N; i>=0; i--){

 int res = mistery(N, i); // 🡺 Θ(N2) or O(N2)

 for(k=N; k>=1; k=k-1)

 printf("B"); // 🡺 Θ(1) or O(1)

}

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| for-k TC1iter(k) = O(1)+O(1)+O(1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | K |  | TC1iter(k) = O(1) |
|  | N |  | 1 |
|  | N-1 |  | 1 |
|  | N-2 |  | 1 |
|  |  |  |  |
|  | K |  | 1 |
|  |  |  |  |
|  | 1 |  | 1 |
| TCfork = 1+1+…+1 = rows\*1 = N \* 1 = O(N) |

  | for-i: TC1iter(i) = O(1)+O(N2)+O(N)+O(1) = O(N2)

|  |  |  |  |
| --- | --- | --- | --- |
|  | i |  | TC1iter(i) = O(i2) |
|  | N |  | N2  |
|  | N-1 |  | N2 |
|  | N-2 |  | N2 |
|  |  |  |  |
|  | i |  | N2 |
|  |  |  |  |
|  | 1 |  | N2  |
| TCfori = N2 + N2 + … + N2 + … + N2 = rows \* N2 = N \* N2 = O(N3)  |

 |

Final answer: O(N3)

A) 7.

// int mistery(int N, int v); has O(N2)

for(i=N; i>=0; i--){

 int res = mistery(i, i); // 🡺 O(i2)

 for(k=N; k>=1; k=k-1)

 printf("B");

}

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| for-k TC1iter(k) = O(1)+O(1)+O(1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | k |  | TC1iter(k) = O(1) |
|  | N |  | 1 |
|  | N-1 |  | 1 |
|  | N-2 |  | 1 |
|  |  |  |  |
|  | k |  | 1 |
|  |  |  |  |
|  | 1 |  | 1 |
| TCfork = 1+1+…+1 = rows\*1 = N \* 1= O(N) |

  | for-i: TC1iter(i) = O(1)+O(i2)+O(N)+O(1) = O(i2)

|  |  |  |  |
| --- | --- | --- | --- |
|  | i |  | TC1iter(i) = O(i2) |
|  | N |  | N2  |
|  | N-1 |  | (N-1)2 |
|  | N-2 |  | (N-2)2 |
|  |  |  |  |
|  | i |  | i2 |
|  |  |  |  |
|  | 1 |  | 12  |
| TCfori = 12 + 22 + … + i2 + … + (N-2)2 + (N-1)2 + N2 = N(N+1)(2N+1)/2 = (2N3 + 3N2 + N)/2 = O(N3)  |

 |

Final answer: O(N3)

A) 8

for (i = 101; i<=(100+N); i++)

 for (k=1; k<=i; k = k+1)

 printf("B ");

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| for-k TC1iter(k) = O(1)+O(1)+O(1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | k |  | TC1iter(k) = O(1) |
|  | i |  | 1 |
|  | i-1 |  | 1 |
|  | i-2 |  | 1 |
|  |  |  |  |
|  | k |  | 1 |
|  |  |  |  |
|  | 1 |  | 1 |
| TCfork = 1+1+…+1 = rows\*1 = i\*1 = O(i) |

 | for-i: TC1iter(i) = O(1)+O(i)+O(1) = O(i)

|  |  |  |  |
| --- | --- | --- | --- |
| r | i | i = fct(r) | TC1iter(i) = O(i) |
| 1 | 101 | =100+1 | 101  |
| 2 | 102 | =100+2 | 102 |
| 3 | 103 | =100+3 | 103 |
|  |  |  |  |
| r | i | =100+r | i |
|  |  |  |  |
| rlast | 100+N | =100+rlast | 100+N  |
| TCfori = 101 + 102 + 103 + … + i + … (100+N)= (100+1)+(100+2)+(100+3)+…+(100+r)+ … +(100+N) =(100+100+…+100+…+100) + (1+2+3+..+r+…N) = =100\*rows + N(N+1)/2 = 100\*N + (N2+N)/2 = O(N2) |

 |

**Final answer: O(N2)**