

Heaps, Heapsort - Solution

If in the exam you are asked to delete or remove or extract from a heap, it means delete the max element (if a max-heap) or the min element (in a min-heap). If asked to delete x (or remove x or extract x) then you must delete the element x .

Any question that would ask to modify/adapt an algorithm, would provide the original code/pseudocode for that algorithm.

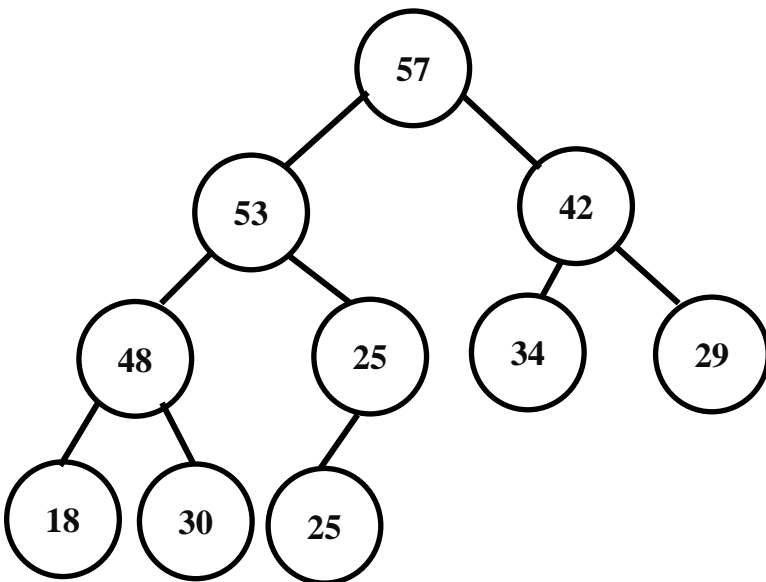
H1. Book (CLRS) – heaps, heapsort

- 6.1-1, 6.1-4, 6.1-5, 6.1-6 (CLRS 3-rd edition, page 153)
- From 6.2-1 to 6.2-6 (inclusive) (CLRS, page 156)
- 6.3-1, 6.3-2
- 6.4-1, 6.4-3
- 6.5-1, 6.5-2, 6.5-6, 6.5-7, 6.5-8, 6.5-9. Also questions 6.5-3 and 6.5-4 (the code for the methods referred in the questions, would be provided in the exam in this case).

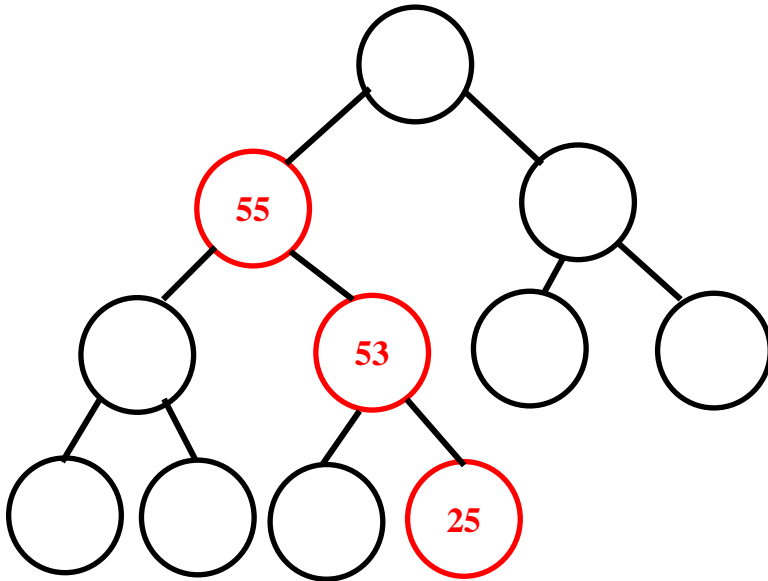
H2. (Heap operations. No code needed.)

A1) (7 points) Insert **55** in the max-heap below. Show all the changes to the heap.

Redraw the heap and leave white the nodes whose value did not change and show values only in the modified nodes.



Solution:



A2) Show the array that stores the above heap. Start at index 1.

A3) Rewrite the array (for the above heap) after each change it goes through during the insert 55 operation.

Answer for A1 and A2. Cells that were not modified are left blank.

	0	1	2	3	4	5	6	7	8	9	10	11
Orig		57	53	42	48	25	34	29	18	30	25	
Step 1												55
Step 2						55						25
Step 3			55			53						

A4) Online-type answers for A1

Version 1 – lists modified nodes and their value (index,newValue) , in increasing order of index.

Consider the heap AFTER 55 was inserted. Give the pairs (index,newValue) for every element of the heap that was modified (that is that is now different from what it was before the insertion). List the pairs in increasing order of the index. Separate them by comma. Do not put any empty spaces.

(2,55),(5,53),(11,25)

Version 2 – lists swaps performed, (value1, value2), in order in which they were performed.

Give the pairs of values that were swapped as part of performing the insert 55 operation. Separate the pairs by comma, do not put any empty spaces. List the pairs in the order in which the swaps were performed.

(25,55),(53,55)

B1) (7 points) Perform one remove () operation on the heap below.

Step 2		53	25							
Step 3			48		25					
Step 4					30				25	

B4) **Online-type answer for this question:**

Version 1) Consider the heap AFTER the `remove()` operation. Give the pairs (index,value) for every element of the heap that was modified (that is that is now different from what it was before `remove()`). List the pairs in increasing order of the index. Separate them by comma. Do not put any empty spaces.

(1,53)(2,48),(4,30),(9,25),(10,x)

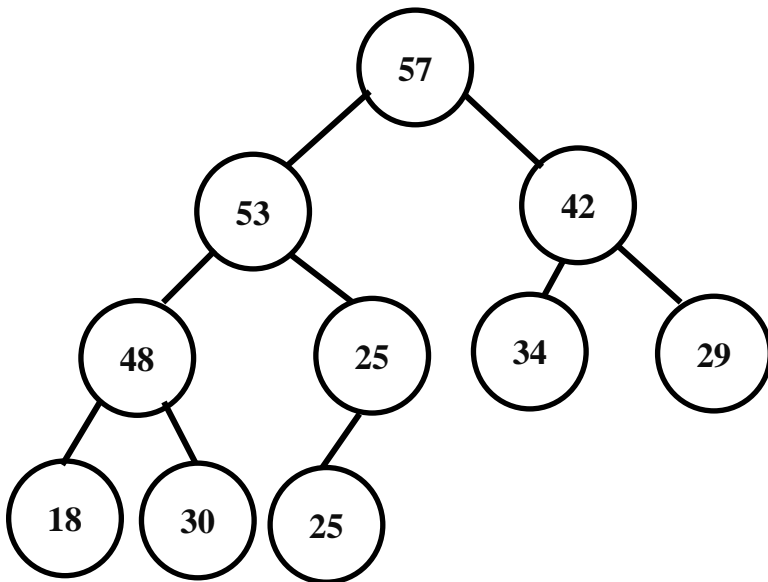
In the answer above, pair (10,x) indicates that the element at index 10 is not part of the heap anymore.

Version 2) Give the pairs of values that were swapped as part of performing the `remove()` operation. Separate the pairs by comma, do not put any empty spaces. List the pairs in the order in which the swaps were performed.

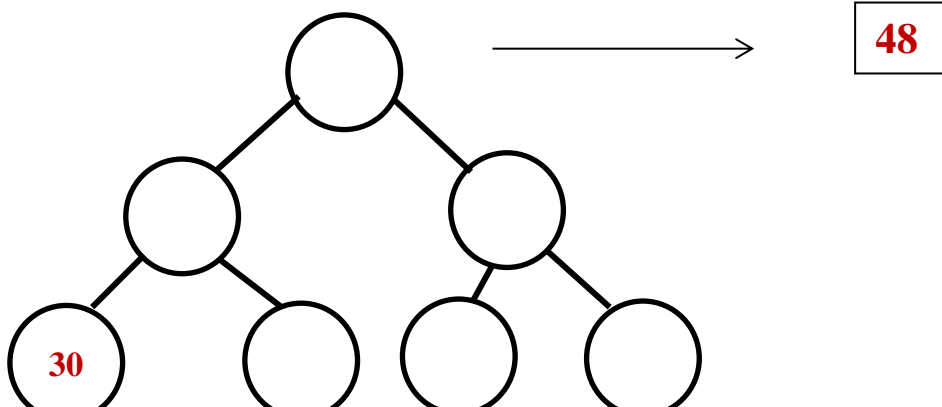
(57,25),(25,53),(25,48),(25,30) In this answer:

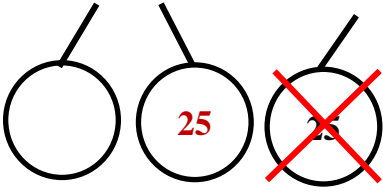
- the first pair, (57,25), indicates that 25 was copied in the root node.
- the next pair, (25,53), indicates that 53 moved to the root node and 25 went in place of 53
- the next pair, (25,48), indicates that 25 and 48 were swapped and so on.

C1) Perform a `remove(48)` on the heap below. Show the changes in drawing and as an array as done for B1, B2, B3.



Solution





	0	1	2	3	4	5	6	7	8	9	10
Orig		57	53	42	48	25	34	29	18	30	25
Step 1					25						48
Step 2					30					25	

C4) Online-type answer:

Version 1) Give the (index,value) pairs to show the heap changes.

(4,30),(9,25),(10,x) Here (10,x) indicates that the element at index 10 is not part of the heap anymore.

Version 2) Give the pairs of values that were swapped to show the heap changes:

(48,25),(25,30) here the swap (48,25) indicates copying 25 in place of 48.

H3. (14 pts) HEAPS

a) (8 pts) Does this array represent a Max-Heap with the root at index 1? (Ignore the data at index 0.)

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Data	12	78	50	30	8	65	22	28	4	6	12	2	13

1) (4 pts) Draw the heap like a tree.



2) (4 pts) Is this a valid heap? Yes / No **Briefly** justify your answer:

No. It would be a Max-heap, but at least one node fails the relationship property: node 50 is smaller than its right child, 65.