COMPRESSED MAGNETIC RESONANCE IMAGING BASED ON WAVELET SPARSITY AND NONLOCAL TOTAL VARIATION

Junzhou Huang

University of Texas at Arlington
Department of Computer Science and Engineering

Fei Yang

Rutgers University
Department of Computer Science

ABSTRACT

This paper introduces an efficient algorithm for the compressed MR image reconstruction problem, which is formulated as the minimization of a linear combination of three terms corresponding to a least square data fitting, nonlocal total variation (NLTV) and wavelet sparsity regularization. In our method, the original minimization problem is decomposed into wavelet sparsity and NLTV norm regularization subproblems respectively. Then, these two subproblems are efficiently solved by existing techniques. Finally, the reconstructed image is obtained from the weighted average of solutions from two subproblems in an iterative framework. Experiments with improved performance over previous methods demonstrate the superior performance of the proposed algorithm for compressed MR image reconstruction.

Index Terms**—** Compressive Sensing, MRI, Wavelet Sparsity, Nonlocal Total Variation

1. INTRODUCTION

Recent developments in compressive sensing theory[1] show that it is possible to accurately reconstruct the Magnetic Resonance (MR) images from highly undersampled K-space data and therefore significantly reduce the scanning time [2]. Suppose x is a MR image and R is a partial Fourier transform, the sampling measurement b of x in K-space is defined as $b = Rx$. The compressed MR image reconstruction problem is to reconstruct x giving the measurement b and the sampling matrix R. Motivated by the compressive sensing theory, Lustig et al. [2] proposed their pioneering work for the MR image reconstruction. In their work, this problem is formulated as follows:

$$\hat{x} = \arg\min_{x}\{\frac{1}{2}\|Rx - b\|^2 + \alpha\|x\|_{TV} + \beta\|\Phi x\|_1\} \qquad (1)$$

where α and β are two positive parameters, b is the undersampled measurements of K-space data, R is a partial Fourier transform and Φ is a wavelet transform. It is based on the fact that the piecewise smooth MR images of organs can be sparsely represented by the wavelet basis and should have small total variations. The TV was defined discretely as $\|x\|_{TV} = \sum_i \sum_j ((\nabla_1 x_{ij})^2 + (\nabla_2 x_{ij})^2)$ where ∇_1 and ∇_2 denote the forward finite difference operators on the first and second coordinates, respectively. This can be denoted as the local TV.

It is a difficult optimization problem because both $L1$ and TV norm regularization terms are nonsmooth. The conjugate gradient (CG) and PDE were used to attack this problem in [2][3]. However, they are very slow for real MR images. Ma et al. proposed an operator-splitting algorithm (TVCMRI) to solve this problem [4]. A variable splitting method (RecPF) was also proposed for the same formulation [5]. Both of them can replace iterative linear solvers with Fourier domain computations, which can gain substantial time savings. Recently, the Fast Composite Splitting Algorithm was proposed to efficiently solve this problem in [6, 7]. It decomposed the original problem into two easier subproblem: L1 regularization and TV regularization subproblems respectively and obtained the reconstructed results by iteratively averaging the solutions of two subproblems. It has been proved to be the best algorithm so far for compressed MR image reconstruction in terms of reconstruction accuracy and computational complexity.

In this paper, a novel method is proposed for compressed MR imaging based on the wavelet sparsity and periodic nonlocal total variation regularization. Followed the FCSA [6, 7], we decompose the hard composite regularization problem (1) into two simpler regularization subproblems: L1 norm regularization subproblems and nonlocal total variation regularization subproblem. Here, we use the NLTV instead of TV in FCSA [6, 7] because the NLTV is far better than total variation for improving the signal-to-noise ratio in practical application. Considering that the computational complexity of NLTV is higher, we do not perform it in each iteration but periodically. We call it the NLTV-MRI. Numerous experiments have been conducted on real MR images to show the advantages of the proposed method over previous methods in terms of reconstruction accuracy and computation complexity.

2. NONLOCAL TOTAL VARIATION

Nonlocal total variation regularization [8, 9, 10, 11, 12, 13] has been studied to address the issue of blocky effect by employing nonlocal pixels for calculating the gradients in the regularization term. It has been thought as an effective tool instead of total variation for improving the signal-to-noise ratio in practical application. Recently, it has been successfully used for 4D computed tomography reconstruction from few-projection data [14]. The NLTV regularization is formulated as:

$$\hat{x} = \arg\min_x \{ \frac{1}{2} \|x - x_0\|^2 + \alpha \|x\|_{NLTV} \} \quad (2)$$

Here, $||x||_{NLTV}$ is defined as nonlocal total variation norm and computed from:

$$\|x\|_{NLTV} = \sum_u \sqrt{\sum_v [x(u) - x(v)] w(u, v)} \quad (3)$$

where $w(u, v)$ is the graph weight function, $x(u)$ and $x(v)$ are the image values in pixel u and v. The graph weight function $w(u, v)$ denotes how much the difference between pixels u and v is penalized in the images. The more similar the neighborhoods of u and v are, the more the difference should be penalized, and vice versa. Given an image x, the graph weight function is calculated by:

$$w(u, v) = \frac{1}{Z_x} \exp^{\frac{-\|q_x(u) - q_x(v)\|^2}{2\sigma^2}} \quad (4)$$

where $q_x(u)$ denotes a small patch in image x centering at the coordinate u, $q_x(v)$ denotes a small patch in image x centering at the coordinate v and Z_x is a normalization factor. The scale parameter σ controls to what extent similarity between patches is enforced. Many recent works have shown that the NLTV is far better than previous TV for improving the signal-to-noise ratio in practical application [8, 9, 13].

3. ALGORITHM

In this section, we use the NLTV instead of TV for compressed MR image reconstruction to address the issue of blocky effect with TV-regularization. In other applications, the NLTV has demonstrated its superior performance to the TV [8, 9, 13, 14]. In NLTV, the gradient for the regularization term is calculated with pixels belonging to the whole image, instead of only the nearest neighboring pixels as used in TV regularization. In addition, a weighted graph between the current pixel and all image pixels is used in calculating the gradient. These differences allow the NLTV regularization to effectively remove noise without destroying the salient features of the original image. Our in vivo results demonstrate that the proposed NLTV regularization is able to preserve more details and fine structures than the existing regularization methods while suppressing noise in applications of compressed MR image reconstruction.

The compressed MR image reconstruction is formulated as follows:

$$\hat{x} = \arg\min_x \{ \frac{1}{2} \|Rx - b\|^2 + \alpha \|x\|_{NLTV} + \beta \|\Phi x\|_1 \} \quad (5)$$

where α and β are two positive parameters, b is the undersampled measurements of K-space data, R is a partial Fourier transform and Φ is a wavelet transform. The only different between it and Equation 1 is that we use the NLTV instead of the TV. We will show that this change is very important and deserved for improving the image reconstruction performance.

Before proposing our algorithm, we introduce some notations:

Gradient: $\nabla f(x)$ denotes the gradient of the function f at the point x.

The proximal map: given a continuous convex function $g(x)$ and any scalar $\rho > 0$, the proximal map associated to function g is defined as follows [15][16]:

$$prox_\rho(g)(x) := \arg\min_u \{ g(u) + \frac{1}{2\rho} \|u - x\|^2 \} \quad (6)$$

Instead of the TV term with the NLTV in the FCSA [6, 7], a new algorithm, NLTV-FCSA, is proposed for MR image reconstruction problem. In practice, we found that a small iteration number in the NLTV regularization is enough to obtain good reconstruction results. Numerous experimental results in the next section will show that it is good enough for real MR image reconstruction.

Algorithm 1 outlines the proposed algorithm for problem 5. A key feature of the NLTV-FCSA is its fast convergence performance borrowed from the FCSA. As shown in [6, 7], the FCSA can obtain an ϵ-optimal solution in $\mathcal{O}(1/\sqrt{\epsilon})$ iterations. In the step $x^k = project(x^k, [l, u])$, the function $x = project(x, [l, u])$ is defined as: 1) $x = x$ if $l \leq x \leq u$; 2)$x = l$ if $x < u$; and 3) $x = u$ if $x > u$, where $[l, u]$ is the range of x. For example, in the case of MR image reconstruction, we can let $l = 0$ and $u = 255$ for 8-bit gray MR images.

Another key feature of the proposed algorithm is that the cost of each iteration is approximately $\mathcal{O}(p \log(p))$ where p is the pixel number of the reconstructed image. It can be confirmed by the following observations. The step 4, 5 and 6 only involve adding vectors or scalars, thus cost only $\mathcal{O}(p)$ or $\mathcal{O}(1)$. In step 1, $\nabla f(r^k = R^T(Rr^k - b)$ since $f(r^k) = \frac{1}{2}\|Rr^k - b\|^2$ in this case. Thus, this step only costs $\mathcal{O}(p \log(p))$. The step $x^k = prox_\rho(2\beta\|\Phi x\|_1)(x_g)$ has a close form solution and can be computed with cost $\mathcal{O}(p \log(p))$. As introduced above, the step can be solved iteratively $x^k = prox_\rho(2\alpha\|x\|_{NLTV})(x_g)$ [8]. Although it generally has higher costs, we only need to run it for limited $\mathcal{O}(1)$ times. Thus, the total cost of each iteration in the FCSA is approximately $\mathcal{O}(p \log(p))$. Compared to the TV in previous methods, the proposed NLTV step can effectively avoid the blocky effects and preserve the fine structures while removing the artifacts.

With these two key features, the NLTV-FCSA efficiently solves the MR image reconstruction problem (1) and obtains better reconstruction results in terms of both the reconstruc-

Algorithm 1 NLTV-FCSA

Input: $\rho = 1/L$, $t^1 = 1$, $x^0 = r^1$, $s = 0$, τ
for $k = 1$ **to** K **do**
  $s = s + 1$, $x_g = r^k - \rho \nabla f(r^k)$
  if $s = \tau$ **then**
    $x_1 = prox_\rho(2\alpha \|x\|_{NLTV})(x_g)$, $s = 0$
  end if
  $x_2 = prox_\rho(2\beta \|\Phi x\|_1)(x_g)$
  $x^k = (x_1 + x_2)/2$; x^k=project(x^k, [l, u])
  $t^{k+1} = \frac{1+\sqrt{1+4(t^k)^2}}{2}$
  $r^{k+1} = x^k + \frac{t^k - 1}{t^{k+1}}(x^k - x^{k-1})$
end for

tion accuracy and computation complexity. The experimental results in the next section demonstrate its superior performance compared with all previous methods for compressed MR image reconstruction.

4. EXPERIMENTS

4.1. Experiment Setup

For fair comparisons, we follow the experimental setup used in previous work [6, 7][4][5]. Suppose a MR image x has p pixels, the partial Fourier transform R in problem (1) consists of m rows of a $p \times p$ matrix corresponding to the full 2D discrete Fourier transform. The m selected rows correspond to the acquired b. The sampling ratio is defined as m/p. The scanning duration is shorter if the sampling ratio is smaller. In MR imaging, we have certain freedom to select the rows, which correspond to certain frequencies. In the k-space, we randomly obtain more samples in low frequencies and less samples in higher frequencies. This sample scheme has been widely used for compressed MR image reconstruction and the same of those used in [2][4][5][6, 7].

All experiments are conducted on a 2.4GHz PC in Matlab environment. We compare the proposed NLTV-FCSA with the fastest methods FCSA [6, 7]. We also compare it with the classic MR image reconstruction method based on the CG [2], TVCMRI [4] and RecPF [5]. For fair comparisons, we download the codes from their websites and carefully follow their experiment setup. For example, the observation measurement b is synthesized as $b = Rx + \mathbf{n}$, where $\mathbf{n}$ is Gaussian white noise with standard deviation $\sigma = 0.01$. The regularization parameter α and β are set as 0.001 and 0.035. R and b are given as inputs, and x is the unknown target. For quantitative evaluation, we compute the Signal-to-Noise Ratio (SNR) for each reconstruction result.

4.2. Visual Comparisons

We apply all methods on four 2D MR images: cardiac, brain, chest and artery respectively. They have been used for validation in [6, 7]. For convenience, they have the same size of 256×256. The sample ratio is set to be approximately 20%. To perform fair comparisons, all methods run 100 iterations except that the CG runs only 20 iterations due to its higher computational complexity. The NLTV-FCSA obtains the best visual effects on all MR images in less CPU time. Figure 1 shows the visual comparisons of the reconstructed results by different methods in the brain image. The classical CG [2] is far worse than others because of its higher cost in each iteration. The FCSA is better than the TVCMRI and RecPF. These results are consistent with observations in [4, 5, 6, 7].

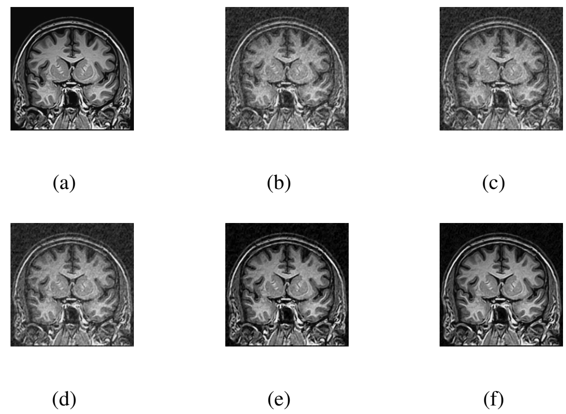

Fig. 1: Brain MR image reconstruction from 20% sampling (a) Original image; (b), (c), (d), (e) and (f) are the reconstructed images by the CG [2], TVCMRI [4], RecPF [5], FCSA [6, 7] and proposed method. Their SNR are 9.19, 12.23, 13.35, 15.42 and 17.90 (db). Their CPU time are 6.52, 5.65, 5.58, 4.70 and 4.76 (s).

4.3. CPU Time and SNRs

Our experiments in the last subsection confirmed the conclusion in [6, 7] that the FCSA is better than the TVCMRI [4] and RecPF [5] and far better than the classic CG [2]. Since the CG method is far less efficient than other methods, we will not include it in this experiment. We gives the performance comparisons between different methods in terms of the CPU time over the SNR. To reduce the randomness, we run each experiments 100 times for each parameter setting of each method. The NLTV-FCSA always obtains the best reconstruction results on all MR images by achieving the highest SNR in less CPU time. Figure 2 gives the performance comparisons between different methods in terms of the CPU time over the SNR. The FCSA is always inferior to the NLTV-FCSA, which shows the effectiveness of NLTV in the proposed algorithm for the MR image reconstruction. While the RecPF is slightly better than the TVCMRI, both of them is inferior to the FCSA. This is consistent to observations in [6, 7], [4] and [5]. In Figure 2(b), we may find that the FCSA obtains reconstructed results with higher SNR compared to the NLTV-FCSA. This is not strange because the NLTV has higher computational complexity although it can obtain results with higher SNR in each iteration. However, the NLTV-FCSA is always better than the FCSA with more iterations. Note that the NLTV regularization is not required to run for each iteration in the NLTV-FCSA. We only runs the NLTV regularization periodically. From the above experiments, we can easily find that the NLTV-FCSA is better than all previous method for compressed MR image reconstruction in terms of both reconstruction accuracy and computational complexity.

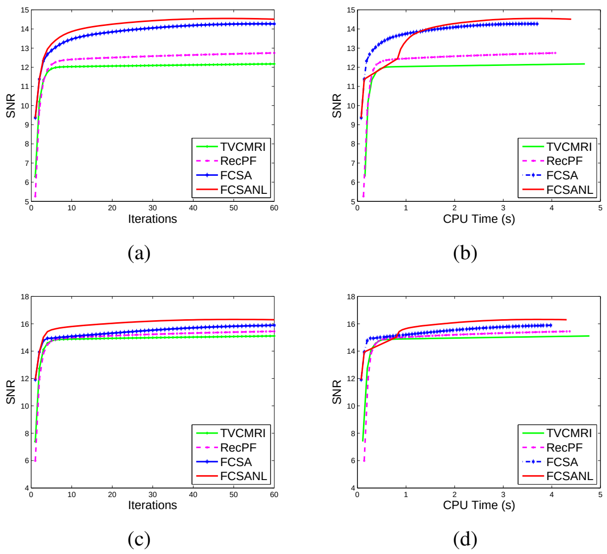

Fig. 2: Performance comparisons: a) Iterations vs. SNR on Brain image; (b) CPU Time vs. SNR on Brain image; (C) Iterations vs. SNR on Chest image and (d) CPU Time vs. SNR on Chest image.

5. CONCLUSION

An efficient algorithm is proposed for the compressed MR image reconstruction based on wavelet sparsity and periodic nonlocal total variation regularization. Our work has the following contributions. First, the proposed NLTV-FCSA can efficiently solve a composite regularization problem including both NLTV term and wavelet sparsity based L1 norm term, which can effectively avoid blocky artifacts caused by traditional TV regularization. Second, it inherits the strong convergence properties of the FCSA and its computational complexity is also approximately $\mathcal{O}(n\log(n))$ for each iteration due to the periodic scheme. Finally, we conduct numerous experiments to compare different reconstruction methods. Our method is shown to impressively outperform the classic methods and the fastest method so far in terms of both accuracy and complexity.

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