

# Supplemental Material for "Efficient MR Image Reconstruction for Compressed MR Imaging"

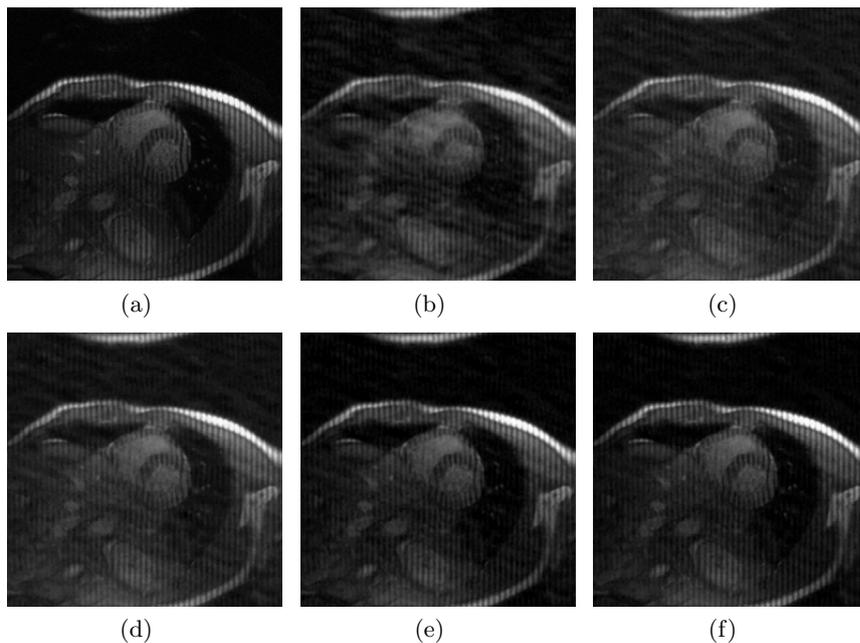
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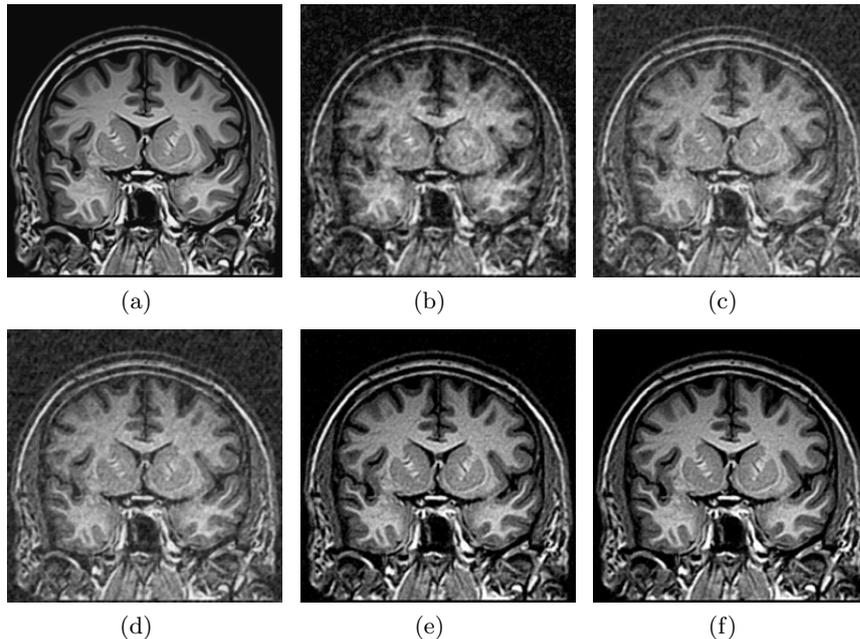
## 1 More Experiment Results

### 1.1 Visual Comparisons

We apply all methods on four 2D MR images: cardiac, brain, chest and artery respectively. Figure 1, 2, 3 and 4 shows the visual comparisons of the reconstructed results. The sample ratio is set to be approximately 20%. To perform fair comparisons, all methods run 50 iterations except that the CG runs only 8 iterations due to its higher complexity.



**Fig. 1.** Cardiac MR image reconstruction from 20% sampling (a) Original image; (b), (c), (d) (e) and (f) are the reconstructed images by the CG [1], TVCMRI [2], RecPF [3], CSA and FCSA. Their SNR are 9.86, 14.43, 15.20, 16.46 and 17.57 (db). Their CPU time are 2.87, 3.14, 3.07, 2.22 and 2.29 (s).



**Fig. 2.** Brain MR image reconstruction from 20% sampling (a) Original image; (b), (c), (d) (e) and (f) are the reconstructed images by the CG [1], TVCMRI [2], RecPF [3], CSA and FCSA. Their SNR are 8.71, 12.12, 12.40, 18.68 and 20.35 (db). Their CPU time are 2.75, 3.03, 3.00, 2.22 and 2.20 (s).

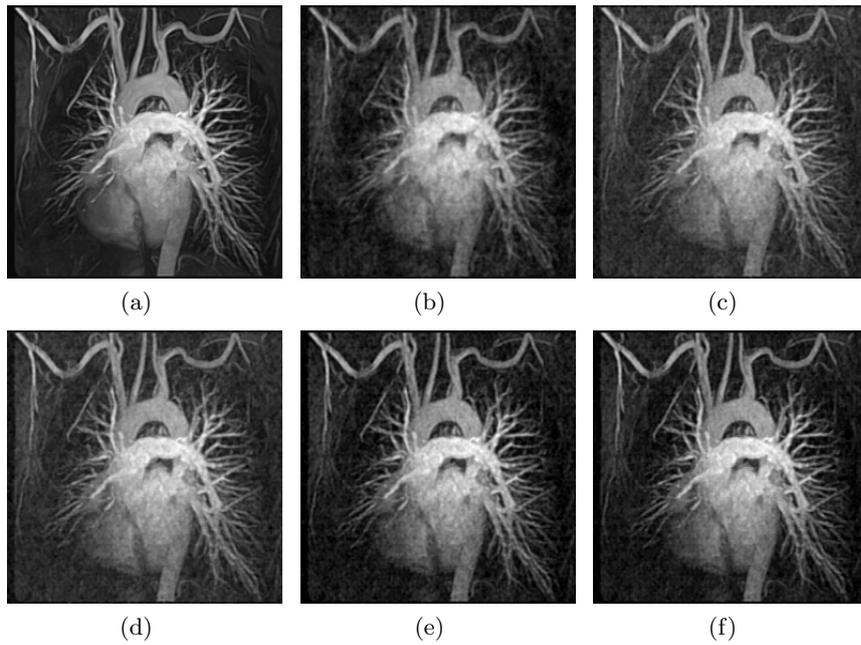
To test the efficiency of the proposed method, we further perform experiments on a full body MR image with size of  $924 \times 208$ . Each algorithm runs 50 iterations. Since we have shown that the CG method is far less efficiency than other methods, we will do not include it in the following experiments. The sample ratio is set to be approximately 25%.

The examples of the original and recovered images by different algorithms are shown in Figure 5. The results obtained by the FCSA are not only visibly better, but also superior in terms of both the SNR and CPU time.

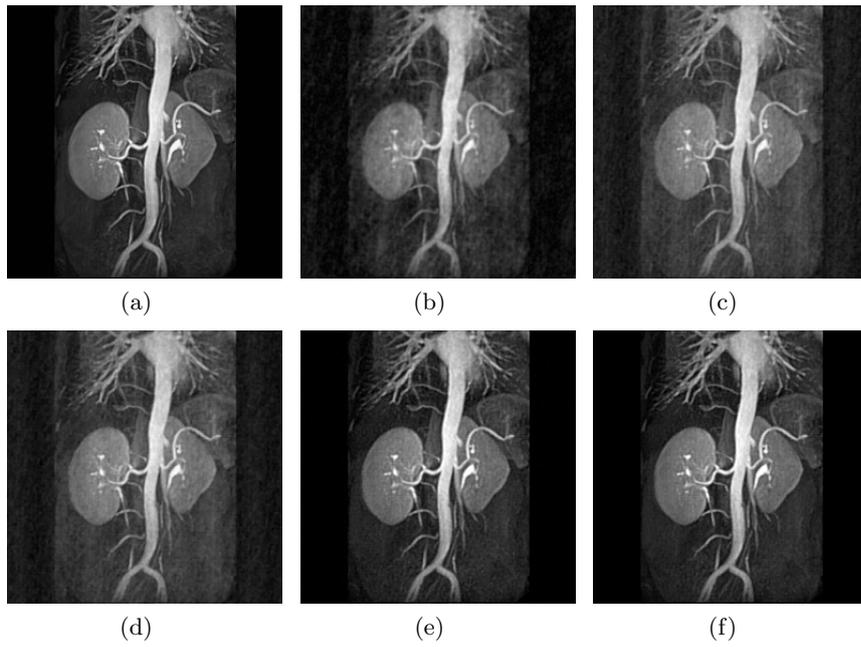
## 1.2 Different Sampling Ratios

To evaluate reconstruction performance with different sampling ratio, we use sampling ratio 36%, 25% and 20% to obtain the measurement  $b$  respectively. Different methods are then used to perform reconstruction. To reduce the randomness, we run each experiments 100 times for each parameter setting and each of methods. We trace the SNR and CPU time in each iteration for each methods.

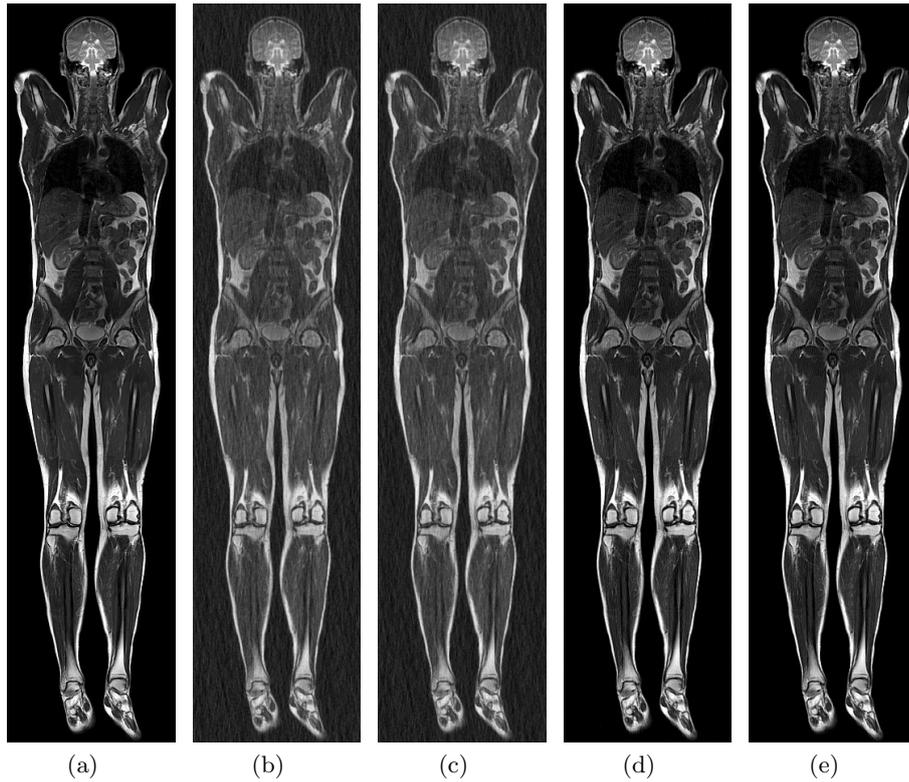
Figure 6 gives the performance comparisons between different methods in terms of the CPU time and SNR when the sampling ratio is 36%. Figure 7 gives



**Fig. 3.** Chest MR image reconstruction from 20% sampling (a) Original image; (b), (c), (d) (e) and (f) are the reconstructed images by the CG [1], TVCMRI [2], RecPF [3], CSA and FCSA. Their SNR are 11.80, 15.06, 15.37, 16.53 and 16.07 (db). Their CPU time are 2.95, 3.03, 3.00, 2.29 and 2.234 (s).

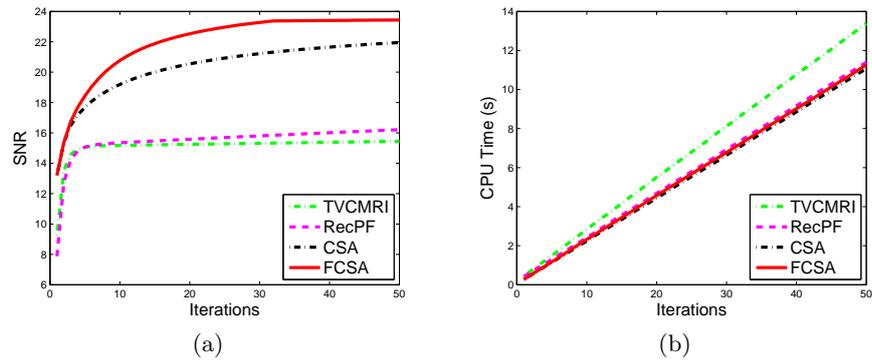


**Fig. 4.** Artery MR image reconstruction from 20% sampling (a) Original image; (b), (c), (d) (e) and (f) are the reconstructed images by the CG [1], TVCMRI [2], RecPF [3], CSA and FCSA. Their SNR are 11.73, 15.49, 16.05, 22.27 and 23.70 (db). Their CPU time are 2.78, 3.06, 3.20, 2.22 and 2.20 (s).

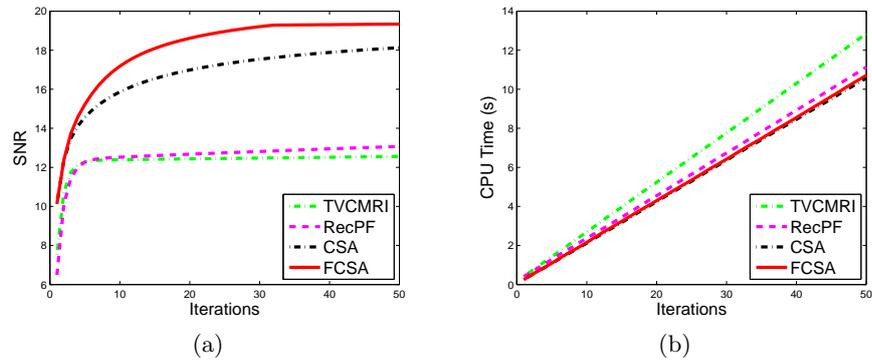


**Fig. 5.** Full Body MR image reconstruction from 25% sampling (a) Original image; (b), (c), (d) and (e) are the reconstructed images by the TVCMRI [2], RecPF [3], CSA and FCSA. Their SNR are 12.56, 13.06, 18.21 and 19.45 (db). Their CPU time are 12.57, 11.14, 10.20 and 10.64 (s).

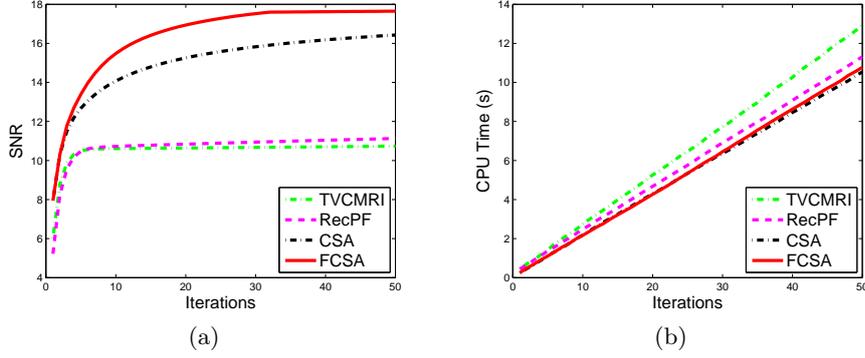
the performance comparisons between different methods in terms of the CPU time and SNR when the sampling ratio is 25%. Figure 8 gives the performance comparisons between different methods in terms of the CPU time and SNR when the sampling ratio is 20%. The reconstructed results produced by the FCSA are far better than those produced by the CG, TVCMRI and RecPF. The reconstruction performance of the FCSA is always the best in terms of both the reconstruction accuracy and the computational complexity, which further demonstrate the effective and efficiency of the FCSA for the compressed MR image construction.



**Fig. 6.** Performance comparisons with sampling ratio 36%: a) Iterations vs. SNR (db) and (b) Iterations vs. CPU Time (s).



**Fig. 7.** Performance comparisons with sampling ratio 25%: a) Iterations vs. SNR (db) and (b) Iterations vs. CPU Time (s).



**Fig. 8.** Performance comparisons with sampling ratio 20%: a) Iterations vs. SNR (db) and (b) Iterations vs. CPU Time (s).

## 2 Proof of Theorem

**Proposition 1.** (Theorem 3.4 in [4]): Let  $\mathcal{H}$  be a real Hilbert space, and let  $g = \sum_{i=1}^m g_i$  in  $\Gamma_0(\mathcal{H})$  such that  $\text{dom}g_i \cap \text{dom}g_j \neq \emptyset$ . Let  $r \in \mathcal{H}$  and  $\{x_j\}$  be generated by the Algorithm 1. Then,  $x_j$  will converge to  $\text{prox}(g)(r)$ .

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**Algorithm 1** Algorithm 3.1 in [4]

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**Input:**  $\rho, \{z_i\}_{i=1,\dots,m} = r, \{w_i\}_{i=1,\dots,m} = 1/m,$   
**for**  $j = 1$  **to**  $J$  **do**  
  **for**  $i = 1$  **to**  $m$  **do**  
     $p_{i,j} = \text{prox}_\rho(g_i/w_i)(z_j)$   
  **end for**  
   $p_j = \sum_{i=1}^m w_i p_{i,j}$   
   $q^{j+1} = z_j + p_j - x_j$   
   $\lambda_j \in ]0, 2[$   
  **for**  $i = 1$  **to**  $m$  **do**  
     $z_{i,j+1} = z_{i,j} + \lambda_j (2p_j - x_j - p_{i,j})$   
  **end for**  
   $x_{j+1} = x_j + \lambda_j (p_j - x_j)$   
**end for**

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**Theorem 1.** Suppose  $\{x^j\}$  be the sequence generated by the CSD, then,  $x^j$  will converge to  $\text{prox}_\rho(\alpha\|x\|_{TV} + \beta\|\Phi x\|_1)(x_g)$ , which means that we have  $x^j \rightarrow \text{prox}_\rho(\alpha\|x\|_{TV} + \beta\|\Phi x\|_1)(x_g)$ .

**Sketch Proof:** In Algorithm 1, we suppose  $g_1(x) = \alpha\|x\|_{TV}$ ,  $g_2(x) = \beta\|x\|_1$ ,  $m = 2$ ,  $w_1 = w_2 = 1/2$  and  $\lambda_j = 1$ , we can obtain the proposed CSD algorithm.

According to the Proposition 1, we know that  $x_j$  will converge to  $prox(g)(r)$ , where  $g = g_1 + g_2 = \alpha\|x\|_{TV} + \beta\|\Phi x\|_1$ .

## References

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