CSE 4308 / CSE 5360 - *Artificial Intelligence I*

Homework 3- Fall 2013

Sample Solution

Problems marked with a * are required only for students in the graduate section (CSE 5360). They will be graded for extra credit for students of CSE 4308.

Propositional Logic

1. Translate the following facts into propositional logic sentences. Make sure you list all the propositions and their meanings.

   a) Insects have six legs.
   $I \rightarrow S$
   Definitions: I - Insect
   S - has 6 legs

   b) If insects had eight legs they would be related to spiders.
   $(I \wedge E) \rightarrow R$
   Definitions: E - has eight legs
   R - related to spiders

   c) Animals that are related to Spiders are also related to Scorpions.
   $(I \wedge R) \rightarrow C$
   Definitions: C - related to scorpions

   d) If insects are six legged animals then they are not related to scorpions.
   $(I \rightarrow S) \rightarrow \neg C$

2. Show that the following sentences follow from the given knowledge base using equivalence and inference rule. (Give all the steps of your proof and indicate what rule you used).

$$
\begin{aligned}
KB: \quad & S_1: && P \wedge Q \\
& S_2: && P \Rightarrow (R \vee S) \\
& S_3: && (A \vee Q) \Rightarrow (P \Rightarrow S) \\
& S_4: && \neg((A \wedge S) \vee (A \wedge P)) \vee (Q \wedge R) \\
& S_5 && A \vee (\neg P \wedge \neg Q) \\
& S_6 && TRUE \Rightarrow (A \wedge (P \Rightarrow Q)) \\
& S_7 && V \vee \neg P \vee \neg U
\end{aligned}
$$

a) $P \wedge A$

$S_8:$	P	S_1 $\wedge$ - elimination
$S_9:$	Q	S_1 $\wedge$ - elimination
$S_{10}:$	$(A \vee \neg P) \wedge (A \vee \neg Q)$	S_5 distributivity
$S_{11}:$	$A \vee \neg P$	S_{10} $\wedge$ elimination
$S_{12}:$	$A \vee \neg Q$	S_{10} $\wedge$ elimination
$S_{13}:$	$\neg P \vee A$	S_{11} commutativity
$S_{14}:$	$P \Rightarrow A$	S_{13} implication
$S_{15}:$	A	S_8, S_{14} modus ponens
	$P \wedge A$	S_8, S_{15} $\wedge$ - introduction

b) $V \Rightarrow S$

$S_{16}:$	$A \vee Q$	S_{15} $\vee$ - introduction
$S_{17}:$	$P \Rightarrow S$	S_{16}, S_3 modus ponens
$S_{18}:$	S	S_{17}, S_8 modus ponens
$S_{19}:$	$S \vee \neg V$	S_{18} $\vee$ - introduction
$S_{20}:$	$\neg V \vee S$	S_{19} commutativity
	$V \Rightarrow S$	S_{20} implication

c)* $TRUE \Rightarrow Q$

$S_{21}:$	$Q \vee \neg TRUE$	S_9 $\vee$ - introduction
$S_{22}:$	$\neg TRUE \vee Q$	S_{21} commutativity
$S_{23}:$	$\neg\neg TRUE \Rightarrow Q$	S_{22} implication
	$TRUE \Rightarrow Q$	S_{23} double negation

d)* $(Q \wedge U) \Rightarrow V$

$S_{24}:$	$\neg P \vee \neg U \vee V$	S_7 commutativity
$S_{25}:$	$\neg P \vee (\neg U \vee V)$	S_{21} associativity
$S_{26}:$	$P \Rightarrow (\neg U \vee V)$	S_{25} implication
$S_{27}:$	$\neg U \vee V$	S_{26}, S_8 modus ponens
$S_{28}:$	$\neg U \vee V \vee \neg Q$	S_{27} $\vee$ - introduction
$S_{29}:$	$\neg Q \vee \neg U \vee V$	S_{28} commutativity
$S_{30}:$	$(\neg Q \vee \neg U) \vee V$	S_{29} associativity
$S_{31}:$	$\neg(Q \wedge U) \vee V$	S_{30} de Morgan
	$(Q \wedge U) \Rightarrow V$	S_{31} implication

First-order Logic

3. Translate the following facts into sentences in first-order logic.

   a) Dogs and cats eat meat.
   $\forall x \exists z((Dog(x) \vee Cat(x)) \Rightarrow Eats(x, z))$

   b) Every bottle that is filled contains liquid.
   $\forall x \forall y(((Bottle(x) \wedge Filled(x, y)) \Rightarrow (Liquid(y) \wedge Contains(x, y)))$

   c) Since Jim and Jack take the same classes, Jack works on the same assignments as Jim.
   $(\forall x(Class(x) \Rightarrow (Takes(Jim, x) \Leftrightarrow Takes(Jack, x)))) \Rightarrow (\forall x(Assignment(x) \Rightarrow (Workon(Jim, x) \Leftrightarrow Workon(Jack, x))))$

   d) If Mary is John's daughter then Mary is younger than John.
   $Daughter(Mary, John) \Rightarrow Jounger(Mary, John)$

4. Determine for each of the following pairs of sentences if they can be unified and if they can, give the most general unifier.

   a) $(Aunt(x, y) \wedge \neg Man(x) \vee Uncle(x, y))$
   $(Aunt(Mary, z) \vee \neg Man(John) \vee Uncle(v, z))$
   Not unifyable due to syntax differences

   b) $((Father(x, Jack) \wedge Mother(y, Jack)) \Rightarrow Married(x, y))$
   $((Father(y, z) \wedge Mother(Mary, z)) \Rightarrow Married(y, Mary))$
   $\{x/Mary, y/Mary, z/Jack\}$

c) $((Son(x,x) \wedge Sister(Mary, Jack)) \Rightarrow (Daughter(x, Mary) \wedge Brother(Jack, Mary)))$
$((Son(Jack,x) \wedge Sister(z,x)) \Rightarrow (Daughter(z, f(x)) \wedge Brother(y,z)))$
Not unifiable because you can not substitute $Mary$ and $f(x)$

d) $((Married(x,y) \wedge Father(x, Mary)) \Rightarrow Man(x))$
$((Married(z, f(Jack)) \vee Father(z,v)) \Rightarrow Man(z))$
Not unifiable due to difference in syntax

5. Transform the following sentences into conjunctive normal form.

a) $P(John) \Rightarrow \exists x(Q(x) \wedge R(John,x))$
$\neg P(John) \vee \exists x(Q(x) \wedge R(John,x))$
$\neg P(John) \vee (Q(g) \wedge R(John,g))$
$(\neg P(John) \vee Q(g)) \wedge (\neg P(John) \vee R(John,g))$

b) $\forall x(\exists y(P(x) \wedge \neg Q(x,y) \wedge R(y)) \Rightarrow \forall y \neg S(x,y))$
$\forall x(\neg \exists y(P(x) \wedge \neg Q(x,y) \wedge R(y)) \vee \forall y \neg S(x,y))$
$\forall x(\forall y \neg (P(x) \wedge \neg Q(x,y) \wedge R(y)) \vee \forall y \neg S(x,y))$
$\forall x(\forall y(\neg P(x) \vee Q(x,y) \vee \neg R(y)) \vee \forall y \neg S(x,y))$
$\forall x(\forall y(\neg P(x) \vee Q(x,y) \vee \neg R(y)) \vee \forall z \neg S(x,z))$
$\neg P(x) \vee Q(x,y) \vee \neg R(y) \vee \neg S(x,z)$

c) $\neg(\exists x \forall y(P(x) \Rightarrow \forall y Q(y)) \wedge \forall z R(z))$
$\neg(\exists x \forall y(\neg P(x) \vee \forall y Q(y)) \wedge \forall z R(z))$
$\forall x \neg(\forall y(\neg P(x) \vee \forall y Q(y)) \wedge \forall z R(z))$
$\forall x \exists y \neg((\neg P(x) \vee \forall y Q(y)) \wedge \forall z R(z))$
$\forall x \exists y(\neg(\neg P(x) \vee \forall y Q(y)) \vee \neg \forall z R(z))$
$\forall x \exists y((P(x) \wedge \neg \forall y Q(y)) \vee \neg \forall z R(z))$
$\forall x \exists y((P(x) \wedge \exists y \neg Q(y)) \vee \exists z \neg R(z))$
$\forall x \exists y((P(x) \wedge \exists u \neg Q(u)) \vee \exists z \neg R(z))$
$\forall x((P(x) \wedge \neg Q(g(x))) \vee \neg R(h(x)))$
$(P(x) \wedge \neg Q(g(x))) \vee \neg R(h(x))$
$(P(x) \vee \neg R(h(x))) \wedge (\neg Q(g(x)) \vee \neg R(h(x)))$

d)* $\exists x(\neg \forall y(P(x,y) \Rightarrow Q(x)) \wedge \forall y(R(y) \Rightarrow \exists z P(y,z)))$
$\exists x(\neg \forall y(\neg P(x,y) \vee Q(x)) \wedge \forall y(\neg R(y) \vee \exists z P(y,z)))$
$\exists x(\exists y \neg((\neg P(x,y) \vee Q(x)) \wedge \forall y(\neg R(y) \vee \exists z P(y,z))))$
$\exists x(\exists y(\neg(\neg P(x,y) \vee Q(x)) \vee \neg \forall y(\neg R(y) \vee \exists z P(y,z))))$
$\exists x(\exists y((P(x,y) \wedge Q(x)) \vee \exists y \neg(\neg R(y) \vee \exists z P(y,z))))$
$\exists x(\exists y((P(x,y) \wedge Q(x)) \vee \exists y(R(y) \wedge \neg \exists z P(y,z))))$
$\exists x(\exists y((P(x,y) \wedge Q(x)) \vee \exists y(R(y) \wedge \forall z \neg P(y,z))))$
$\exists x(\exists y((P(x,y) \wedge Q(x)) \vee \exists u(R(u) \wedge \forall z \neg P(u,z))))$

$(P(g,h) \wedge Q(g)) \vee (R(f) \wedge \forall z \neg P(f,z))$
$(P(g,h) \wedge Q(g)) \vee (R(f) \wedge \neg P(f,z))$
$(P(g,h) \vee (R(f) \wedge \neg P(f,z))) \wedge (Q(g)) \vee (R(f) \wedge \neg P(f,z)))$
$(P(g,h) \vee R(f)) \wedge (P(g,h) \vee \neg P(f,z)) \wedge (Q(g)) \vee R(f)) \wedge (Q(g) \vee$
$\neg P(f,z))$

6. Use resolution with refutation to show that the following queries can be inferred from the given knowledge base. At each resolution step also indicate the corresponding unifier.

$$
\begin{aligned}
KB: \quad & Father(John, Jack) \\
& Married(John, Jane) \\
& Man(Jack) \\
& Father(x_1, y_1) \Rightarrow Man(x_1) \\
& Mother(x_2, y_2) \Rightarrow Woman(x_2) \\
& Married(x_3, y_3) \wedge Father(x_3, z_3) \Rightarrow Mother(y_3, z_3) \\
& Father(x_4, y_4) \wedge Mother(z_4, y_4) \Rightarrow Married(x_4, z_4) \vee Divorced(x_4) \\
& Divorced(John) \Rightarrow False \\
& Mother(Mary, Jack) \\
& Married(x_5, y_5) \wedge Son(z_5, x_5) \Rightarrow Son(z_5, y_5) \\
& Father(x_6, y_6) \wedge Man(y_6) \Rightarrow Son(y_6, x_6)
\end{aligned}
$$

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$$
\begin{aligned}
KB: \quad S_1: \quad & Father(John, Jack) \\
S_2: \quad & Married(John, Jane) \\
S_3: \quad & Man(Jack) \\
S_4: \quad & \neg Father(x_1, y_1) \vee Man(x_1) \\
S_5: \quad & \neg Mother(x_2, y_2) \vee Woman(x_2) \\
S_6: \quad & \neg Married(x_3, y_3) \vee \neg Father(x_3, z_3) \vee Mother(y_3, z_3) \\
S_7: \quad & \neg Father(x_4, y_4) \vee \neg Mother(z_4, y_4) \vee Married(x_4, z_4) \vee Divorced(x_4) \\
S_8: \quad & \neg Divorced(John) \\
S_9: \quad & Mother(Mary, Jack) \\
S_{10}: \quad & \neg Married(x_5, y_5) \vee \neg Son(z_5, x_5) \vee Son(z_5, y_5) \\
S_{11}: \quad & \neg Father(x_6, y_6) \vee \neg Man(y_6) \vee Son(y_6, x_6)
\end{aligned}
$$

S_i indicates true sentences (facts)
T_i indicates sentences derived from incorrect assumptions (and can thus not be reused later)

a) $Married(John, Mary)$

$S_{12}:$	$\neg Father(John, y_4) \vee \neg Mother(z_4, y_4) \vee Married(John, z_4)$	S_8, S_7 $\{x_4/John\}$
$S_{13}:$	$\neg Mother(z_4, Jack) \vee Married(John, z_4)$	S_{12}, S_1 $\{y_4/Jack\}$
$S_{14}:$	$Married(John, Mary)$	S_{12}, S_1 $\{z_4/Mary\}$
	$FALSE$	$S_{14}, \neg Query$

b) $Woman(Mary) \wedge Woman(Jane)$

$S_{15}:$	$Woman(Mary)$	S_5, S_9 $\{x_2/Mary, y_2/Jack\}$
$S_{16}:$	$\neg Father(John, z_3) \vee Mother(Jane, z_3)$	S_6, S_2 $\{x_3/John, y_3/Jane\}$
$S_{17}:$	$Mother(Jane, Jack)$	S_{16}, S_1 $\{z_3/Jack\}$
$S_{18}:$	$Woman(Jane)$	S_5, S_{17} $\{x_2/Jane, y_2/Jack\}$
$T_1:$	$\neg Woman(Jane)$	$S_{15}, \neg Query$
	$FALSE$	T_1, S_{18}

c)* $Son(Jack, Mary) \wedge Son(Jack, Jane)$

$S_{19}:$	$\neg Son(z_5, John) \vee Son(z_5, Jane)$	S_{10}, S_2 $\{x_5/John, y_5/Jane\}$
$S_{20}:$	$\neg Father(x_6, Jack) \vee Son(Jack, x_6)$	S_{11}, S_3 $\{y_6/Jack\}$
$S_{21}:$	$Son(Jack, John)$	S_{20}, S_1 $\{x_6/John\}$
$S_{22}:$	$Son(Jack, Jane)$	S_{19}, S_{21} $\{z_5/Jack\}$
$S_{23}:$	$\neg Son(z_5, John) \vee Son(z_5, Mary)$	S_{10}, S_{14} $\{x_5/John, y_5/Mary\}$
$S_{24}:$	$Son(Jack, Mary)$	S_{19}, S_{21} $\{z_5/Jack\}$
$T_1:$	$\neg Son(Jack, Mary)$	$S_{22}, \neg Query$
	$FALSE$	T_1, S_{24}

7. Translate the knowledge base of problem 6 into a formula list for *Prover9* (a common theorem prover) and use it to perform a proof by refutation of the following queries. You can download and install *Prover9* and access on-line documentation at *http://www.cs.unm.edu/∼mccune/prover9*. To install it on omega, download the Linux code and compile it using the given instructions. The install directory will also contain examples (additional ones can be found on the web site). For each proof, include a printout of the output of

Prover9.

```
formulas(sos).
Father(John,Jack).
Married(John,Jane).
Man(Jack).
Father(x1,y1) -> Man(x1).
Mother(x2,y2) -> Woman(x2).
Married(x3,y3) & Father(x3,z3) -> Mother(y3,z3).
Father(x4,y4) & Mother(z4,y4) -> Married(x4,z4) | Divorced(x4).
Divorced(John) -> $F.
Mother(Mary,Jack).
Married(x5,y5) & Son(z5,x5) -> Son(z5,y5).
Father(x6,y6) & Man(y6) -> Son(y6,x6).
end_of_list.
```

a) $Woman(Mary) \wedge Son(Jack, Mary)$

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formulas(goals).
Woman(Mary) & Son(Jack,Mary).
end_of_list.
```

b) $Married(John, Mary) \wedge Married(John, Jane)$

```
formulas(goals).
Married(John,Mary) & Married(John,Jane).
end_of_list.
```

c)* $\forall x(Married(John, x) \Rightarrow Mother(x, Jack))$

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formulas(goals).
all x (Married(John, x) -> Mother(x, Jack)).
end_of_list.
```

d)* $Son(Jack, Jane) \vee \neg Married(John, Jane)$

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formulas(goals).
Son(Jack,Jane) | -Married(John,Jane).
end_of_list.
```