CSE 4345 / CSE 5315 - *Computational Methods*

Homework 5- Fall 2011

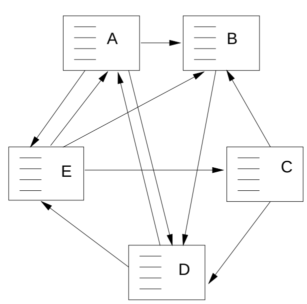
Due Date: Dec. 11 2011

This homework assignment is optional and the final homework grade will be calculated as the better of the assignments with or without this homework. Problems marked with * are required only for students of CSE 5315 but will be graded for extra credit for students of CSE 4345.

Eigenvalues and Eigenvectors

1. Power Iteration for Eigenvalue Problems In Homework 2 we used LU factorization to solve a simplified version of the Page rank procedure for ranking web pages based on their interconnectivity. In general this was defined in terms of the link matrix L, modeling the link structure of a set of web pages, and a popularity of the web pages, $\vec{r}$. Using this, the popularity (rank) of the pages was represented as $L\vec{r} = \vec{r}$ which we converted into a root finding problem. However, in this form it is more appropriately an eigenvector problem with the popularity being an eigenvector with eigenvalue 1 (which, since all the columns add up to 1) turns out to be the largest eigenvalue of L.

   Again consider the link graph from Homework 2:

   a) Implement power iteration with normalization for this problem and use it to determine the page rank vector $\vec{r}$ for the web pages in the link graph

   b) Besides determining the page rank (popularity) for the pages as the eigenvector with the largest eigenvalue, we can also ask the question if there are other eigenvalues/eigenvectors by reformulating the problem into $L\vec{q} = \lambda\vec{q}$. While the other eigenvalues and eigenvectors do not determine the page rank, they can provide important information about the scalability of the eigenvalue problem and the robustness of the page rank with respect to attempts at manipulating the rank of pages through spurious links.

      Implement Inverse iteration and use it to determine the eigenvector with the smallest eigenvalue (and the corresponding eigenvalue). You can use an existing implementation of matrix inversion or equation solving to compute the inverse needed for Inverse iteration.

c) Implement Rayleigh Quotient iteration and apply it to the problem. Compare the convergence rate of Rayleigh iteration with the one of Inverse iteration.

d)* One of the most significant eigenvalues (besides the largest one) is actually the second largest one which can not be determined using Power iteration or Inverse iteration. However, deflation with Householder transforms can be used together with Power iteration to successively derive the largest eigenvalue/eigenvector, then the second largest, and so on.

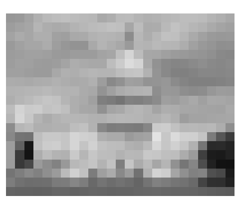
Implement deflation with Householder transforms for the above problem and apply it, together with Power iteration to determine all the real-valued eigenvalues and the corresponding eigenvectors for the link matrix corresponding to the above link graph.

Singular Values

2. Singular Value Decomposition Singular Value Decomposition (SVD) can be used for noise reduction or compression by reducing the resulting system to the one described by the main singular values (and the corresponding left and right singular vectors).

One application of this is in image (or generally sensor data) compression where an image (or set of sensor data sequences) can be reduced to its major components and only those are transmitted (resulting in some image loss but a smaller image).

For this question, consider very low resolution grayscale images of 25x20 pixels (2 samples are provided on the course site but you can use your own if you want.), represented as a matrix of pixel values (Note: the images are in ASCII PGM format which basically consists of a header of the form ”P2 width height maxpixval” followed by the grayscale values (between 0 and maxpixval) of all the pixels in the image. This makes it easy to transform into a matrix - and to transform a matrix into an image)).




a) Use the fact that singular values are the eigenvalues of AA^T and that left singular vectors and right singular vectors are the eigenvectors of AA^T and A^TA, respectively to determine the largest singular value and corresponding left and right singular vectors for the two images of your choice.

As part of this, implement your procedure to determine these values (you can use power iteration for this) and show the image resulting from the first singular value and the corresponding singular vectors.

b) Implement SVD using eigenvalue determination on AA^T and show the image representation for different numbers of singular values that are preserved. For this you can use an existing implementation of QR iteration to determine all the eigenvectors of AA^T, i.e. all singular values.

c)* Implement SVD for this problem using your own implementation of eigenvalue and eigenvector determination. You can implement either simultaneous iteration (such as QR iteration) or use Power iteration with Householder deflation.