Name:	ID#:

CSE 4345 / CSE 5315 - *Computational Methods*

Practice Exam 3- Fall 2011

Closed book, 6 pages of notes

Date: Dec. 13 2011, 2:00 pm - 4:30 pm

Problems marked with * are required only for students of CSE 5315 but will be graded for extra credit for students of CSE 4345.

Eigenvalues and Singular Values

1. Inverse iteration computes the smallest eigenvalue and the corresponding eigenvector by computing the largest eigenvalue of A^{-1}. Show one iteration of normalized inverse iteration for the following transformation matrix, A, and starting vector, x_0.

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}, x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2. When computing eigenvectors and eigenvalues, a number of transformations can be applied to the original matrix A without loosing the ability to compute its eigenvalues and eigenvectors.

   a) Shift (the subtracting of a constant from the diagonal terms, i.e. $B = A - \sigma I$) and Similarity Transformations (i.e. transformations of the form $B = T^{-1}AT$) are important problem transformations. Indicate how these transformations influence the eigenvalues and eigenvectors and discuss how they can be used to simplify the eigenvalue problem.

   b) Eigenvalues and eigenvectors of diagonal and triangular matrices are significantly simpler to compute than for general matrices. Discuss why this is the case and how they can be determined.

3. Power iteration and inverse iteration are limited to computing the largest and the smallest eigenvalue (and corresponding eigenvector).

   a) Both, Deflation and Simultaneous Iteration can be used to address this limitation and determine more than just the largest and smallest eigenvectors. Discuss some of the differences of these methods and some of their advantages and disadvantages.

   b)* Simple simultaneous iteration becomes ill-conditioned relatively fast. Discuss why this is the case and how QR factorization in QR iteration addresses this problem.

4. Singular values are important for a number of important problems related to the transformation A and the space it describes. List at least four of the characteristics of the transformation that can be answered using the result of singular value decomposition.

Randomness and Monte Carlo Methods

5. Randomized algorithms can be effective ways to approximate solutions to highly complex problems. To apply them it is important to be able to generate random numbers. There are differnt types of random numbers based on their general properties. List 3 different types of random numbers and indicate one way to generate them for each of them.

6. Pseudo-random number generators can be very sensitive to the parameter settings.

   a) Briefly discuss why this is and what some of the effect of bad parameter choices can be.

   b)* For the following, simple settings of a congruential random number generator, compute the first 5 pseudo-random numbers for a seed of 4. Base $m = 100$, Multiplier $a = 17$, Shift $c = 0$.

Errors

7. Briefly discuss the difference between stability and sensitivity. What do they measure and what are they influenced by ?

8. For the following equations which suffer from a loss of significance (cancellation) for $x \to 0$, provide a reformulation that avoids this problem.

   a) $\frac{(x^2-4)^2-16}{x^4}$

   b) $\frac{\sqrt{x+9}-3}{2x}$

Root Finding

9. Briefly explain the operation of the interval bisection method for equation solving with single equations in one variable. Illustrate the operation by performing the first two iterations of the bisection method on the function $f(x) = x^2 - 2x + 1$ starting with the bracket $[0, 2]$. Show the new points, their values, and the brackets for each of the four iterations.

10. The Multivariate Newton method solves a system of nonlinear equations by iteratively solving a sequence of linear systems of equations. List the basic operation steps of the Multivariate Newton method.

Interpolation

11. There are generally an infinite number of possible piecewise quadratic or cubic interpolation functions for a set of data points. Provide a piecewise cubic interpolation for the data points $(0, 1), (1, 1), (2, -2)$. (Note that it does not have to be differentiable or smooth.)

12.* Interpolate the data points of problem 17. using polynomial interpolation with monomial basis functions.

Optimization

13. First and second-order optimality conditions are necessary conditions that allow to characterize optima. In particular, the first order optimality condition states that in a continuous differentiable function, the first derivative of the function at an optimum has to be 0.

    a) Fulfilling the first-order optimality condition is not sufficient for a point to be identified as an optimum. Discuss why this is the case.

    b) How does the second order optimality condition (i.e. information about the Hessian - the second derivatives of the function) address this ?

14. In multi-dimensional optimization, a variety of optimization methods can be used that are variations on Newton's method. Briefly discuss the differences between Newton's method and quasi-Newton methods and list some of the advantages of quasi-Newton methods ?

15. Linear programs are a special type of constrained optimization problem. Briefly discuss what makes a linear programming problem easier to solve than other, more general constrained optimization problems.

16. Define the Lagrange function for the following constrained optimization problem with equality constraints.
Objective function: $f(x, y) = x^2 + y^2 - 2xy + 7$, Constraints: $g_1(x, y) = 7x + 2y = 0$, $g_2(x, y) = x^2 - xy = 0$