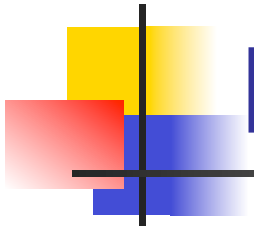
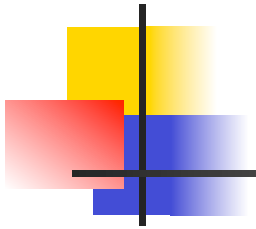


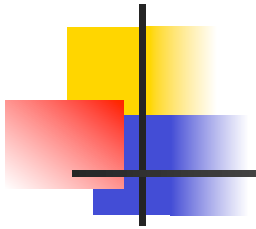
Reasoning with Uncertainty

Fuzzy Logic



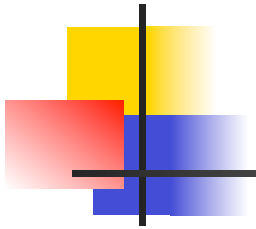
Fuzzy Logic

- Fuzzy Logic is a multivalued logic
  - Rule-based inference system
  - Membership values indicate degree of truth of predicates
  - Fuzzy set operations permit reasoning with membership values
- Fuzzy Logic has been applied very successfully to a number of control problems



Fuzzy Logic - Applications

- Many everyday applications use Fuzzy Logic control
  - Microwaves
  - ABS brakes
  - Camera image stabilization
  - Cruise control
  - Air conditioning control
  - Washing machine control

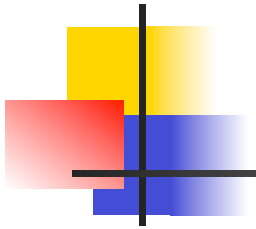


Fuzzy Sets

- A Fuzzy set A is a set of items with membership values
  - A is a subset of the universe (all possible objects)
  - There is a membership function

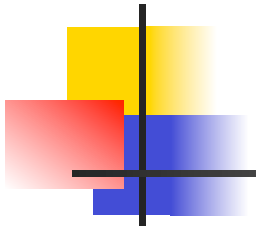
    $$\mu_A(x) \in [0..1]$$

    indicating the degree to which x belongs to set A



Fuzzy Set Operations

- Union of two Fuzzy Sets
  - $\mu_{A \cup B}(x) = \mu_A(x) \oplus \mu_B(x)$
    Often max $(\mu_A(x), \mu_B(x))$
- Intersection of two Fuzzy Sets
  - $\mu_{A \cap B}(x) = \mu_A(x) \otimes \mu_B(x)$
    Often min $(\mu_A(x), \mu_B(x))$
- Inversion of a Fuzzy Set
  - $\mu_{\neg A}(x) = 1 - \mu_A(x)$



Fuzzy Inference (Control)

- Fuzzy Logic uses logic inference rules and defuzzification
  - Inference rules are of the form:

    If a in A and b in B then c in C

    Where *A*, *B*, and *C* are Fuzzy sets, a, b, and c, are elements from the universe of discourse.
  - Multiple rules for the same set are combined using $\oplus$
  - A value for the variable c in C is extracted by defuzzification - Often as the center of mass of the membership function

Fuzzy Inference (Control)

- Inference rules derive a membership function for the resulting fuzzy set
  - Rule 1: If a in A and b in B then c in C
    - Results in a membership function for c which is limited by the degree of truth of the rule's antecedents and the membership function for

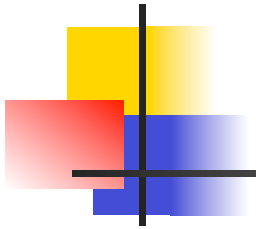
      $\mu_{Rule\ 1}(c) = min\ (\ \mu_A(a) \otimes \mu_B(b)\ ,\ \mu_C(c)\ )$
  - All inference rules that have the same variable as a consequent

    $\mu_{Result}(c) = \mu_{Rule\ 1}(c) \oplus \mu_{Rule\ 2}(c) \oplus \ldots$

Fuzzy Inference (Control)

- A value for the variable in the consequent is derived from the resulting membership profile using defuzzification
  - Defuzzification often uses the center of mass of the membership function
    - *c is the point where*

$$\int_{-\infty}^{c} \mu_{Result}(x)\,dx = \int_{c}^{\infty} \mu_{Result}(x)\,dx$$



Fuzzy Logic

- Advantages
  - Simple inference system
  - Easy to design
  - Good for simple control
- Problems
  - Problems with strings of inference
  - Non-symmetric inference
  - Difficulty interpreting resulting membership values.