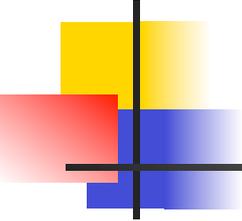


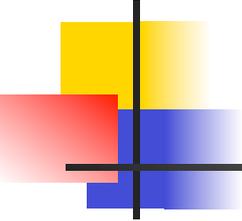
Reasoning with Uncertainty

Probability



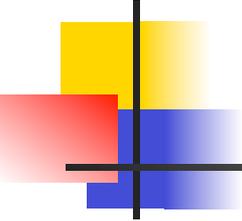
Probability

- Bayesian probabilities summarize the effects of uncertainty on the state of knowledge
 - Probabilities represent the values of statistics
 - $P(o) = (\# \text{ of times of outcome } o) / (\# \text{ of outcomes})$
 - All types of uncertainty are incorporated into a single number
$$P(H | E)$$
 - Probabilities follow a set of strict axioms



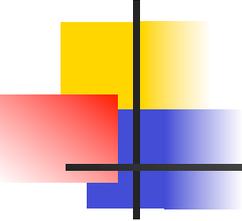
Probability

- Random variables define the entities of probability theory
 - Propositional random variables:
 - E.g.: IsRed, Earthquake
 - Multivalued random variables:
 - E.g.: Color, Weather
 - Potentially Real-Valued
 - E.g.: Height, Weight



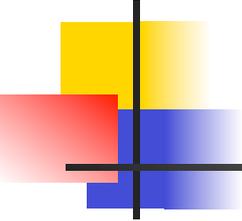
Axioms of Probability

- Probability follows a fixed set of rules
 - Propositional random variables:
 - $P(A) \in [0..1]$
 - $P(T) = 1, P(F) = 0$
 - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
 - $P(A \wedge B) = P(A) P(B|A)$
 - $\sum_{x \in \text{Values}(X)} P(X=x) = 1$



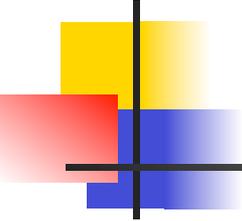
Probability Syntax

- Unconditional or prior probabilities represent the state of knowledge before new observations or evidence
 - $P(H)$
- A probability distribution gives values for all possible assignments to a random variable
- A joint probability distribution gives values for all possible assignments to all random variables



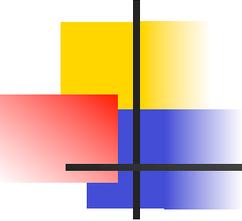
Conditional Probability

- Conditional probabilities represent the probability after certain observations or facts have been considered
 - $P(H/E)$ is the posterior probability of H after evidence E is taken into account
 - Bayes rule allows to derive posterior probabilities from prior probabilities
 - $P(H | E) = P(E | H) P(H)/P(E)$



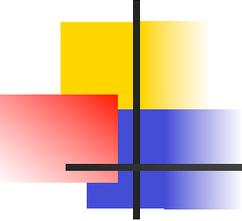
Conditional Probability

- Probability calculations can be conditioned by conditioning all terms
 - Often it is easier to find conditional probabilities
- Conditions can be removed by marginalization
 - $P(H) = \sum_E P(H|E) P(E)$



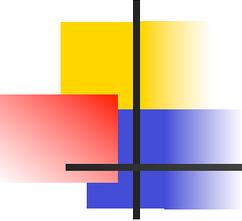
Joint Distributions

- A joint distribution defines the probability values for all possible assignments to all random variables
 - Exponential in the number of random variables
 - Conditional probabilities can be computed from a joint probability distribution
 - $P(A/B) = P(A \wedge B)/P(B)$



Inference

- Inference in probabilistic representation involves the computation of (conditional) probabilities from the available information
 - Most frequently the computation of a posterior probability $P(H/E)$ from a prior probability $P(H)$ and new evidence E



Probabilistic Inference

- Benefits:
 - Precise quantitative measure of uncertainty
 - Inference can be automated
- Problems:
 - *Worst case time complexity:*
 - $O(d^n)$: d = arity, n = #of random variables
 - *Worst case space complexity:*
 - $O(d^n)$