Reasoning with Uncertainty

Representing Uncertainty

Reasoning with Uncertainty

- The goal of reasoning is usually to:
  - Determine the state of the world
  - Determine what actions to take
  - Determine the how the world behaves
- Uncertainty makes reasoning more complex
  - Multiple possible states
  - Multiple possible outcomes or conclusions
  - Simple logical representations are often insufficient to represent uncertain information

Traditional Reasoning

- Traditional reasoning methods
  - Formal reasoning:
    - Rule-based inference
    - Logic knowledge representations
  - Procedural reasoning
    - Use procedures that specify how to solve problems
  - Reasoning by analogy
    - Infer solution from similar instances (very difficult for computers)

Uncertainty

- Sources of Uncertainty:
  - Noise:
    - Observations of the environment are not precise
  - Uncertain change:
    - The world does not behave deterministically
    - No single successor state can be predicted
  - Incompleteness ("ignorance"):
    - Information is missing
- Uncertainty leads to inconsistencies in many traditional reasoning systems

Example Problems

- Determine the state of the world
  - Given a set of observations, determine what state (or situation) the world (or part of it) is in.
- Predict the outcome of an action
  - Given the state (or situation) of the world, determine what effect an action would have.
- Determine optimal actions
  - Given the state (or situation) of the world, what actions would lead to the best outcome.

Note: The results of reasoning with uncertainty are not always intuitive

Uncertainty Reasoning Methods

- Symbolic Methods
  - Non-monotonic reasoning
    - Use logic framework but accept inaccuracies. E.g.:
      - Ignore unlikely possibilities
      - Draw conclusions that are merely possible
- Statistical Methods
  - Certainty factors
    - Use logic rules augmented with measures of certainty
  - Probability
  - Dempster-Shafer
    - Reason with Belief and Plausibility
- Fuzzy Logic Methods

Symbolic Methods

- Extend traditional logic to non-monotonic logics:
  - Addition of knowledge does not strictly increase the set of known things and thus does not strictly reduce the set of possible consequences
- Extension can take multiple forms:
  - Allow for changing sets of facts and conclusions
    - Default assumptions about unknown facts
    - Changing assumptions due to new information
  - Allow for additional belief in truth values:
    - True, False, neither True nor False / True and False
  - Include information about knowledge of facts

Symbolic Methods

- Examples:
  - Default reasoning
    - Include default assumptions about unknown facts
  - Abductive reasoning
    - Allow to infer the most likely explanations
  - Autoepistemic logic
    - Reason explicitly about the knowledge behind facts
  - Belief revision
    - Dynamically change the belief / available facts
  - Paraconsistent logics
    - Allow for facts to be true and false at the same time

Symbolic Methods

- Advantages:
  - Use of a well defined reasoning and inference framework
  - Mostly well understandable and interpretable steps
  - Simple mechanism
- Problems:
  - No quantitative notion of uncertainty
  - Chains of inferences potentially loose consistency and order can not be changed

Statistical Methods: Certainty Factors

- Proposed for the MYCIN system
  - Augment rules with certainty factors
  - Perform logic inference while tracking certainty facors
- Certainty Factors
  - A measure of belief and disbelief

    $B(H \mid E), D(H \mid E)$

    $C(H \mid E) = B(H \mid E) - D(H \mid E)$
  - Inference maintains both belief measures
    - Evidence is combined as in probability
    - Hypotheses are combined using an independence assumption

Statistical Methods: Certainty Factors

- Advantages:
  - Rule-based inference mechanism
  - Follows probabilistic rules
  - More tractable than general probaility
- Problems
  - Rules have to be independent (very difficult to do in practice)
  - Certainty factors have to be available

Statistical Methods: Probability

- Probability distributions over states represent the current situation of the world
- Conditional probabilities are used as models of the world
- Bayes theorem allows probabilistic inference:

  $P(H|E) = P(E \mid H) * P(H) / P(E)$
- Advantages:
  - Quantitative measure of uncertainty
- Problems:
  - High complexity of inference

Statistical Methods: Dempster-Shafer

- Distinguishes between incompleteness ("ignorance") and other uncertainty
- Represent Belief and Plausibility
  - Belief: represents all evidence available for a hypothesis
  - Plausibility: represents all evidence that is compatible and not inconsistent with the hypothesis
- Belief and Plausibility represent an interval within which the true probability lies with a certain confidence

Statistical Methods: Dempster-Shafer

- Advantages:
  - Represents the actual state of knowledge more precisely
  - Distinguishes randomness from missing information
- Problems:
  - Very complex mathematically
  - Has to be calculated over all possible sets of states

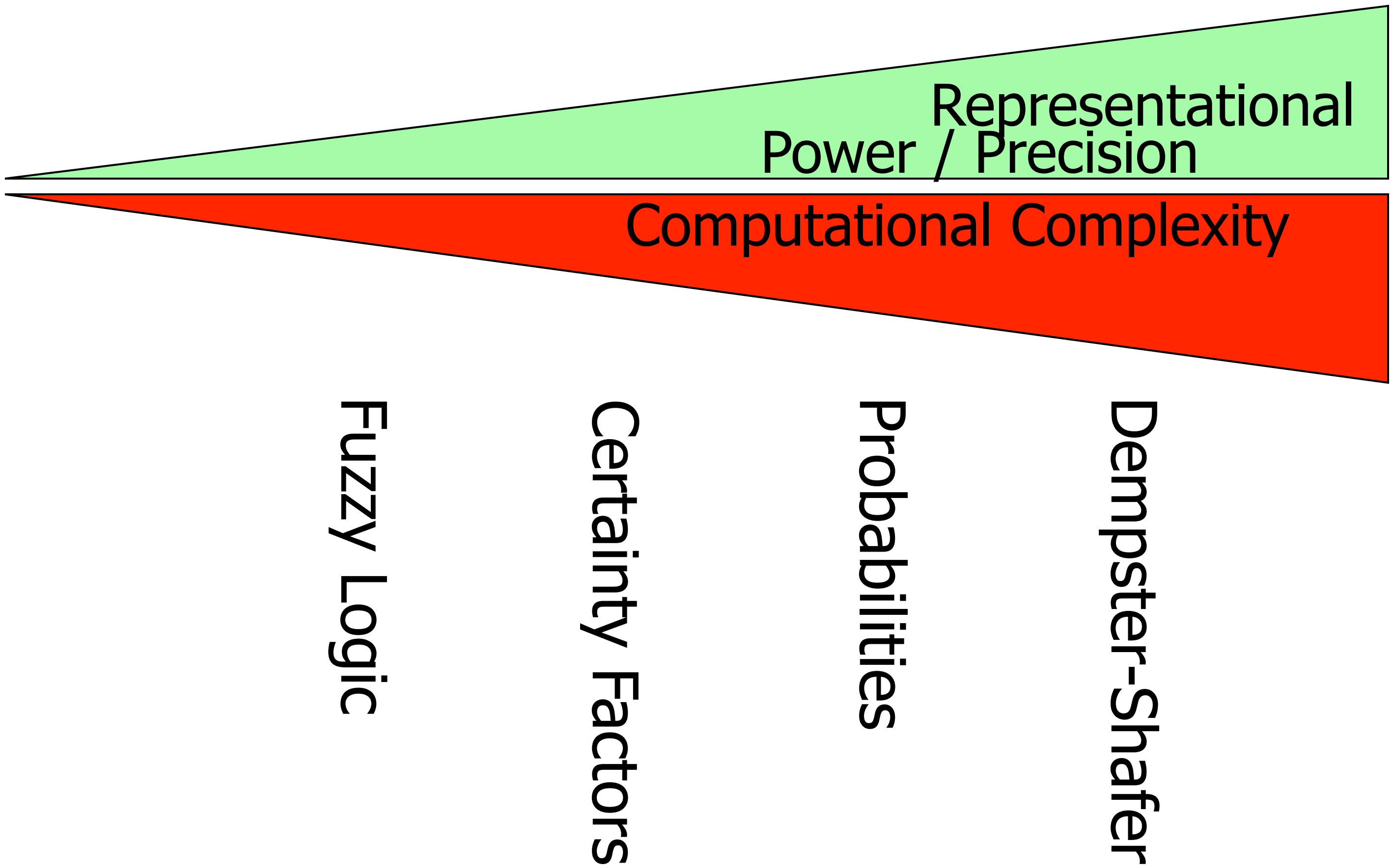
Fuzzy Methods: Fuzzy Logic

- Fuzzy Logic models the degree of truth or membership to a class.
  - Redefines set theory with a real-valued membership function
    - Membership function $\mu_A(x)$ represents the degree to which x belongs to set A
    - Set operations are defined as:
      - $\mu_{A \cup B}(x) = \mu_A(x) \oplus \mu_B(x)$ (often defined as *max*)
      - $\mu_{A \cap B}(x) = \mu_A(x) \otimes \mu_B(x)$ (often defined as *min*)
  - Inference through the application of rules and defuzzification
    - Defuzzification extracts a value from a membership function (usually as the weighted mean or center of mass)

Fuzzy Methods: Fuzzy Logic

- Advantages:
  - Easy to design
  - Relatively intuitive rules
  - Relatively robust controllers
- Problems:
  - Longer inference chains can be problematic
  - The order of inference steps matters
  - After inference it can be difficult to exactly interpret the membership value

Choosing a Mechanism



Choosing a Mechanism

- The choice of reasoning mechanism depends on the given requirements and the problem
  - Fuzzy logic works well for simple control problems that do not require precise knowledge of uncertainty
  - Certainty factors work well if rules can be engineered to be independent and if certainties are available
  - Probability is best for situations where a precise measure of uncertainty is required
  - Dempster-Shafer provides additional information about the degree to which information is available

Choosing a Mechanism

- Bayesian Probability is the most commonly used and best developed mechanism
  - Formal mathematical framework exists
  - Different modeling mechanisms are available
  - Optimal modeling and decision methods exist

This course will largely focus on this framework for the representation of uncertainty.