CSE 6369 - *Multiagent Systems*

Homework 1- Spring 2018

Due Date: Feb. 21 2018

Utility Theory and Rational Decisions

1. Consider the following game of chance in a casino: To enter the game, a player has to pay $\$x$ and put \$1 on the table. He then rolls two dice repeatedly until they end up with an even score (i.e. both dice add up to an even number) and the player wins the money on the table. Every time the dice show an odd score, the bank doubles the money on the table.

   a) If the player has $\$y$ when entering the casino, can play only once, and wants to maximize the expected amount of money that he leaves the casino with, what is the rational upper limit on the cover charge x that he should be willing to pay ? (include the rationale/proof for your answer)

   b) We saw in class that for humans money does not behave like a utility function. If we assume that the utility of $\$z$ of money for the player behaves logarithmically according to $u(z) = \frac{1}{y}\log(z)$, what would the rational upper limit for the cover charge x be ? (include the rationale/proof for your answer)

Games in Normal Form and Equilibria

2. Two teenagers decide to play a game of chicken on their bikes. On a narrow path they race towards each other deciding that whoever moves off the path first is chicken and thus looses. However, if none of them deviated, they will crash and injure themselves. Assuming there is only one place where the two bikes can move off the path, this yields the following two player game:

	go straigth	turn
go straight	-10,-10	2,-1
turn	-1, 2	0, 0

   a) Find all pure strategy Nash equilibria.

   b) Find all mixed strategy Nash Equilibria.

   c) Find all pure pareto optimal strategy profiles.

   d) Calculate the minmax strategy for player 1.

3. Provide the linear programs for the Nash equilibrium strategies for the two players in the following zero-sum game:

	A	B	C
D	-5,5	-2,2	2,-2
E	-1, 1	-4,4	2,-2
F	-2, 2	-3,3	4,-4

4. Consider the following game:

	A	B	C
D	-5,5	-2,2	2,-2
E	-1, 3	-1,2	2,-2
F	-2,-1	-2,3	4,-4

   a) Iteratively eliminate all strictly dominated pure strategies.
   b) Determine the Nash equilibria for the reduced game resulting from the iterated elimination in part a).

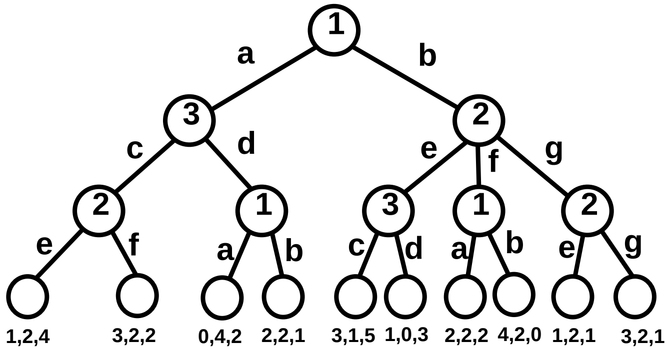
5. Consider the following game:

	A	B	C
D	-2,1	6,2	-2,-1
E	-2, 3	-1,3	2,6
F	2, -1	3,1	2,-2

   a) Derive a linear program to compute a correlated equilibrium for this game
   b) Show that [0.5:(D,B) ; 0.5:(E,C)] is a correlated equilibrium.
   c) Show that [[0.5:D ; 0.5:E] ; [0.5:B ; 0.5:C]] is not a Nash equilibrium.

Extensive Form Games

6. Consider the following three-player game in extensive form:



a) List all pure strategies for the three agents.

b) Derive the induced normal form for this game. (Note: you can list separate tables for each of the strategies of agent 3)

c) Use the induced normal form game from part b) to derive a Nash equilibrium for this game.

d) Use backward induction to derive a subgame perfect Nash equilibrium for the extensive form game.