



Reasoning with Uncertainty

Fuzzy Logic



Fuzzy Logic

- Fuzzy Logic is a multivalued logic
 - Rule-based inference system
 - Membership values indicate degree of truth of predicates
 - Fuzzy set operations permit reasoning with membership values
- Fuzzy Logic has been applied very successfully to a number of control problems



Fuzzy Logic - Applications

- Many everyday applications use Fuzzy Logic control
 - Microwaves
 - ABS brakes
 - Camera image stabilization
 - Cruise control
 - Air conditioning control
 - Washing machine control



Fuzzy Sets

- A Fuzzy set A is a set of items with membership values
 - A is a subset of the universe (all possible objects)
 - There is a membership function

$$\mu_A(x) \in [0..1]$$

indicating the degree to which x belongs to set A



Fuzzy Set Operations

- Union of two Fuzzy Sets
 - $\mu_{A \cup B}(x) = \mu_A(x) \oplus \mu_B(x)$
Often $\max(\mu_A(x), \mu_B(x))$
- Intersection of two Fuzzy Sets
 - $\mu_{A \cap B}(x) = \mu_A(x) \otimes \mu_B(x)$
Often $\min(\mu_A(x), \mu_B(x))$
- Inversion of a Fuzzy Set
 - $\mu_{\neg A}(x) = 1 - \mu_A(x)$



Fuzzy Inference (Control)

- Fuzzy Logic uses logic inference rules and defuzzification

- Inference rules are of the form:

If a in A and b in B then c in C

Where A , B , and C are Fuzzy sets, a , b , and c , are elements from the universe of discourse.

- Multiple rules for the same set are combined using \oplus
- A value for the variable c in C is extracted by defuzzification - Often as the center of mass of the membership function



Fuzzy Inference (Control)

- Inference rules derive a membership function for the resulting fuzzy set
 - Rule 1: If a in A and b in B then c in C
 - Results in a membership function for c which is limited by the degree of truth of the rule's antecedents and the membership function for
- All inference rules that have the same variable as a consequent

$$\mu_{Result}(c) = \mu_{Rule\ 1}(c) \oplus \mu_{Rule\ 2}(c) \oplus \dots$$



Fuzzy Inference (Control)

- A value for the variable in the consequent is derived from the resulting membership profile using defuzzification
 - Defuzzification often uses the center of mass of the membership function
 - *c is the point where*

$$\int_{-\infty}^c \mu_{Result}(x) dx = \int_c^{\infty} \mu_{Result}(x) dx$$



Fuzzy Logic

- Advantages

- Simple inference system
- Easy to design
- Good for simple control

- Problems

- Problems with strings of inference
- Non-symmetric inference
- Difficulty interpreting resulting membership values.