

CSE 6392 - Reasoning with Uncertainty

Homework 2- Fall 2006

Due Date: Nov. 7 2006, 7:00 pm

Monte Carlo Estimation

1. You are given the following Markov process and observation model:

State space: $S = \{1, 2, 3, 4, 5, 6\}$

$$\text{Transition probabilities: } P(s|s') = T_{s,s'}, T = \begin{pmatrix} 0.4 & 0.6 & 0.0 & 0.0 & 0.0 & 0.3 \\ 0.3 & 0.2 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.2 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.2 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.2 & 0.3 \\ 0.3 & 0.0 & 0.0 & 0.0 & 0.6 & 0.4 \end{pmatrix}$$

Observations: $O = \{a, b, c, d\}$

$$\text{Observation probabilities: } P(o|s) = B_{o,s}, B = \begin{pmatrix} 0.2 & 0.0 & 0.0 & 0.9 & 0.0 & 0.0 \\ 0.7 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 & 0.0 & 0.8 & 0.7 \\ 0.0 & 0.1 & 0.8 & 0.0 & 0.1 & 0.2 \end{pmatrix}$$

- a) Show three iterations of a particle filter with 15 samples for the following conditions:
 Start state: $s_0 = 3$ (with probability 1)
 Observed sequence (the first observation is made after the first time step - i.e. after the first model update): d, c, b, a
- b) Use the result of your filter to determine an estimate for the expected value of the properties $f(s)$ and $g(s)$ (interpreting the numbers representing states as continuous values) after 4 model steps.
 $f(s) = s, g(s) = (s - \hat{s})^2$, where \hat{s} is the expected value for the state in the given distribution (i.e. $E[g(s)]$ is (approximately) the variance of the distribution).

Hidden Markov Models

2. You are given the following Hidden Markov Models for two differently biased coins:

Model 1:

State space: $S_1 = \{s_1, s_2\}$

Observations: $O_1 = \{H, T\}$

$$\text{Transition probabilities: } P_1(s_i|s_j) = T_{i,j}, T = \begin{pmatrix} 0.3 & 0.3 \\ 0.7 & 0.7 \end{pmatrix}$$

Observation probabilities: $P_1(o|s) = B_{o,s}, B = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$

Prior probabilities: $\pi_1(s) = \Pi_s, \Pi = \begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$

Model 2:

State space: $S_2 = \{s_1, s_2\}$

Observations: $O_2 = \{H, T\}$

Transition probabilities: $P_2(s_i|s_j) = T_{i,j}, T = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$

Observation probabilities: $P_2(o|s) = B_{o,s}, B = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$

Prior probabilities: $\pi_2(s) = \Pi_s, \Pi = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

- Use the Forward algorithm to determine which of the two models better describes an actual coin used which produced the observation sequence $H, T, H, H, H, T, T, T, H, T, T$. List all the α values involved in the calculations.¹
3. Build a Hidden Markov Model for the following (unrealistically simplified) problem of detecting break-ins in a computer network:

The state of the network can be either normal or hacked and the only observable information is the average bandwidth utilization within the network. This information is provided once every 30 minutes and, based on past data, it falls into 5 groups: $a : 0 - 5\%$, $b : 5 - 15\%$, $c : 15 - 30\%$, $d : 30 - 60\%$, and $e : 60 - 100\%$.

Previously measurements were taken during normal operation and while the network was compromised and it was observed that bandwidth usage observations were made with the following frequencies:

- Normal Network: $a : 20\%$ of the time, $b : 30\%$ of the time, $c : 30\%$ of the time, $d : 10\%$ of the time, and $e : 10\%$ of the time.
- Compromised Network: $a : 10\%$ of the time, $b : 20\%$ of the time, $c : 40\%$ of the time, $d : 20\%$ of the time, and $e : 10\%$ of the time.

Further study of these past events also revealed that a a break-in in the network is happening on average every 5 hours and lasts for an average of 1.5 hours before the intruder leaves.

- a) Build a Hidden Markov Model that represents the scenario described above. Assume that at the beginning of the observations the network is operating normally.
- b) Given the following observation sequence $a, d, b, c, d, b, c, d, c, d, a, b$, taken between 6:00 pm and 12:00 am, determine if there were likely any break-ins by determining the most likely interpretation (i.e. corresponding state sequence) using the Viterbi algorithm. List the intermediate values (δ and Ψ) of the Viterbi algorithm.¹

¹Rather than computing all the values by hand it might be simpler to write a short program to compute the values.