

RATIONAL DECISIONS

CHAPTER 16

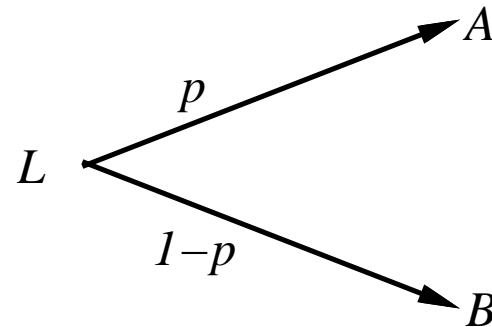
Outline

- ◇ Rational preferences
- ◇ Utilities
- ◇ Money
- ◇ Multiattribute utilities
- ◇ Decision networks
- ◇ Value of information

Preferences

An agent chooses among prizes (A , B , etc.) and lotteries, i.e., situations with uncertain prizes

Lottery $L = [p, A; (1 - p), B]$



Notation:

- $A \succ B$ A preferred to B
- $A \sim B$ indifference between A and B
- $A \not\succeq B$ B not preferred to A

Rational preferences

Idea: preferences of a rational agent must obey constraints.

Rational preferences \Rightarrow

behavior describable as maximization of expected utility

Constraints:

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p [p, A; 1 - p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A; 1 - p, C] \sim [p, B; 1 - p, C]$$

Monotonicity

$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; 1 - p, B] \succsim [q, A; 1 - q, B])$$

Rational preferences contd.

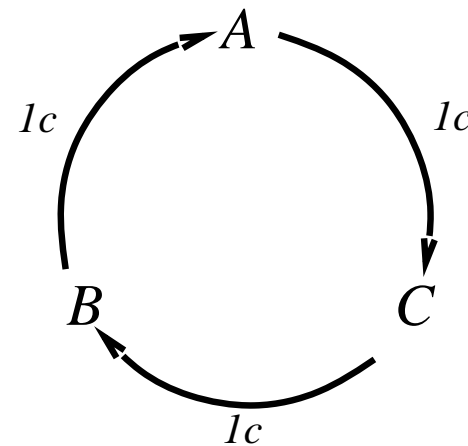
Violating the constraints leads to self-evident irrationality

For example: an agent with intransitive preferences can be induced to give away all its money

If $B \succ C$, then an agent who has C would pay (say) 1 cent to get B

If $A \succ B$, then an agent who has B would pay (say) 1 cent to get A

If $C \succ A$, then an agent who has A would pay (say) 1 cent to get C



Maximizing expected utility

Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):

Given preferences satisfying the constraints
there exists a real-valued function U such that

$$U(A) \geq U(B) \Leftrightarrow A \succsim B$$
$$U([p_1, S_1; \dots; p_n, S_n]) = \sum_i p_i U(S_i)$$

MEU principle:

Choose the action that maximizes expected utility

Note: an agent can be entirely rational (consistent with MEU)
without ever representing or manipulating utilities and probabilities

E.g., a lookup table for perfect tictactoe

Utilities

Utilities map states to real numbers. Which numbers?

Standard approach to assessment of human utilities:

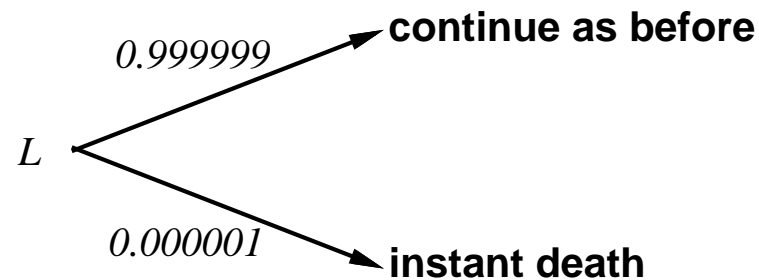
compare a given state A to a **standard lottery** L_p that has

“best possible prize” u_{\top} with probability p

“worst possible catastrophe” u_{\perp} with probability $(1 - p)$

adjust lottery probability p until $A \sim L_p$

pay \$30 \sim



Money

Money does **not** behave as a utility function

Given a lottery L with expected monetary value $EMV(L)$, usually $U(L) < U(EMV(L))$, i.e., people are risk-averse

Utility curve: for what probability p am I indifferent between a prize x and a lottery $[p, \$M; (1 - p), \$0]$ for large M ?

Typical empirical data, extrapolated with risk-prone behavior:

