



Reasoning with Uncertainty

Representing Uncertainty



Reasoning with Uncertainty

- The goal of reasoning is usually to:
 - Determine the state of the world
 - Determine what actions to take
 - Determine the how the world behaves
- Uncertainty makes reasoning more complex
 - Multiple possible states
 - Multiple possible outcomes or conclusions
 - Simple logical representations are often insufficient to represent uncertain information



Traditional Reasoning

- Traditional reasoning methods
 - Formal reasoning:
 - Rule-based inference
 - Logic knowledge representations
 - Procedural reasoning
 - Use procedures that specify how to solve problems
 - Reasoning by analogy
 - Infer solution from similar instances (very difficult for computers)



Uncertainty

- Sources of Uncertainty:
 - Noise:
 - Observations of the environment are not precise
 - Uncertain change:
 - The world does not behave deterministically
 - No single successor state can be predicted
 - Incompleteness (“ignorance”):
 - Information is missing
- Uncertainty leads to inconsistencies in many traditional reasoning systems



Example Problems

- Determine the state of the world
 - Given a set of observations, determine what state (or situation) the world (or part of it) is in.
- Predict the outcome of an action
 - Given the state (or situation) of the world, determine what effect an action would have.
- Determine optimal actions
 - Given the state (or situation) of the world, what actions would lead to the best outcome.

Note: The results of reasoning with uncertainty are not always intuitive



Uncertainty Reasoning Methods

- Symbolic Methods
 - Non-monotonic reasoning
 - Use logic framework but accept inaccuracies. E.g.:
 - Ignore unlikely possibilities
 - Draw conclusions that are merely possible
- Statistical Methods
 - Certainty factors
 - Use logic rules augmented with measures of certainty
 - Probability
 - Dempster-Shafer
 - Reason with Belief and Plausibility
- Fuzzy Logic Methods



Symbolic Methods

- Extend logic reasoning with belief as
 - True, False, or neither True or False (unknown)
- Advantages:
 - Use of a well defined reasoning and inference framework
 - Simple mechanism
- Problems:
 - No quantitative notion of uncertainty
 - Chains of inferences potentially loose consistency and order can not be changed



Statistical Methods: Certainty Factors

- Proposed for the MYCIN system
 - Augment rules with certainty factors
 - Perform logic inference while tracking certainty factors
- Certainty Factors
 - A measure of belief and disbelief
 - $B(H | E)$, $D(H | E)$
 - $C(H | E) = B(H | E) - D(H | E)$
 - Inference maintains both belief measures
 - Evidence is combined as in probability
 - Hypotheses are combined using an independence assumption



Statistical Methods: Certainty Factors

- Advantages:
 - Rule-based inference mechanism
 - Follows probabilistic rules
 - More tractable than general probability
- Problems
 - Rules have to be independent (very difficult to do in practice)
 - Certainty factors have to be available



Statistical Methods: Probability

- Probability distributions over states represent the current situation of the world
- Conditional probabilities are used as models of the world
- Bayes theorem allows probabilistic inference:
$$P(H|E) = P(E | H) * P(H) / P(E)$$
- Advantages:
 - Quantitative measure of uncertainty
- Problems:
 - High complexity of inference



Statistical Methods: Dempster-Shafer

- Distinguishes between incompleteness (“ignorance”) and other uncertainty
- Represent Belief and Plausibility
 - Belief: represents all evidence available for a hypothesis
 - Plausibility: represents all evidence that is compatible and not inconsistent with the hypothesis
- Belief and Plausibility represent an interval within which the true probability lies with a certain confidence



Statistical Methods: Dempster-Shafer

- Advantages:
 - Represents the actual state of knowledge more precisely
 - Distinguishes randomness from missing information
- Problems:
 - Very complex mathematically
 - Has to be calculated over all possible sets of states



Fuzzy Methods: Fuzzy Logic

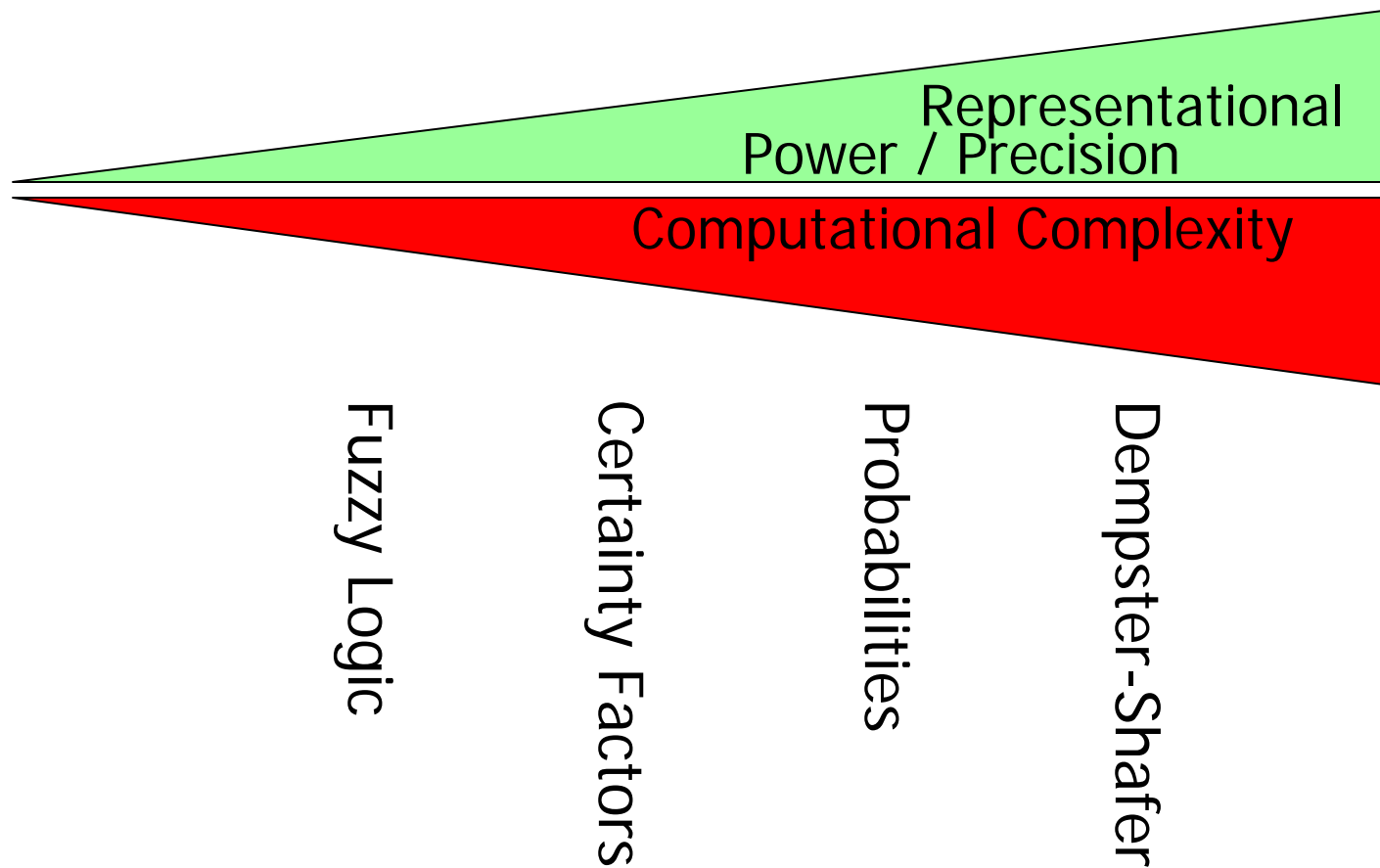
- Fuzzy Logic models the degree of truth or membership to a class.
 - Redefines set theory with a real-valued membership function
 - Membership function $\mu_A(x)$ represents the degree to which x belongs to set A
 - Set operations are defined as:
 - $\mu_{A \cup B}(x) = \mu_A(x) \oplus \mu_B(x)$ (often defined as *max*)
 - $\mu_{A \cap B}(x) = \mu_A(x) \otimes \mu_B(x)$ (often defined as *min*)
 - Inference through the application of rules and defuzzification
 - Defuzzification extracts a value from a membership function (usually as the weighted mean or center of mass)



Fuzzy Methods: Fuzzy Logic

- Advantages:
 - Easy to design
 - Relatively intuitive rules
 - Relatively robust controllers
- Problems:
 - Longer inference chains can be problematic
 - The order of inference steps matters
 - After inference it can be difficult to exactly interpret the membership value

Choosing a Mechanism





Choosing a Mechanism

- The choice of reasoning mechanism depends on the given requirements and the problem
 - Fuzzy logic works well for simple control problems that do not require precise knowledge of uncertainty
 - Certainty factors work well if rules can be engineered to be independent and if certainties are available
 - Probability is best for situations where a precise measure of uncertainty is required
 - Dempster-Shafer provides additional information about the degree to which information is available



Choosing a Mechanism

- Bayesian Probability is the most commonly used and best developed mechanism
 - Formal mathematical framework exists
 - Different modeling mechanisms are available
 - Optimal modeling and decision methods exist

This course will largely focus on this framework for the representation of uncertainty.