Computer Graphics
Spring 2014 Midterm

NAME:

Prob #	1	2	3	4	5	6
Points	12	16	12	20	20	20

Time: 80 Minutes

NOTES:

a. **Credit is only given to the correct numerical values.**
b. **All numerical values must be calculated with three digits of accuracy after the decimal point.**
c. **Do not write on the back side of the papers.**

1. Equation of a line is given as: $\begin{cases} x(t) = 4t - 1 \\ y(t) = 5t \\ z(t) = 3t - 7 \end{cases}$

Find the equation of this line after it has been rotated 90 degrees around the z axis.

Rotation around Z matrix $R_z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotating direction vector: $\overrightarrow{N'} = R_z \cdot \vec{N} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 3 \end{bmatrix}$

Rotating reference point: $\overrightarrow{P'} = R_z \cdot \vec{P} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ -7 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -7 \end{bmatrix}$

Rotated line equation = $\begin{cases} \boldsymbol{x(t) = -5t} \\ \boldsymbol{y(t) = 4t - 1} \\ \boldsymbol{z(t) = 3t - 7} \end{cases}$

2. Line AB and point C(1,2,4) are on the same plane. Equation of the line is given as:

$$\begin{cases} x(t) = 4t - 1 \\ y(t) = 5t \\ z(t) = 3t - 7 \end{cases}$$

Find the equation of this plane

Two points on AB: $t = 0 \rightarrow P = (-1,0,-7)$ and $t = 1 \rightarrow Q = (3,5,-4)$

$$PC = C - P = \begin{bmatrix} 2 \\ 2 \\ 11 \end{bmatrix} \text{ and } QC = C - Q = \begin{bmatrix} -2 \\ -3 \\ 8 \end{bmatrix}$$

$$\vec{N} = PC \times QC = \begin{bmatrix} 16 - (-33) \\ -22 - (-16) \\ -6 - (-4) \end{bmatrix} = \begin{bmatrix} 49 \\ -38 \\ -2 \end{bmatrix}$$

Plane equation: $49x - 38y - 2z + D = 0$,
Put P in the equation: $49 \times -1 - 38 \times 0 - 2 \times -7 + D = 0 \rightarrow D = 35$
Plane equation $\mathbf{49x - 38y - 2z + 35 = 0}$

3. Point **A(-4, 12)** is given in a two dimensional world coordinate system. Find the coordinates of the point A on the screen after it is mapped from window to viewport.
   x_{wmin} = -5 y_{wmin} = 10 x_{wmax} = 15 y_{wmax} = 30
   Normalized device coordinate of the viewport:
   x_{vmin} = 0.1 y_{vmin} = 0.2 x_{vmax} = 0.5 y_{vmax} = 0.7
   The origin of the screen coordinate system is defined in the **upper left** corner of the screen and the screen resolution is 800 by 600.
   Use rounding to convert from float to integer.

$$S_x = \frac{0.5 - 0.1}{15 - (-5)} = 0.02 \rightarrow A_x = [0.1 + 0.02(-4 - (-5))] \times 800 = \mathbf{96}$$
$$S_y = \frac{0.7 - 0.2}{30 - (10)} = 0.025 \rightarrow A_y = [0.2 + 0.025(30 - (12))] \times 600 = \mathbf{390}$$

Screen coordinates of point A after mapping are: $(\mathbf{96}, \mathbf{390})$

4. The viewing parameters for a perspective projection are given as
VRP(WC)=(**0,0,0**) VPN(WC)=(**0, 0,1**)
VUP(WC)=(**0,1,0**) PRP (VRC)=(**5,12,-5**)

u_{min} (VRC) = **-9** u_{max} (VRC) = **11**
v_{min} (VRC) = **-19** v_{max} (VRC) = **-15**
n_{min} (VRC) = **-3.5** n_{max} (VRC) = **-1**

Find the sequence of transformations which will transform this viewing volume into a standard perspective view volume which is bounded by the planes: x=z ; x=-z ; y=z ; y=-z ; z=1 ; z=zmin

a. Find the **Shear matrix** (Matrix #6)
b. Find the **scale matrices** (Matrix #7 and Matrix #8).
c. Find the **zmin** after all transformations are done.

Matrix #2: Rx

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #1: Translate

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #4: Rz

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #3: Ry

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #6: Shear

1	**0**	**0.8**	**0**
0	**1**	**5.8**	**0**
0	**0**	**1**	**0**
0	**0**	**0**	**1**

Matrix #5: Translate

1	0	0	-5
0	1	0	-12
0	0	1	5
0	0	0	1

Scale2

0.25	**0**	**0**	**0**
0	**0.25**	**0**	**0**
0	**0**	**0.25**	**0**
0	**0**	**0**	**1**

Scale1

0.5	**0**	**0**	**0**
0	**2.5**	**0**	**0**
0	**0**	**1**	**0**
0	**0**	**0**	**1**

Zmin= 0.375

5. Equation of a cubic curve is given as:

$x(t) = 2t^3 + 6t^2 - 7t + 10$
$y(t) = t^3 - 12t + 9$
$z(t) = t^3 + 3t^2 - 5$

Find the numerical values of the Hermite geometry vector for this curve.

$$G_{H_x} = [P_{1_x} \quad P_{4_x} \quad R_{1_x} \quad R_{4_x}] = [x(0) \quad x(1) \quad x'(0) \quad x'(1)] = [10 \quad 11 \quad -7 \quad 11]$$

$$G_{H_y} = [P_{1_y} \quad P_{4_y} \quad R_{1_y} \quad R_{4_y}] = [y(0) \quad y(1) \quad y'^{(0)} \quad y'(1)] = [9 \quad -2 \quad -12 \quad -9]$$

$$G_{H_z} = [P_{1_z} \quad P_{4_z} \quad R_{1_z} \quad R_{4_z}] = [z(0) \quad z(1) \quad z'(0) \quad z'(1)] = [-5 \quad -1 \quad 0 \quad 9]$$

$$G = \begin{bmatrix} 10 & 11 & -7 & 11 \\ 9 & -2 & -12 & -9 \\ -5 & -1 & 0 & 9 \end{bmatrix}$$

6. Clip line AB **A(9.8, -6.9, 6.2)** , **B(-10.2 , 7.1 , -5.8)** against the four planes **x = -z** ; **x=z** **;** **y=z** ; and $\mathbf{z=z_{min}}$ in the standard canonical perspective viewing volume. Assume $z_{min} = 0.6$

Note: You do not need to clip against the other two planes in the standard volume.

Equation of line AB
$\begin{cases} x(t) = -20t + 9.8 \\ y(t) = 14t - 6.9 \\ z(t) = -12t + 6.2 \end{cases}$

Plane	t	Intersection point (x,y,z)	Accept or Reject	Reason to accept or reject
x = -z	$\mathbf{0.5}$	$(-0.2, 0.1, 0.2)$	R	$z < z_{min}$
x = z	$\mathbf{0.45}$	$(0.8, -0.6, 0.8)$	A	$z_{min} < z < 1$; $\lvert y\rvert < z$
y=z	$\mathbf{0.504}$	$(-0.277, 0.154, 0.154)$	R	$z < z_{min}$
z=0.6	$\mathbf{0.467}$	$(0.467, -0.367, 0.6)$	A	$\lvert x\rvert < z_{min}$ $\lvert y\rvert < z_{min}$

$$R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_x(\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$M_{Hermite}=\begin{bmatrix}2 & -3 & 0 & 1\\ -2 & 3 & 0 & 0\\ 1 & -2 & 1 & 0\\ 1 & -1 & 0 & 0\end{bmatrix}\qquad M_{Bezier}=\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 3 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

Rotate a vector around x axis until it lies in the xz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_x=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0\\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotate a vector around y axis until it lies in the yz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_y=\begin{bmatrix}\frac{c}{\sqrt{a^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+c^2}} & 0\\ 0 & 1 & 0 & 0\\ \frac{a}{\sqrt{a^2+c^2}} & 0 & \frac{c}{\sqrt{a^2+c^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotate a vector around z axis until it lies in the yz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_x=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0\\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

How to convert a general perspective view volume into canonical perspective volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Translate PRP (COP) to the origin
Step 6: Shear such that the center line of the view volume becomes the z axis
Step 7: Scale such that the sides of the view volume become 45 degrees
Step 8: Scale such that the view volume becomes the canonical perspective volume