



NAME:

Prob #	1	2	3	4	5	6
Points	12	16	24	18	12	18

Time: 80 Minutes

NOTES:

- a. Credit is only given to the correct numerical values.
 - b. All numerical values must be calculated with three digits of accuracy after the decimal point.
 - c. Do not write on the back side of the papers.
1. Points **A(-1,12)** and **B(60,6)** are given in a two dimensional world coordinate system. Find the coordinates of the points A and B on the screen after they have been mapped from window to viewport.

$$x_{wmin} = -40 \quad y_{wmin} = 5 \quad x_{wmax} = 360 \quad y_{wmax} = 15$$

Normalized device coordinate of the viewport:

$$x_{vmin} = 0.25 \quad y_{vmin} = 0.1 \quad x_{vmax} = 0.5 \quad y_{vmax} = 0.6$$

The origin of the screen coordinate system is defined in the **upper left** corner of the screen and the screen resolution is 800 by 600.
Use **truncation** to convert from float to integer.

Screen coordinates of point A after mapping are:

Screen coordinates of point B after mapping are:

2. Equations of plane P and line L are given as:

Plane P : $5x + 6y + 7z - 11 = 0$

Line L :
$$\begin{cases} x(t) = 2 \\ y(t) = -3t \\ z(t) = 4t + 3 \end{cases}$$

a. Find the point of intersection of line L with plane P .

Intersection point of line L and plane p is: _____

b. Find the equation of plane P after it has been translated by

$$dx = 6, \quad dy = 9, \quad dz = -2$$

Equation of plane P after translation is:

_____ x + _____ y + _____ z + _____ = 0

3. The viewing parameters for a perspective projection are given as

$$\text{VRP(WC)}=(2,6,3)$$

$$\text{VPN(WC)}=(6,0,0)$$

$$\text{VUP(WC)}=(0,3,4)$$

$$\text{PRP (VRC)}=(3,6,-10)$$

$$u_{\min}(\text{VRC}) = 1$$

$$u_{\max}(\text{VRC}) = 11$$

$$v_{\min}(\text{VRC}) = -1$$

$$v_{\max}(\text{VRC}) = 3$$

$$n_{\min}(\text{VRC}) = 10$$

$$n_{\max}(\text{VRC}) = 40$$

- Find the sequence of transformations which will transform this viewing volume into a standard perspective view volume which is bounded by the planes: $x=z$; $x=-z$; $y=z$; $y=-z$; $z=1$ (Complete the blank matrices)
- Find the z_{\min} after all transformations are done.

Matrix #2: Rx

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Matrix #1: Translate

1	0	0	-2
0	1	0	-6
0	0	1	-3
0	0	0	1

Matrix #4: Rz

Matrix #3: Ry

Matrix #6 Shear

Matrix #5: Translate

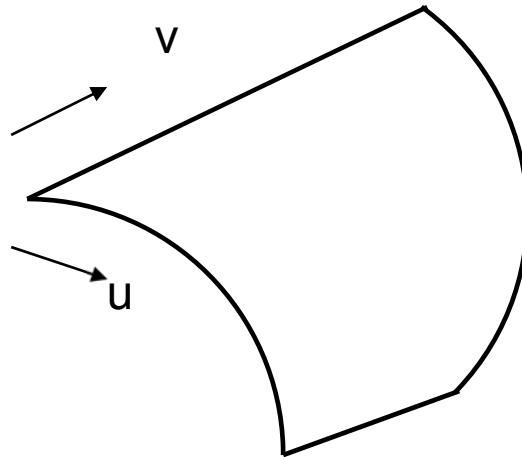
1	0	0	-3
0	1	0	-6
0	0	1	10
0	0	0	1

Matrix #8 Scale

Matrix #7 Scale

Zmin=

4. A curved surface is quadric in the u direction and linear in the v direction



The parametric equation of the curve corresponding to $v=0$ is given as:

$$C(u) = 9u^2 + 4u - 2$$

The parametric equation of the curve corresponding to $v=1$ is given as:

$$C(u) = -7u^2 + 8u + 12$$

Find the coefficient matrix C for this surface.

5. Equation of a cubic curve is given as:

$$C(t) = 6t^3 + 9t^2 - 12t + 8$$

Find the numerical values of the Bezier geometry vector for this curve.

6. Given the equation of a parametric surface

$$x(u,v) = 40u^2v + 20uv + 9$$

$$y(u,v) = 8u^2 - 10uv + 11$$

$$z(u,v) = 20u^2v - 10u^2 + 6$$

Find the normal to this surface at $u=0.5$ and $v=0.2$

Hint: Find the tangent vectors in u and v directions and then find the cross product of these two vectors.

Tangent vector in the u direction at point $u=0.5$ and $v=0.2$ is: _____

Tangent vector in the v direction at point $u=0.5$ and $v=0.2$ is: _____

Normal vector at point $u=0.5$ and $v=0.2$ is: _____

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{Hermite} = \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \quad M_{Bezier} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Rotate a vector around x axis until it lies in the xz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \quad R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate a vector around y axis until it lies in the yz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \quad R_y = \begin{bmatrix} \frac{c}{\sqrt{a^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{\sqrt{a^2+c^2}} & 0 & \frac{c}{\sqrt{a^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate a vector around z axis until it lies in the yz plane

$$V = \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} \quad R_z = \begin{bmatrix} \frac{b}{\sqrt{a^2+b^2}} & \frac{-a}{\sqrt{a^2+b^2}} & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How to convert a general perspective view volume into canonical perspective volume

- Step 1: Translate VRP to origin
- Step 2: Rotate VPN around x until it lies in the xz plane with positive z
- Step 3: Rotate VPN around y until it aligns with the positive z axis.
- Step 4: Rotate VUP around z until it lies in the yz plane with positive y
- Step 5: Translate PRP (COP) to the origin
- Step 6: Shear such that the center line of the view volume becomes the z axis
- Step 7: Scale such that the sides of the view volume become 45 degrees
- Step 8: Scale such that the view volume becomes the canonical perspective volume