Computer Graphics
Fall 2017 Midterm

NAME:

Prob #	1	2	3	4	5	6
Points	6	18	24	16	18	18

Time: 80 Minutes

NOTES:

a. **Credit is only given to the correct numerical values.**
b. **All numerical values must be calculated with three digits of accuracy after the decimal point.**

1. Line AB and point C(3,2,4) are on the same plane. Equation of the line is given as:

$$\begin{cases} x(t) = +5t - 1 \\ y(t) = -2t + 3 \\ z(t) = -3t + 6 \end{cases}$$

Find the equation of this plane

$AB: (5, -2, -3)$
$AC: (3 - (-1), 2 - 3, 4 - 6)$ (2 points)
$AB \times AC = (1, -2, 3)$
$x - 2y + 3z + C = 0$ (3 points)
⇨ $C = -11$ (1 points)

2. Given the point **A (3,1,4)** and plane P: $\mathbf{5x - 2y + z - 2 = 0}$
   a. Find the equation of the line L which is passing through point A and is perpendicular to the plane P.

Equation of the line L is:

$$\begin{cases} x(t) = +5t + 3 \\ y(t) = -2t + 1 \\ z(t) = +1t + 4 \end{cases}$$ (6 points)

   b. Find the intersection point of the line L with plane P:

The intersection point is:
$\mathbf{5(5t+3) - 2(-2t+1) + (t+4) - 2 = 0 \Rightarrow t = -0.5}$ (3 points)
$\mathbf{(0.5, 2, 3.5)}$ (3 points)

   c. Find the equation of plane P after it has been translated by

$$dx = 3, \quad dy = 1, \quad dz = -5$$

Equation of plane P after translation is:
$\mathbf{5(x - dx) - 2(y - dy) + (z - dz) - 2 = 0}$
⇨ $\mathbf{5(x-3) - 2(y-1) + (z+5) - 2 = 5x - 2y + z - 10 = 0}$ (6 points)

3. The viewing parameters for a parallel projection are given as

VRP(WC)=(1,3,4) **VPN(WC)=(8,0,6)**
VUP(WC)=(10,0,0) **PRP (VRC)=(0,4,5)**
u_{min} (VRC) = 13 **u_{max} (VRC) = 17**
v_{min} (VRC) = -7 **v_{max} (VRC) = 3**
n_{min} (VRC) = 12 **n_{max} (VRC) = 16**

Find the matrix transformations which will transform this viewing volume into a standard parallel view volume which is bounded by the planes: x=-1 ; x=1 ; y=-1 ; y=1 ; z=0 ; z=1

Matrix #1: Translate

1.0	0.0	0.0	-1.0
0.0	1.0	0.0	-3.0
0.0	0.0	1.0	-4.0
0.0	0.0	0.0	1.0

Matrix #2: Rx

1.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #3: Ry (4 points)

0.6	0.0	-0.8	0.0
0.0	1.0	0.0	0.0
0.8	0.0	0.6	0.0
0.0	0.0	0.0	1.0

Matrix #4: Rz (4 points)

0.0	-1.0	0.0	0.0
1.0	0.0	0.0	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #5: Shear (6 points)

1.0	0.0	3.0	0.0
0.0	1.0	-1.2	0.0
0.0	0.0	1.0	0.0
0.0	0.0	0.0	1.0

Matrix #6: Translate (4 points)

1.0	0.0	0.0	-15.0
0.0	1.0	0.0	2.0
0.0	0.0	1.0	-12.0
0.0	0.0	0.0	1.0

Matrix #7: Scale (6 points)

0.5	0.0	0.0	0.0
0.0	0.2	0.0	0.0
0.0	0.0	0.25	0.0
0.0	0.0	0.0	1.0

4. The equation of a parametric bi-cubic curved surface is given:

$$S(u,v) = u^3v - 4uv^3 - 2v^2 + 7uv - 2u - 9v + 5$$

Find the numerical value of the Hermite geometry matrix for this surface.

$\frac{dS}{du} = 3u^2v - 4v^3 + 7v - 2$ (3 points)

$\frac{dS}{dv} = u^3 - 12uv^2 - 4v + 7u - 9$ (3 points)

$\frac{dS}{dudv} = 3u^2 - 12v^2 + 7$ (2 points)

$$[G_S] = \begin{bmatrix} S_{00} & S_{01} & \frac{dS_{00}}{dv} & \frac{dS_{01}}{dv} \\ S_{10} & S_{11} & \frac{dS_{10}}{dv} & \frac{dS_{11}}{dv} \\ \frac{dS_{00}}{du} & \frac{dS_{01}}{du} & \frac{dS_{00}}{dudv} & \frac{dS_{01}}{dudv} \\ \frac{dS_{10}}{du} & \frac{dS_{11}}{du} & \frac{dS_{10}}{dudv} & \frac{dS_{11}}{dudv} \end{bmatrix}$$

$$[G_S] = \begin{bmatrix} 5 & -6 & -9 & -13 \\ 3 & -4 & -1 & -17 \\ -2 & 1 & 7 & -5 \\ -2 & 4 & 10 & -2 \end{bmatrix}$$ (8 points)

5. Clip line AB **A(0.9, -0.7,0. 6)** , **B(-1.1 , 0.7 , -0.4)**

against the three planes **x = -z** ; **x=z** ; **y=-z** ; and **$z=z_{min}$** in the standard canonical perspective viewing volume. Assume z_{min} = 0.2

Note: You do not need to clip against the other three planes in the standard volume.

Equation of line AB	
$$\begin{cases} x(t) = -2t + 0.9 \\ y(t) = 1.4t - 0.7 \\ z(t) = -1t + 0.6 \end{cases}$$	(6 points)

Plane	t	Intersection point (x,y,z)	Accept or Reject	Reason to accept or reject
x = z	0.3	$(0.3, -0.28, 0.3)$	A	$\|y\| \leq z$ $z_{min} \leq z \leq 1$ $0 \leq t \leq 1$
y= -z	0.25	$(0.4, -0.35, 0.35)$	R	$\|x\| \nleq z$
Z=0.2	0.4	$(0.1, -0.14, 0.2)$	A	$\|y\| \leq z$ $\|x\| \leq z$ $0 \leq t \leq 1$
	(6 points)		(6 points)	

6. A quadratic parametric curve is passing through points A(1,4,2) at t=0 and B(2,5,3) at t=1. The tangent vector to this curve at point A is (6,1,-2).
   Find the parametric equations of this curve for x(t), y(t), and z(t).

$$\begin{cases} x(t) = a_1 t^2 + b_1 t + c_1 \\ y(t) = a_2 t^2 + b_2 t + c_2 \\ z(t) = a_3 t^2 + b_3 t + c_3 \end{cases}$$ (4.5 points)

$$@t = 0, \begin{cases} x(0) = 1 \\ y(0) = 4 \\ z(0) = 2 \end{cases} => \begin{cases} c_1 = 1 \\ c_2 = 4 \\ c_3 = 2 \end{cases}$$ (4.5 points)

$$@t = 0, \begin{cases} x'(0) = 6 \\ y'(0) = 1 \\ z'(0) = -2 \end{cases} => \begin{cases} b_1 = 6 \\ b_2 = 1 \\ b_3 = -2 \end{cases}$$ (4.5 points)

$$@t = 1, \begin{cases} x(1) = 2 \\ y(1) = 5 \\ z(1) = 3 \end{cases} => \begin{cases} a_1 + 6 + 1 = 2 \\ a_2 + 1 + 4 = 5 \\ a_3 - 2 + 2 = 3 \end{cases} => \begin{cases} a_1 = -5 \\ a_2 = 0 \\ a_3 = 3 \end{cases}$$ (4.5 points)

$$\begin{cases} x(t) = -5t^2 + 6t + 1 \\ y(t) = +0t^2 + 1t + 4 \\ z(t) = +3t^2 - 2t + 2 \end{cases}$$

$$R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_x(\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$M_{Hermite}=\begin{bmatrix}2 & -3 & 0 & 1\\ -2 & 3 & 0 & 0\\ 1 & -2 & 1 & 0\\ 1 & -1 & 0 & 0\end{bmatrix}$$

Rotate a vector around x axis until it lies in the xz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_x=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0\\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotate a vector around y axis until it lies in the yz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_y=\begin{bmatrix}\frac{c}{\sqrt{a^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+c^2}} & 0\\ 0 & 1 & 0 & 0\\ \frac{a}{\sqrt{a^2+c^2}} & 0 & \frac{c}{\sqrt{a^2+c^2}} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotate a vector around z axis until it lies in the yz plane

$$V=\begin{bmatrix}a\\ b\\ c\\ 1\end{bmatrix}\qquad R_z=\begin{bmatrix}\frac{b}{\sqrt{a^2+b^2}} & \frac{-a}{\sqrt{a^2+b^2}} & 0 & 0\\ \frac{a}{\sqrt{a^2+b^2}} & \frac{b}{\sqrt{a^2+b^2}} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

How to convert a general parallel view volume into canonical parallel volume

Step 1: Translate VRP to origin
Step 2: Rotate VPN around x until it lies in the xz plane with positive z
Step 3: Rotate VPN around y until it aligns with the positive z axis.
Step 4: Rotate VUP around z until it lies in the yz plane with positive y
Step 5: Shear DOP such that it aligns with vpn
Step 6: Translate the lower corner of the view volume to the origin
Step 7: Scale such that the volume becomes a 2x2x1 standard parallel view volume