CSE-4303 CSE5365 Computer Graphics
Practice Problems 03

Assuming a right-handed 3-d coordinate system, solve the following problems.

1. Find the parametric equation of a line in a 3-dimensional Cartesian coordinate system which is passing through points $A\ (2, 5, -4)$ and point $B\ (-3, 10, 1)$

Considering B as reference point we have $\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = B = \begin{bmatrix} -3 \\ 10 \\ 1 \end{bmatrix}$,

and $\overrightarrow{BA} = A - B = \begin{bmatrix} 5 \\ -5 \\ -5 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$

Thus, the parametric equation of line passing A and B is $\begin{cases} \boldsymbol{x(t) = 5\,t - 3} \\ \boldsymbol{y(t) = -5\,t + 10} \\ \boldsymbol{z(t) = -5\,t + 1} \end{cases}$

Or by considering A as the reference point: $\begin{cases} \boldsymbol{x(t) = -5\,t + 2} \\ \boldsymbol{y(t) = 5\,t + 5} \\ \boldsymbol{z(t) = 5\,t - 4} \end{cases}$

2. Equation of line AB is given as: $\begin{cases} x(t) = 3\,t + 9 \\ y(t) = -4\,t - 6 \\ z(t) = 6\,t \end{cases}$

Find the equation of line AB after it has been rotated 90 degrees around the y axis.

90° Rotation around y matrix is $R_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Find 2 points on the line before rotation:

$$t = 0 \rightarrow A = \begin{bmatrix} 9 \\ -6 \\ 0 \end{bmatrix} \ \ \& \ \ t = 1 \rightarrow B = \begin{bmatrix} 12 \\ -10 \\ 6 \end{bmatrix}$$

Find the points after rotation:

$$A' = R_y \times A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 9 \\ -6 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \\ -9 \end{bmatrix}$$

$$B' = R_y \times B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 12 \\ -10 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -12 \end{bmatrix}$$

Considering A' as reference point we have $\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = A' = \begin{bmatrix} 0 \\ -6 \\ -9 \end{bmatrix}$,

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and $\overrightarrow{B'A'} = A' - B' = \begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \rightarrow \overline{B'A'} = \begin{cases} x(t) = 6t \\ y(t) = -4\,t - 6 \\ z(t) = -3\,t - 9 \end{cases}$ or $\overline{A'B'} = \begin{cases} x(t) = -6t + 6 \\ y(t) = 4\,t - 10 \\ z(t) = 3\,t - 12 \end{cases}$

3. Given point $A\ (3,6,-2)$ and plane P as: $3x + 6y - 5z - 30 = 0$. Find the equation of a line which is passing through point A and is perpendicular to plane P.

Considering A as reference point we have $\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = A = \begin{bmatrix} 3 \\ 6 \\ -2 \end{bmatrix}$,

Normal to the P is $\vec{N} = \begin{bmatrix} 3 \\ 6 \\ -5 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$

Thus, the parametric equation of line passing A and $\perp P$ is $\begin{cases} x(t) = 3\,t + 3 \\ y(t) = 6\,t + 6 \\ z(t) = -5\,t - 2 \end{cases}$

4. Point $A(3,-4)$ is given in a two dimensional world coordinate system. Find the coordinates of the point A on the screen after it is mapped from window to viewport.

Window coordinates:

$$x_{wmin} = 1\ , \quad y_{wmin} = -6\ , \quad x_{wmax} = 10\ , \quad y_{wmax} = 15$$

Normalized device coordinate of the viewport:

$$x_{vmin} = 0.1\ , \quad y_{vmin} = 0.25\ , \quad x_{vmax} = 0.6\ , \quad y_{vmax} = 0.8$$

The origin of the screen coordinate system is defined in the upper left corner of the screen and the screen resolution is 1600 by 1200. Use truncation to convert from float to integer.

$s_x = \frac{x_{vmax}-x_{vmin}}{x_{wmax}-x_{wmin}} = \frac{0.6-0.1}{10-1} = \frac{0.5}{9} = \frac{1}{18}\ ,\ s_y = \frac{y_{vmax}-y_{vmin}}{y_{wmax}-y_{wmin}} = \frac{0.8-0.25}{15-(-6)} = \frac{0.55}{21} = \frac{55}{2100}$

$A'_x = x_{vmin} + s_x(A_x - x_{wmin}) = 0.1 + \frac{1}{18} \times (3-1) = \frac{19}{90} \simeq 0.21111$

$A'_y = y_{vmin} + s_y(y_{wmax} - A_y) = 0.25 + \frac{55}{2100} \times (15 - (-4)) \simeq 0.747619$

$A'_{xpixel} = 0.21111 \times 1600 = 337.77 \quad , \quad A'_{ypixel} = 0.747619 \times 1200 = 897.14$

After truncation: Pixel Coordinate = $(337, 897)$

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5. Find the equation of a plane which is passing through points
   $A\ (1,4,2)\ ,\ B\ (4,-3,6)\ ,C\ (2,8,\ -4)$

   Calculate normal to plane:

   $\overrightarrow{AB} = B - A = \begin{bmatrix} 3 \\ -7 \\ 4 \end{bmatrix}$ and $\overrightarrow{AC} = C - A = \begin{bmatrix} 1 \\ 4 \\ -6 \end{bmatrix}$

   $\vec{N} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 42-16 \\ 4+18 \\ 12+7 \end{bmatrix} = \begin{bmatrix} 26 \\ 22 \\ 19 \end{bmatrix}$ (see question 9)

   Plane equation: $26x + 22y + 19z + D = 0,$

   Put A in the equation: $26 \times 1 + 22 \times 4 + 19 \times 2 + D = 0 \rightarrow D = -152$

   Plane equation $26x + 22y + 19z - 152 = 0$

6. Equation of plane P is given as $4x - 5y + z + 10 = 0$. Find the equation of this plane after it has been rotated 45 degrees around z axis.

   Normal vector to the plane: $\vec{N} = (4,-5,1)$

   Find a point on the plane: $A = (0,0,-10)$

   Find rotation of points around z-axis

   $$R_z = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Rotation of the normal vector:

   $$\overrightarrow{N'} = R_z \times \vec{N} = \left(2\sqrt{2} + \frac{5\sqrt{2}}{2}, 2\sqrt{2} - \frac{5\sqrt{2}}{2}, 1\right) = \left(\frac{9\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)$$

   $A' = R_z \times A = (0,0,-10)$,

   Plane equation: $\frac{9\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y + z + D = 0,$

   Put A in the equation $\rightarrow D = 10$

   Plane equation after rotation $\frac{9\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y + z + 10 = 0$

7. Equation of plane P is given as $4x - 5y + z + 10 = 0$. Find the equation of this plane after it has been translated by $dx = 2\ , dy = \ -5\ ,\ dz\ =\ 6$

   Translation $x' = x + 2, y' = y - 5, z' = z + 6 \rightarrow$

   $x = x' - 2, y = y' + 5, z = z' - 6$

   Put in plane equation: $4(x' - 2) - 5(y' + 5) + (z' - 6) + 10 = 0 \rightarrow$ **$4x' - 5y' + z' - 29 = 0$**

8. Given point $A = \ (4,2,-1)$ and plane P as $2x - y - 5z - 7 = 0$. Find the distance of point A from plane P.

   Distance of point $A = (x_0, y_0, z_0)$ to plane $P: ax + by + cz + d = 0$ is defined as:

   $$D = \frac{ax_0 + by_0 + cz_0 + d}{\sqrt{a^2 + b^2 + c^2}}$$

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In this problem,

$$D = \frac{2(4) + (-1)(2) + (-5)(-1) + (-7)}{\sqrt{2^2 + (-1)^2 + (-5)^2}} = \frac{4}{\sqrt{30}} \rightarrow \boldsymbol{D = \frac{2\sqrt{30}}{15}}$$

9. Find the cross product of two vectors $v_1 = (3,2,-1)$ and $v_2 = (4,2,3)$

$$a \times b = \begin{bmatrix} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \\ -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \\ \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

In this problem

$$v_1 \times v_2 = \begin{bmatrix} (2 \times 3) - (-1 \times 2) \\ (-1 \times 4) - (3 \times 3) \\ (3 \times 2) - (2 \times 4) \end{bmatrix} = \begin{bmatrix} \mathbf{8} \\ \mathbf{-13} \\ \mathbf{-2} \end{bmatrix}$$

10. Find the equation of a plane which is passing through x axis and point $A\ (3,1,-4)$

Consider two points on x axis which are also on plane P:

$B = (0,0,0)$ and $C = (1,0,0)$

$\overrightarrow{AB} = B - A = \begin{bmatrix} -2 \\ -1 \\ 4 \end{bmatrix}$ and $\overrightarrow{BC} = C - B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \vec{N} = \overrightarrow{AB} \times \overrightarrow{BC} = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$

$P: 0 \times x + 4y + z + D = 0$

Putting C in P: $D = 0 \rightarrow \boldsymbol{P: 4y + z = 0}$