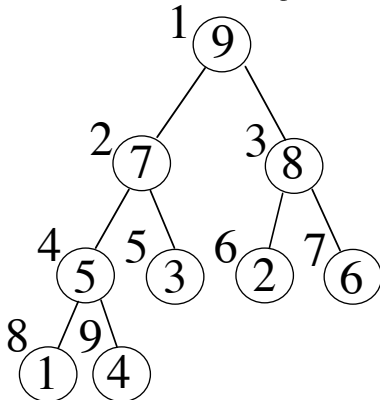Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to compute the sum of the n elements of an integer array is:

   A. $\Theta(n)$ B. $\Theta(n\log n)$ C. $\Theta\left(n^2\right)$ D. $\Theta\left(n^3\right)$

2. What happens in the third phase of counting sort?
   A. Add to appropriate counter for each key
   B. Clear count table
   C. Copy each record to output
   D. Determine first slot for each range value

3. Which of the following sorts is not stable?
          A. Insertion   B. LSD Radix Sort   C. MERGE-SORT D. QUICKSORT

4. Suppose $H_n = \dfrac{137}{60}$. What is the value of n?

          A. 4   B. 5   C. 6   D. 7

5. The function $2n + \log n$ is in which set?
          A. $\Omega(n\log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

6. Which of the following facts cannot be proven using one of the limit theorems?

   A. $n^2 \in O\left(n^3\right)$

   B. $n^2 \in \Omega(n\log n)$

   C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$

   D. $3^n \in \Omega\left(2^n\right)$

7. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?
          A. 31   B. 63   C. 64   D. 127

8. $f(n) = n\lg n$ is in all of the following sets, except

          A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega(n)$ D. $O\left(n^2\right)$

9. The worst-case time for QUICKSORT for n keys is in which set?

          A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n\log n)$ D. $\Theta\left(n^2\right)$

10. How many inversions occur in the sequence 1   3   0   6   5   2   4   7?
          A. 7   B. 8   C. 9   D. 11

11. Suppose that you have correctly determined some c and n_o to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?
    A. c may be decreased   B. c may be increased   C. n_o may be increased   D. $f(n) \in O(g(n))$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that

    $T(n) \in \Omega(\log n)$ and $T(n) \in O\left(n^3\right)$. Which of the following cannot be shown as an improvement?

          A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega\left(n^2\right)$ D. $T(n) \in \Omega\left(n^3\right)$

13. An array with 1000 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the twenty largest values, but there is no requirement that the twenty largest values be ordered. Which of the following is not correct?
   A. If 978 is returned from PARTITION, we are done.
   B. If 979 is returned from PARTITION, we are done.
   C. If 980 is returned from PARTITION, we are done.
   D. If 981 is returned from PARTITION, we must continue.

14. Suppose a minheap that can hold m records is used to produce sorted subfiles for an external mergesort. In the worst case, the number of records in a subfile (except the last one) is:

      A. m B. $2m$ C. $m \ln m$ D. m^2

15. Which of the following will not be true regarding the decision tree for QUICKSORT for sorting n input values?

    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.

    B. The height of the tree is $\Omega(n \log n)$.

    C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.

    D. There will be $n!$ leaves.

Long Answer

1. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication. Give C code for a $\Theta(m + n)$ algorithm to find the set intersection by producing a third array `C` with the values that are common to both `A` and `B` (in asscending order) and sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style are unnecessary.) 15 points

2. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta(\sqrt{n} \log n)$. 10 points

3. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta(\sqrt{n} \log n)$. 10 points

4. Show the result after PARTITION manipulates the following subarray. (4 is the pivot.) 10 points

      1   3   5   7   9   6   8   0   2   4

5. Show the result after performing HEAP-EXTRACT-MAX twice on the following maxheap. 10 points

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Test 2

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

    A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. 2α

2. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:

    A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$

3. Why is it common for a circular queue implementation to waste one table element?
    A. To avoid confusing an empty table with a full table
    B. To have a place to store the tail and head values
    C. To make sure the queue always has at least one element in use
    D. To perform some loops faster

4. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at H?
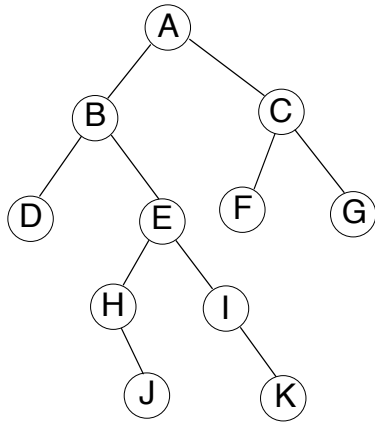
A. A   B. B   C. E   D. J

5. What data structures are needed to implement an infix calculator?
   A. A red-black tree   B. A stack and a queue   C. Two queues   D. Two stacks

6. What is the worst-case time to perform MINIMUM(*L*) for a sorted, doubly-linked list with *n* nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

7. Which type of linked list will be the most convenient if PREDECESSOR and SUCCESSOR operations will be frequent?
   A. ordered, doubly-linked
   B. ordered, singly-linked
   C. unordered, doubly-linked
   D. unordered, singly-linked

8. Which version of rat-in-a-maze finds the shortest path?
   A. queue   B. recursive   C. red-black tree D. stack

9. What is the worst-case time to perform a rotation in an unbalanced binary search tree storing *n* keys given a pointer to a node and the direction of the rotation? Assume that parent pointers are available.
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

10. If POP is implemented as return stack[SP--], then the test for an empty stack is implemented as:
    A. return stack[SP++]   B. return SP==(-1)   C. return SP==0   D. stack[++SP] = X

11. Which of the following binary trees can be legally colored as a red-black tree <u>with its root colored red</u>?
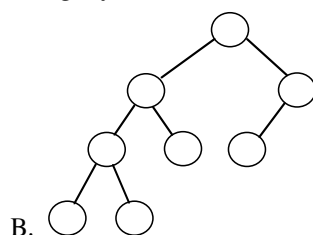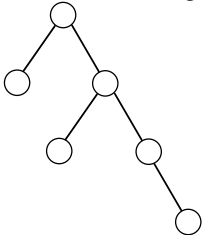
A.   B.
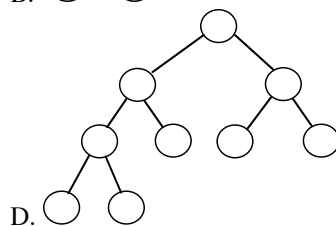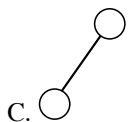
C.   D.

12. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?
    A. hashing with chaining B. open addressing

    C. ordered linked list   D. unbalanced binary search tree

13. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(nodept x)
{
if (x==NULL)
   return;
// process x here
traverse(x->left);
traverse(x->right);
```

```
}
```
    A. inorder   B. postorder   C. preorder   D. search for key x

14. The number of potential probe sequences when using linear probing with a table with m entries (m is prime) is:

    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

15. Assuming that each key stored in a double hash table with $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

    A. 1.2   B. 2   C. 5   D. 10

Long Answer

1. Consider the following hash table whose keys were stored by linear probing using

$h(key, i) = (key + i) \% 19$. Show your work.

```
 0  -1
 1  400
 2  800
 3  -1
 4  -1
 5  100
 6  101
 7  500
 8  -1
 9  -1
10  -1
11  201
12  600
13  -1
14  -1
15  -1
16  301
17  -1
18  -1
```

    a. Suppose 2000 is to be stored (using linear probing). Which slot will be used? (3 points)

    b. Suppose 2001 is to be stored (using linear probing) *after* 2000 has been stored. Which slot will be used? (4 points)

2. Consider the following hash table whose keys were stored by double hashing using

$h_1(key) = key \% 19$ and $h_2(key) = 1 + (key \% 18)$. Show your work.

```
 0  -1
 1  -1
 2  800
 3  402
 4  -1
 5  -1
 6  101
 7  501
 8  -1
 9  -1
10  200
11  -1
12  601
13  -1
14  -1
15  -1
16  301
17  701
18  -1
```

    a. Suppose 2001 is to be inserted (using double hashing). Which slot will be used? (4 points)

    b. Suppose 2002 is to be inserted (using double hashing) *after* 2001 has been stored. Which slot will be used? (4 points)
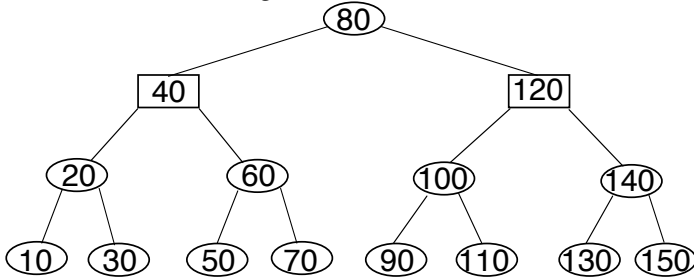
3. Insert 115 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
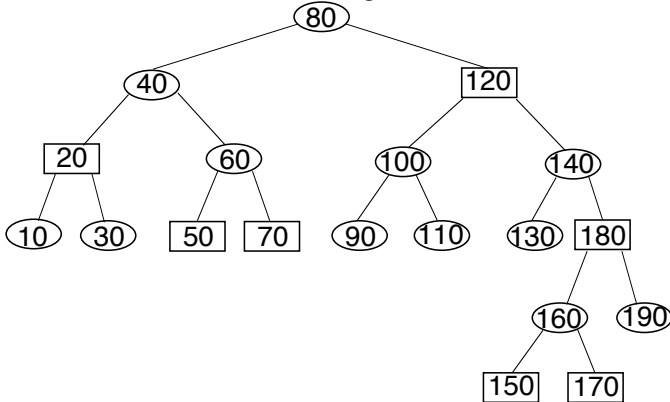
```
              (80)
        [40]        [120]
    (20)   (60)   (100)  (140)
  10  30 50 70  90 110 130 150
```
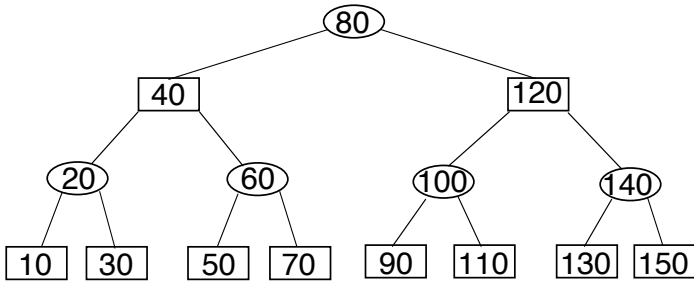
4. Insert 35 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)

```
              (80)
        [40]            (120)
    (20)   (60)     [100]   [140]
  10  30 50 70
```
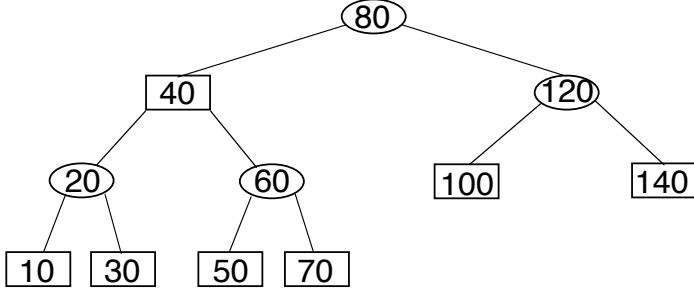
5. Delete 50 from the given red-black tree. Be sure to indicate the cases that you used. (10 points)

```
              (80)
        [40]         [120]
    (20)   (60)   (100)  (140)
 (10)(30)(50)(70)(90)(110)(130)(150)
```

6. Delete 130 from the following red-black tree. Be sure to indicate the case(s) that you used. 10 points

```
              (80)
        (40)          [120]
    [20]    (60)   (100)   (140)
 (10)(30) 50  70 (90)(110)(130)[180]
                        (160) (190)
                       150  170
```

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Test 3
Fall 2006   Student ID # _____
Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each
1. In Dijkstra's algorithm, the final shortest path distance from the source s to a vertex x is known when:
    A. some vertex y moves from T to S and there is an edge from y to x.
    B. x has its entry extracted from the heap.
    C. x is placed on the heap.
    D. x is read from the input file.
Problems 2, 3, and 4 refer to the following network. 0 is the source. 7 is the sink.
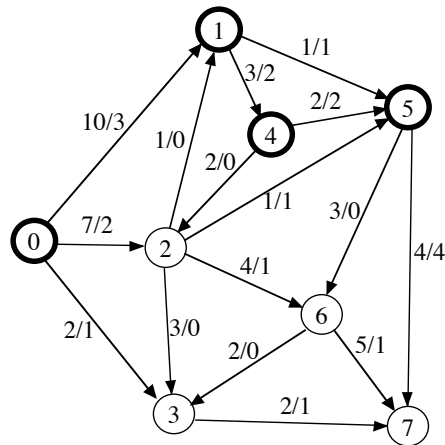
2. The capacity of the indicated cut (S vertices are bold) is:
   A. 6   B. 7   C. 16   D. 18

3. The net flow across the given cut is:
   A. 6   B. 7   C. 16   D. 18

4. Suppose the flow is increased as much as possible using the augmenting path $0 \rightarrow 2 \rightarrow 6 \rightarrow 7$. Which is the critical edge?
   A. $0 \rightarrow 2$ B. $2 \rightarrow 6$ C. $6 \rightarrow 7$ D. Insufficient information

5. Augmenting paths are usually found by:
   A. Breadth-first search on the saturated edges in the residual network.
   B. Breadth-first search on the unsaturated edges in the residual network.
   C. Depth-first search on the unsaturated edges in the residual network.
   D. Dijkstra's algorithm.

6. What is the number of strongly connected components in this graph?
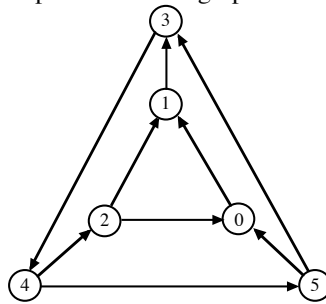
   A. 1   B. 2   C. 3   D. 4

7. Which of the following uses a max-heap?
   A. Dijkstra's algorithm
   B. Huffman code construction
   C. Maximum capacity path
   D. Prim's algorithm

8. Which directed graph representation may use binary search when testing for the presence of a particular edge?
   A. Adjacency list
   B. Adjacency matrix
   C. Compressed adjacency list
   D. None of the above

9. The time to fill in the dynamic programming matrix when computing the LCS for sequences of lengths m and n is:
   A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

10. When a graph is dense, the best way to find a minimum spanning tree is:
    A. Floyd-Warshall algorithm
    B. Prim's algorithm using heap
    C. Prim's algorithm using T-table
    D. Warshall's algorithm

11. If the choice of the augmenting path is arbitrary, an edge could be critical this number of times:
    A. E
    B. V
    C. (V-2)/2
    D. depends on the maximum capacity appearing in the network

12. During a breadth-first search, the status of a black vertex is:
    A. It has been completely processed.
    B. It is in the FIFO queue.
    C. It is in the priority queue.
    D. It is undiscovered.

13. When using two breadth-first searches to find the diameter of a tree, the purpose of the first search is to find:
    A. all vertices that could be an end of a diameter.
    B. both ends of a diameter.
    C. one end of a diameter.
    D. the number of edges in the diameter.

14. Suppose a depth-first search is performed on an undirected graph. There are no cycles if:
    A. no edge is a cross edge or forward edge
    B. both C and D
    C. there are no restarts
    D. there are no back edges

15. The goal of the Huffman coding method is:
    A. Another type of binary search tree.
    B. Maximize the compression for every string.
    C. Minimize the expected bits per symbol.
    D. Store a string within the leaves of a binary tree.

16. Suppose that a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:
    A. Back   B. Cross   C. Forward   D. Tree

17. The number of edges in a maximum bipartite matching for the graph below is:
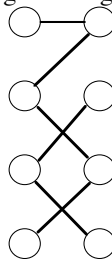

          A. 2   B. 3   C. 4   D. 5

18. Which of the following is not true for the activity scheduling problem?
    A. The activities may have various durations.
    B. The greedy solution is optimal.
    C. There may be several optimal solutions.
    D. The goal is to minimize the number of activities chosen.

19. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

    -1   3   3   3   3
    -1   3   3   3   4
    -1   1   1   1   4
    -1   2   2   2   2
    -1   -1   -1   -1   -1
       A. 0   B. 1   C. 2   D. 3

20. When solving the fractional knapsack problem, the items are processed in the following order.
    A. Ascending order of weight
    B. Ascending order of $$$/lb
    C. Descending order of weight
    D. Descending order of $$$/lb

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.
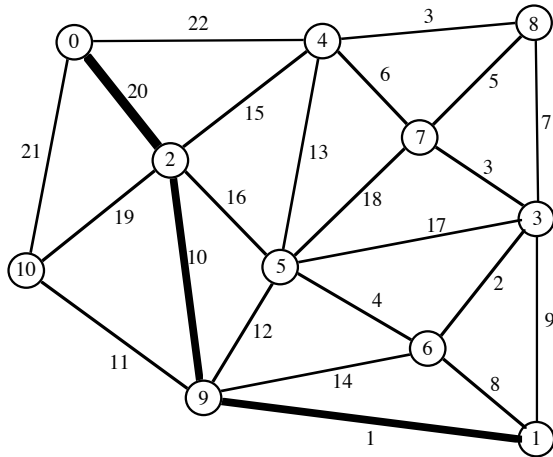
2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points
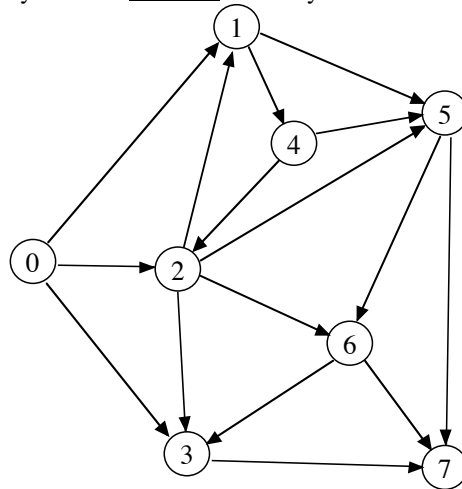

Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	___	0 1	___	2 6	___
1	___	___	0 2	___	3 7	___
2	___	___	0 3	___	4 2	___
3	___	___	1 4	___	4 5	___
4	___	___	1 5	___	5 6	___
5	___	___	2 1	___	5 7	___
6	___	___	2 3	___	6 3	___
7	___	___	2 5	___	6 7	___

3. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following graph. The paths indicated in the final matrix must have **at least one** edge. You are not required to show the intermediate matrices. 10 points.
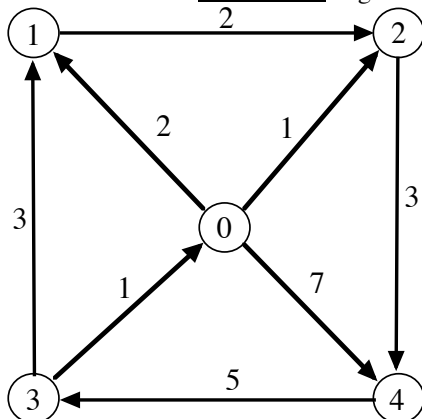
4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.
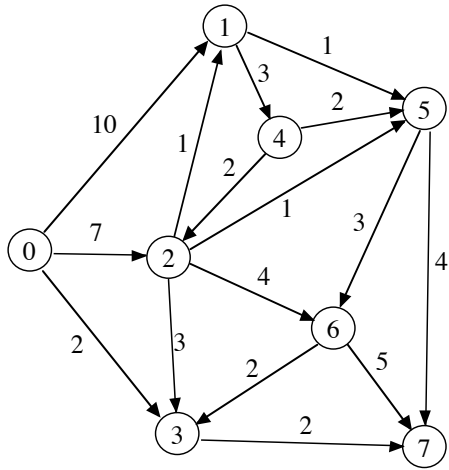
Minimum Cut:

    S vertices: 0

    T vertices: 7

Augmenting Paths and Contribution to Flow:

5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=8
p[1]=6
p[2]=4
p[3]=3
p[4]=5
p[5]=7

```
              1           2           3           4           5
  1     0   0    192   1   216   1   336   3   ???   ?

  2    -------        0   0     72   2   162   3   303   3

  3    -------     -------        0   0     60   3   189   3

  4    -------     -------     -------        0   0    105   4

  5    -------     -------     -------     -------        0   0
```

6. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points

	v_i	p_i	$m(i)$
1	1	0	
2	5	0	
3	3	2	
4	8	1	
5	1	4	
6	2	4	
7	3	5	
8	1	6	
9	4	6	
10	1	8	