Multiple Choice. Write your answer to the LEFT of each problem. 2 points each

1. What is the value of $\sum_{k=0}^{t-1} 2^k$?

    A. 2^k B. $2^t - 1$ C. $2^{t+1} - 1$ D. $2^{t+1} + 1$

2. Suppose that you have correctly determined some c and n_o to prove $f(n) \in O(g(n))$. Which of the following is not necessarily true?

    A. c may be decreased   B. c may be increased   C. n_o may be increased   D. $g(n) \in \Omega(f(n))$

3. Suppose there is a large, unordered table with n integers, possibly with repeated values. How much time is needed to determine the minimum value?

    A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

4. The time to multiply an $m \times n$ matrix and a $n \times p$ matrix is:

    A. $\Theta(n)$ B. $\Theta(\max(m,n,p))$ C. $\Theta(n^3)$ D. $\Theta(mnp)$

5. Which of the following is solved heuristically by a greedy method?
    A. Fractional knapsack   B. Huffman code   C. Unweighted interval scheduling   D. 0/1 knapsack

6. Which of the following is true regarding mergesort?
    A. It is difficult to code without recursion
    B. It is difficult to code to ensure stability
    C. It may be coded to operate in a bottom-up fashion
    D. The input must be preprocessed to exploit ordered subarrays or sublists in the input.

7. Which function is in both $\Omega(2^n)$ and $O(3^n)$, but is not in $\Theta(2^n)$ or $\Theta(3^n)$?

    A. $2^n + n^2$ B. $3^n - n^2$ C. 2.5^n D. $\ln n$

8. The function $3n \log n + n$ is in which set?

    A. $\Omega(n^2)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

9. Which of the following is not true?

    A. $n^2 \in O(n^3)$ B. $n^2 \in \Omega(n \log n)$

    C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $\log n \in O(\log \log n)$

10. Suppose a binary search is to be performed on an ordered table with 40 elements. The maximum number of elements that could be examined (probes) is:
    A. 4   B. 5   C. 6   D. 7

11. $\log(n!)$ is in all of the following sets, except

    A. $\Omega(\log n)$ B. $\Theta(n \log n)$ C. $O(n^2)$ D. $\Omega(n^2)$

12. The cost function for the optimal matrix multiplication problem is:

    A. $C(i,j) = \min_{i \le k < j} \{C(i,k) + C(k,j) + P_{i-1} P_k P_j\}$ B. $C(i,j) = \min_{i \le k < j} \{C(i,k) + C(k+1,j) + P_i P_k P_j\}$

    C. $C(i,j) = \min_{i \le k < j} \{C(i,k) + C(k+1,j) + P_{i-1} P_k P_j\}$ D. $C(i,j) = \max\{C(i,j-1), C(i-1,j)\}$ if $x_i \ne y_j$

13. The recursion tree for mergesort has which property?
    A. each level has the same contribution
    B. it leads to a definite geometric sum
    C. it leads to a harmonic sum
    D. it leads to an indefinite geometric sum

14. Which of the following is the best approximation for H_{mn}? (m and n are positive integers)

    A. $\dfrac{1}{mn}$ B. $\ln m + \ln n$ C. $\ln n$ D. 1

15. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know

$T(n) \in \Omega(n \log n)$ and $T(n) \in O(n^3)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(n \log n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^2 \log n)$

Long Answer

1. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e. each table represents a set). Give Java code for a $\Theta(m + n)$ algorithm to find the **symmetric difference** by producing a third array `C` (in ascending order) with the values that appear in **exactly** one of `A` and `B` **and** sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style **are unnecessary**.) 10 points

2. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

A	0.05
B	0.02
C	0.5
D	0.04
E	0.22
F	0.07
G	0.10

3. Use the recursion-tree method to show that $T(n) = 3T\left(\dfrac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points

4. Use the substitution method to show that $T(n) = 3T\left(\dfrac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points

5. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points


i	v_i	p_i	$m(i)$
1	3	0	
2	3	0	
3	1	2	
4	3	1	
5	2	4	
6	3	4	
7	5	5	
8	3	6	
9	6	6	
10	6	8	
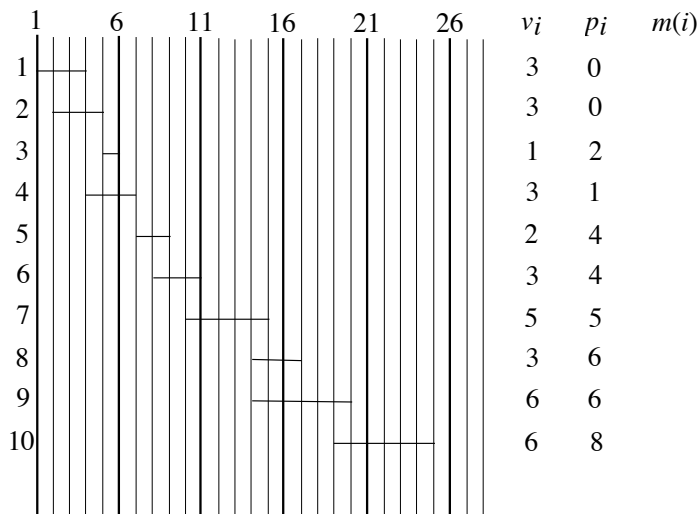
6. Use dynamic programming to find a longest common subsequence of the following sequences. Be sure to provide the DP table and the backtrace for your LCS. 10 points

   ```
   0 1 2 0 1 2
   0 0 1 1 2 2
   ```

7. a. Show the minheap after performing `getmin`. 5 points

Heap (subscript : priority): 1:2, 2:3, 3:5, 4:7, 5:4, 6:10, 7:6, 8:9, 9:8
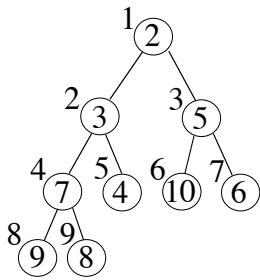
b. Show the maxheap after changing the priority at subscript 8 to 6. 5 points
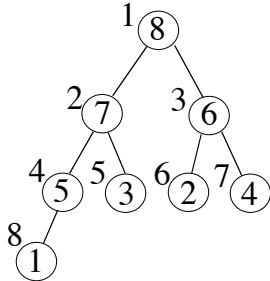
Heap (subscript : priority): 1:8, 2:7, 3:6, 4:5, 5:3, 6:2, 7:4, 8:1

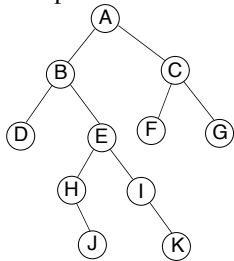CSE 2320   Name _____

Test 2

Fall 2009   Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 2 points each

1. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
    A. return stack[SP++]   B. stack[SP++] = X   C. stack[SP--] = X   D. stack[++SP] = X

2. Suppose the tree below is a binary search tree whose keys <u>are not shown</u>. Which node will contain the key that is the predecessor of the key stored at H?

Tree: A root; A's children B and C; B's children D and E; C's children F and G; E's children H and I; H's child J; I's child K.

    A. A
    B. B
    C. C
    D. G

3. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?

    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.

    B. The height of the tree is $\Omega(n \log n)$.

    C. There will be a path from the root to a leaf with $\Omega\left(n^2\right)$ decisions.

    D. There will be $n!$ leaves.

4. What is the worst-case time to perform MINIMUM(L) for an ordered, doubly-linked list with n nodes?
                A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

5. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with n nodes in ascending order is:
        A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

6. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing n keys? Assume that parent pointers are available.
        A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

7. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:

A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$

8. Which of the following would not be used in implementing rat-in-a-maze in a depth-first fashion?
   A. Circular queue B. Recursion C. Stack D. 2-d array

9. Circular linked lists are occasionally useful because
   A. some operations may be done in constant time.
   B. they are an alternative to red-black trees.
   C. they are useful for implementing circular queues.
   D. they avoid mallocs.

10. In a binary search tree, which element does not have a predecessor?
    A. any one of the leaves B. the maximum C. the minimum D. the root

11. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?

A.

B.

C.

D.
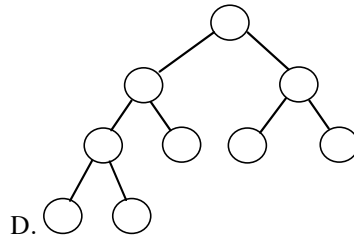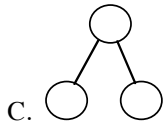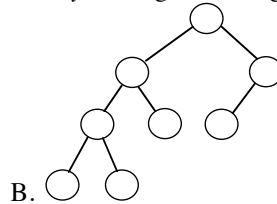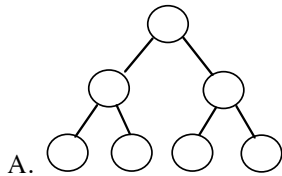
12. Which sort treats keys as several digits and uses a counting sort for each position?
    A. counting B. insertion C. merge D. radix

13. The worst-case number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?

    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta\left(n^2\right)$

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
if (x==null)
   return;
traverse(x.left);
traverse(x.right);
// process x here
}
```
   A. inorder B. postorder C. preorder D. search for key x

15. Suppose that only numbers in 1 ... 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
    A. 600, 100, 550, 540, 500
    B. 10, 200, 300, 100, 500
    C. 100, 1000, 200, 900, 300, 800, 400, 700, 500
   D. 200, 300, 400, 700, 600, 500

Long Answer

1. Twenty million positive integers in the range 0 ... 99,999,999 are to be sorted by LSD radix sort. Compare the performance for using radix 0 ... 9999 and radix 0 ... 9. Show your work. (10 points)

2. Use the dynamic programming solution for subset sums to determine a subset that sums to 10. (10 points)

i	0	1	2	3	4
S_i	0	2	3	6	7

3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

8	2	5	7	4	1	9	0	6	3

   Version: 1 2/Sedgewick

4. Use dynamic programming to solve the following instance of the longest increasing subsequence problem. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. 10 points

1 7 2 9 8 5 10 3 6 4 14 15 12 11 13

5. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
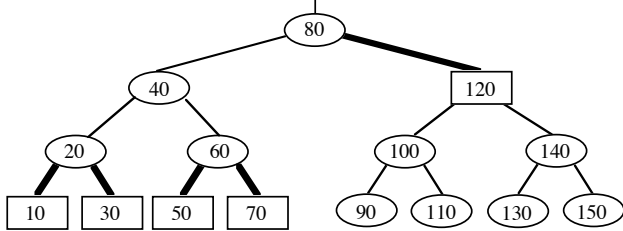
6. Insert 45 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
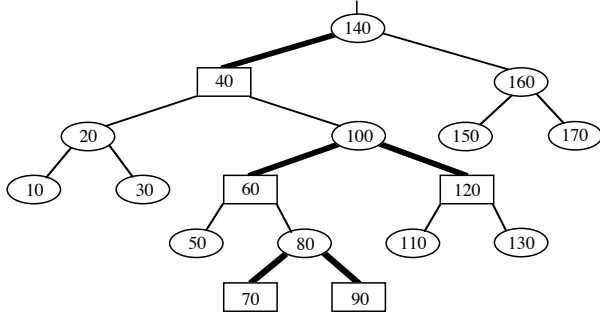
7. Give the unbalanced binary search tree that results when the keys $70, 50, 40, 110, 60, 90, 100, 80, 120$ are inserted, in the given order, into an initially empty tree. (10 points)

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Test 3

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Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Suppose an instance of bipartite matching has 15 vertices in the left column, 15 vertices in the right column, and 25 edges. The number of edges in the corresponding instance of network flow is:
    A. 25
    B. 30
    C. 55
    D. 150

2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:
       A. Back   B. Cross   C. Forward   D. Tree

3. Compressed adjacency lists have the following disadvantage:
    A. Testing whether an edge from X to Y is present will take $\theta(V + E)$ worst-case time.
    B. They are static.
    C. They can only be used for graphs without weights.
    D. They require $\theta(V + E)$ space to store.

4. Augmenting paths are usually found by:
    A. Breadth-first search on the saturated edges in the residual network.
    B. Breadth-first search on the unsaturated edges in the residual network.
    C. Depth-first search on the unsaturated edges in the residual network.
    D. Dijkstra's algorithm.

5. The capacity of any cut is:
    A. A lower bound on the maximum flow.
    B. An upper bound on the maximum flow.
    C. The same as the capacity of all other cuts.
    D. The same as the maximum attainable flow.

6. A topological ordering of a directed graph may be computed by:
    A. Ordering the vertices by descending discovery time after DFS
    B. Ordering the vertices by descending finish time after DFS
    C. Ordering the vertices by ascending discovery time after DFS
    D. Ordering the vertices by ascending finish time after DFS

7. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:
    A. The net flow does not exceed the amount of flow.
    B. There is no relationship.
    C. They are equal.

D. The amount of flow does not exceed the net flow.

8. What is the number of strongly connected components in this graph?
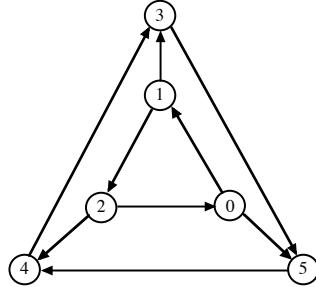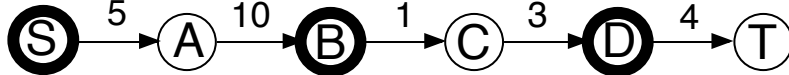

    A. 1   B. 2   C. 3   D. 4

9. The capacity of the following cut is _____. (S vertices are bold.)

    A. 1   B. 10   C. 13   D. 23

10. When a graph is sparse, the best way to find a minimum spanning tree is:
    A. Floyd-Warshall algorithm
    B. Prim's algorithm using heap
    C. Prim's algorithm using T-table
    D. Warshall's algorithm

11. Which of the following is not true regarding the Edmonds-Karp variant.
    A. An augmenting path is found using breadth-first search.
    B. An augmenting path may be used several times.
    C. An edge may go back-and-forth between being saturated and unsaturated.
    D. It solves the network flow problem in polynomial time.

12. When finding the strongly connected components, the number of components is indicated by:
    A. The number of restarts for the first depth-first search.
    B. The number of restarts for the second depth-first search.
    C. The number of back edges found during the first depth-first search.
    D. The number of cross edges found during the second depth-first search.

13. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

```
-1    3    3    3    3
-1    3    3    3    4
-1    1    1    1    4
-1    2    2    2    2
-1   -1   -1   -1   -1
```

    A. 0   B. 1   C. 2   D. 3

14. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

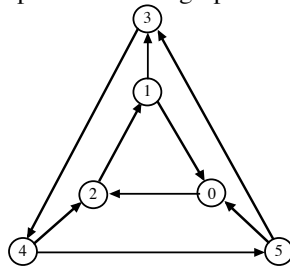   A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

15. What is the number of strongly connected components in this graph?

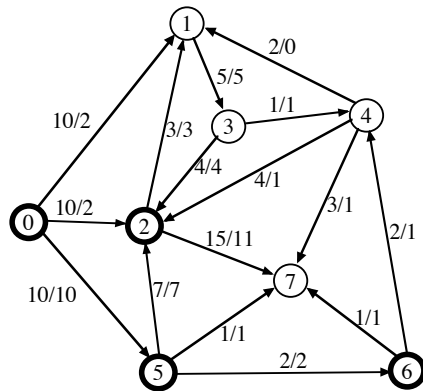   A. 1   B. 2   C. 3   D. 4

Problems 16, 17, and 18 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow.

16. The capacity of the indicated cut (S vertices are bold) is:
    A. 20   B. 30   C. 31   D. 32
17. The net flow across the given cut is:
    A. 14   B. 16   C. 18   D. 20
18. Suppose the flow is increased as much as possible using the augmenting path $0 \to 2 \to 4 \to 7$. Which is the critical edge?
    A. $0 \to 2$ B. $2 \to 4$ C. $4 \to 7$ D. Insufficient information
19. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The number of entries in the two tables are:
    A. m for both
    B. n for both
    C. $n + 1$ and m
    D. n and m
20. Which of the following is not true about probe sequences for an implementation of double hashing?
    A. All slots in the hash table appear in each probe sequence
    B. Every key has a probe sequence different from the probe sequences for other keys
    C. The elements of a probe sequence are subscripts in the hash table
    D. The probe sequence for a key cannot change
21. Suppose a double hash table has $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2   B. 2   C. 5   D. 10
22. Which directed graph representation may use binary search when testing for the presence of a particular edge?
    A. Adjacency list
    B. Adjacency matrix
    C. Compressed adjacency list
   D. None of the above

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using
$h(key) = key \% 13$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	135		86	87			

23. 16 would be inserted into which slot of the given table?
    A. 2   B. 4   C. 7   D. 10
24. 138 would be inserted into which slot of the given table? (without previously inserting 16)
    A. 2   B. 4   C. 7   D. 10
25. If the choice of the augmenting path is arbitrary (Ford-Fulkerson), an edge could be critical this number of times:
    A. E
    B. V
    C. (V-2)/2
    D. depends on the maximum capacity appearing in the network

Long Answer
1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.
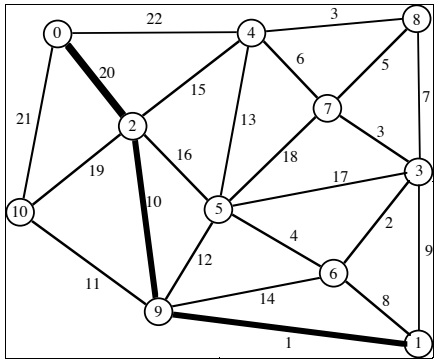
2. Consider the following hash table whose keys were stored by double hashing using
$h_1(key) = key \% 11$ and $h_2(key) = 1 + (key \% 10)$. ***Show your work***.

```
 0  22
 1
 2
 3  17
 4  4
 5  15
 6  28
 7
 8
 9
10  10
```

a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)
b. Suppose 130 is to be inserted (using double hashing) ***after*** 142 has been stored. Which slot will be used? (5 points)
3. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10
points.
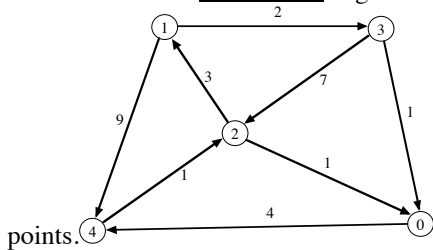
4. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points
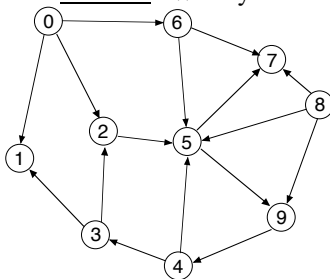
Vertex	Start	Finish	Edge	Type		Edge	Type
0	1		0 1			6 5	
1			0 2			6 7	
2			0 6			8 5	
3			2 5			8 7	
4			3 1			8 9	
5			3 2			9 4	
6			4 3				
7			4 5				
8			5 7				
9			5 9				

5. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.
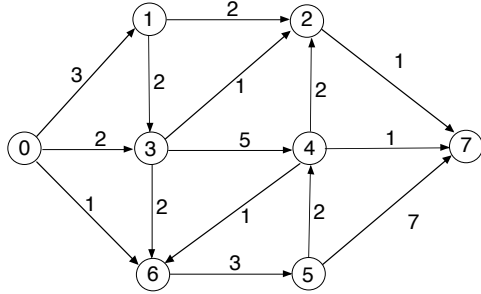
Minimum Cut:

    S vertices: 0

    T vertices: 7

Augmenting Paths and Contribution to Flow: