CSE 2320 Name _____
Test 1
Fall 2014

Multiple Choice. Write the SINGLE LETTER of your answer to the LEFT of each 3 point problem.

1. The time to find the minimum of the n elements of an integer array in ascending order is in:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(1)$ D. $\Theta(n^3)$

2. The goal of the Huffman code tree construction method is:
   A. Construct a max-heap for the symbols in an alphabet
   B. Find the symbols with high probability of occuring.
   C. Maximize the compression for every string.
   D. Minimize the expected bits per symbol.

3. Which of the following is true?

   A. $n^3 \in \Omega(n^2)$ B. $n \log n \in \Omega(n^2)$ C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $3^n \in O(2^n)$

4. Which of the following facts cannot be proven using one of the limit theorems?

   A. $n^2 \in O(n^3)$ B. $n^2 \in \Omega(n \log n)$ C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ D. $3^n \in \Omega(2^n)$

5. The function $2 \log n + \log n$ is in which set?

   A. $\Omega(n \log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

   A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega(n)$ D. $O(n^2)$

7. The number of calls to `mergeAB` while performing `mergesort` on n items is in:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(\log n)$ D. $\Theta(m+n)$

8. What is required when calling `union(i,j)` for maintaining disjoint subsets?
   A. `i` is the ancestor of `j` in one of the trees
   B. `i` and `j` are in the same subset
   C. `i` and `j` are leaders for different subsets
   D. `i` and `j` are leaders for the same subset

9. The time to run the code below is in:

```
sum=1;
for (i=1; i<n; i=3+i)
  sum++;
```

   A. $\Theta(\log n)$ B. $\Theta(\sqrt{n})$ C. $\Theta(n)$ D. $\Theta(n \log n)$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that $T(n) \in \Omega(1)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

11. Suppose a binary search is to be performed on a table with 50 elements. The maximum number of elements that could be examined (probes) is:
    A. 4 B. 5 C. 6 D. 7

12. The time to run the code below is in:

```
for (i=n; i>=0; i--)
  for (j=0; j<n; j+=2)
    sum+=i*j;
```

    A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

13. Which of the following functions is not in $\Omega(n^2)$?

    A. n^3 B. n C. n^2 D. $n^2 \lg n$

14. What is the value of $\sum_{k=0}^{t} 2^k$?

A. 2^k B. 2^t C. $2^{t+1}-1$ D. $2^{t+1}+1$

15. When solving the fractional knapsack problem, the items are processed in the following order.
    A. Ascending order of weight B. Ascending order of $$$/lb
    C. Descending order of weight D. Descending order of $$$/lb

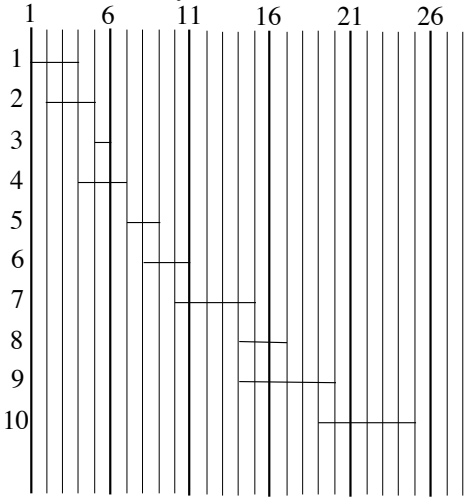
Long Answer

1. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
p[0]=6
p[1]=2
p[2]=4
p[3]=3
p[4]=2
```

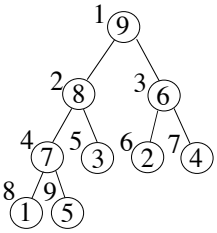
	1	2	3	4
1	0 0	48 1	60 1	??? ?
2	-------	0 0	24 2	36 3
3	-------	-------	0 0	24 3
4	-------	-------	-------	0 0

2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points

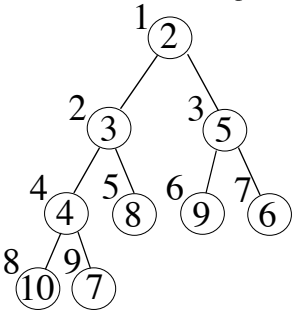


	v_i	p_i	$m(i)$
1	1	0	
2	5	0	
3	3	2	
4	8	1	
5	1	4	
6	2	4	
7	3	5	
8	1	6	
9	4	6	
10	1	8	

3. Give the greedy solution for the unweighted interval scheduling problem using the set of intervals for problem 2. You may simply give the indices for the intervals in the solution. 5 points
4. a. Show the maxheap after performing `PQdelmax`. 5 points



b. Show the minheap after changing (`PQchange`) the priority at subscript 6 to 1. 5 points



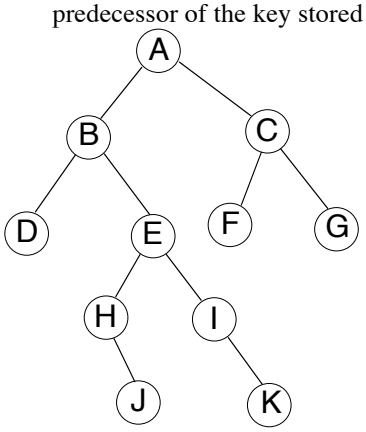
5. Use the recursion-tree method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^3\right)$. 10 points

6. Use the substitution method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^2$ is in $\mathrm{O}\left(n^3\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^3\right)$.) 10 points

CSE 2320-002 Name _____
Test 2
Fall 2014

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. In a binary search tree, which element does not have a predecessor?
   A. any one of the leaves B. the maximum C. the minimum D. the root
2. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at H?



A. A
B. B
C. C
D. G

3. Which of the following will not be true regarding the decision tree for MERGESORT for sorting *n* input values?
   A. There will be a path from the root to a leaf with $\Omega\left(n^2\right)$ decisions.
   B. There will be *n*! leaves.
   C. Every path from the root to a leaf will have $\mathrm{O}\left(n^2\right)$ decisions.
   D. The height of the tree is $\Omega(n \log n)$.
4. What is the worst-case time to perform MAXIMUM(*L*) for an unordered, doubly-linked list with *n* nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. Given a pointer to a node, the worst-case time to delete the node from a doubly-linked list with *n* nodes in ascending order is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
6. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing *n* keys? Assume that parent pointers are available.
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
7. An array with 150 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
   A. If 119 is returned from PARTITION, we must continue.
   B. If 118 is returned from PARTITION, we must continue.
   C. If 131 is returned from PARTITION, we must continue.
   D. If 120 is returned from PARTITION, we are done.
8. In which situation will a sentinel be inappropriate?
   A. Search for a key in an unordered linked list, to simplify and speed-up code
   B. Red-black tree, to simplify code

C. Binary search for a key in an ordered table, to simplify and speed-up code
D. Search for a key in an unordered table, to simplify and speed-up code

9. A more accurate name for the subset sum problem is:
   A. Combination sum B. Indivisible, unbounded knapsack
   C. Maximum achievable sum D. Permutation sum
10. How should the successor of a node without a right child in an unbalanced binary search tree be found?
   A. Examine the ancestors of the node B. Go left, then proceed to the right
   C. Go right, then proceed to the left D. Preorder traversal
11. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
   A. to sort the original input
   B. to assure that the final solution is free of duplicate values
   C. to determine the longest possible increasing subsequence terminated by a particular input value
   D. to search a table that will contain only the LIS elements at termination
12. Recursion is often an alternative to using which data structure?
   A. Linked list B. Queue C. Stack D. 2-d array
13. The expected number of comparisons for finding the *k*th largest of *n* keys using PARTITION is in which asymptotic set?
   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$
14. The time to extract the LCS (for sequences of lengths *m* and *n*) after filling in the dynamic programming matrix is in:
   A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$
15. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(node x)
{
if (x==null)
  return;
traverse(x->left);
// process x here
traverse(x->right);
}
```

   A. inorder B. postorder C. preorder D. search for key `x`

Long Answer

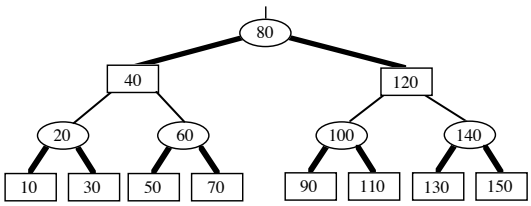
1. Give the unbalanced binary search tree that results when the keys 50, 100, 80, 70, 60, 90, 120 are inserted, in the given order, into an initially empty tree. (5 points)
2. Give the expression describing the asymptotic performance of LSD radix sort and explain the parameters. (5 points)
3. Give the diagrams showing how a stack and a queue may be stored using linked lists (5 points)
4. Provide an LCS by drawing lines between instances of symbols and indicate the corresponding backtrace using arrows in the matrix below. (10 points)

```
   a b c d d c b a a b c d
   a a b b c c d d a b a c
      a  a  b  b  c  c  d  d  a  b  a  c
   0  0  0  0  0  0  0  0  0  0  0  0  0
a  0  1  1  1  1  1  1  1  1  1  1  1  1
b  0  1  1  2  2  2  2  2  2  2  2  2  2
c  0  1  1  2  2  3  3  3  3  3  3  3  3
d  0  1  1  2  2  3  3  4  4  4  4  4  4
d  0  1  1  2  2  3  3  4  5  5  5  5  5
c  0  1  1  2  2  3  4  4  5  5  5  5  6
b  0  1  1  2  3  3  4  4  5  5  6  6  6
a  0  1  2  2  3  3  4  4  5  6  6  7  7
a  0  1  2  2  3  3  4  4  5  6  6  7  7
b  0  1  2  3  3  3  4  4  5  6  7  7  7
c  0  1  2  3  3  4  4  4  5  6  7  7  8
d  0  1  2  3  3  4  4  5  5  6  7  7  8
```

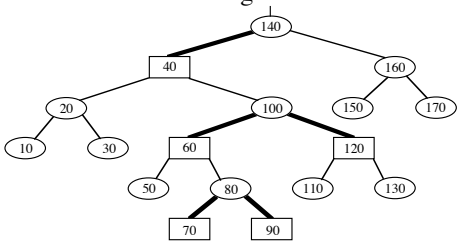
5. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

```
            8     2     5     3     4     1     9     0     7     6
Version:          1                 2/Sedgewick
```

6. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



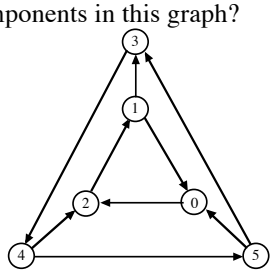
7. Insert 95 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320 Name _____
Test 3
Fall 2014

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Which edge is chosen in a phase of Kruskal's algorithm?
   A. An edge that is on a shortest path from the source
   B. The unprocessed edge (x, y) of smallest weight such that `find(x)==find(y)`
   C. The unprocessed edge (x, y) of smallest weight such that `find(x)!=find(y)`
   D. An edge of maximum-weight in a cycle (to be excluded)
2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, its type will be:
   A. Back B. Cross C. Forward D. Tree
3. Augmenting paths are usually found by:
   A. Breadth-first search on the saturated edges in the residual network.
   B. Breadth-first search on the unsaturated edges in the residual network.
   C. Depth-first search on the unsaturated edges in the residual network.
   D. Dijkstra's algorithm.
4. Which statement is incorrect regarding depth-first search on a directed graph?
   A. Exploring an edge whose head is colored black will cause the edge to be a back edge.
   B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
   C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
   D. The run time is $\Theta(m+n)$, where m is the number of edges and n is the number of vertices.
5. Suppose that an instance of bipartite matching has 10 vertices in the left column, 15 vertices in the right column, and 30 edges. The number of edges in the corresponding instance of network flow is:
   A. 25 B. 30 C. 55 D. 150
6. A topological ordering of a directed graph may be computed by:
   A. Ordering the vertices by descending finish time after DFS
   B. Ordering the vertices by ascending discovery time after DFS
   C. Ordering the vertices by ascending finish time after DFS
   D. Ordering the vertices by descending discovery time after DFS
7. Before searching for a minimum cut in a network, it is useful to do the following:
   A. Find one augmenting path.
   B. Determine the type of each edge using depth-first search.
   C. Find and record augmenting paths until none remains.
   D. Perform a breadth-first search on the input network.
8. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

9. During a breadth-first search, the status of a gray vertex is:
   A. It has been completely processed. B. It is in the FIFO queue.
   C. It is in the priority queue. D. It is undiscovered.
10. When a graph is dense, the best way to find a minimum spanning tree is:
   A. Prim's algorithm using T-table B. Warshall's algorithm
   C. Floyd-Warshall algorithm D. Prim's algorithm using heap
11. Suppose an adjacency matrix represents a directed graph with *V* vertices (numbered 0 .. *V*-1) and an adjacency list representation (with unordered lists) represents a directed graph with the same vertices. How fast can you verify that the two representations are storing the same graph? You are allowed to use additional memory.

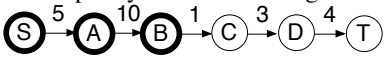
   A. $\Theta(V)$ B. $\Theta(V \log V)$ C. $\Theta\left(V^2\right)$ D. $\Theta\left(V^3\right)$

12. Which of the following cannot occur when edges are included in a directed graph?
   A. The number of strong components may remain the same.
   B. The number of strong components may decrease.
   C. The graph acquires a cycle.
   D. The number of strong components may increase.
13. Suppose Warshall's algorithm is used on a directed graph with vertices 0 . . . 25, but is stopped after column 10 in the matrix is processed. Which paths will be represented in the matrix?
   A. All paths that start at some vertex in 0 . . . 10 and stop at some vertex in 0 . . . 10.
   B. All paths that start at some vertex in 0 . . . 10, stop at some vertex in 0 . . . 10, and have only vertices in 0 . . . 10 in between.
   C. All paths that start at some vertex in 0 . . . 25, stop at some vertex in 0 . . . 25, and have only vertices in 0 . . . 10 in between.
   D. All paths with no more than 12 edges.
14. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:
   A. They are equal. B. The amount of flow does not exceed the net flow.
   C. The net flow does not exceed the amount of flow. D. There is no relationship.
15. Compressed adjacency lists have the following disadvantage:
   A. Testing whether an edge from X to Y is present will take θ(V + E) worst-case time.
   B. They are static.
   C. They can only be used for graphs without weights.
   D. They require θ(V + E) space to store.
16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

```
-1   3   3   3   3
-1   3   3   3   4
-1   1   1   1   4
-1   2   2   2   2
-1  -1  -1  -1  -1
```

   A. 0 B. 1 C. 2 D. 3

17. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:
   A. Use breadth-first search. B. Use Dijkstra's algorithm.
   C. Use the Floyd-Warshall algorithm. D. Use Prim's algorithm.
18. Which of the following is not true regarding the Edmonds-Karp variant.
   A. An augmenting path is found using breadth-first search.
   B. An augmenting path may be used several times.
   C. An edge may go back-and-forth between being saturated and unsaturated.
   D. It solves the network flow problem in polynomial time.
19. The capacity of the following cut is _____. (S vertices are bold.



   A. 1 B. 10 C. 15 D. 23

20. Which of the following is not true about probe sequences for an implementation of double hashing?
   A. Two keys could have the same probe sequence
   B. The probe sequence for a key cannot change
   C. All slots in the hash table appear in each probe sequence
   D. The elements of a probe sequence are possible keys for the hash table

21. Suppose a double hash table has $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2 B. 2 C. 5 D. 10
22. The main disadvantage of compressed adjacency lists is:
    A. Undirected graphs may not be represented B. Directed graphs may not be represented
    C. It is difficult to change the graph D. They waste space

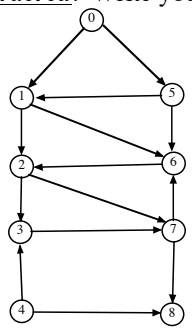
Problems 23 and 24 refer to the following hash table whose keys are stored by double hashing using $h_1(key) = key \% 13$ and $h_2(key) = 1 + (key \% 12)$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			120	186	187	162		122	110			194

23. 266 would be inserted into which slot of the given table?
    A. 0 B. 1 C. 2 D. 7 E. 10 F. 11
24. 313 would be inserted into which slot of the given table?
    A. 0 B. 1 C. 2 D. 7 E. 10 F. 11
25. Dijkstra's algorithm may be viewed as being a generalization of which technique?
    A. BFS B. DFS C. Minimum spanning trees D. Maximum flow

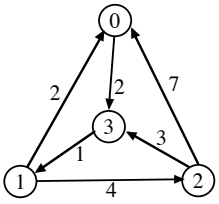
Long Answer

1. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ***ordered***. Write your answer in the tables below. 10 points

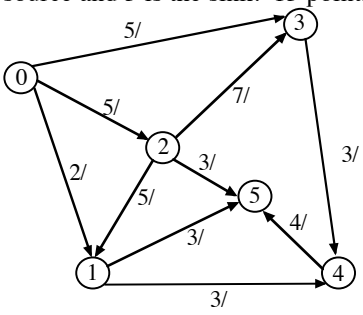


Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	____	0 1	____	5 1	____
1	____	____	0 5	____	5 6	____
2	____	____	1 2	____	6 2	____
3	____	____	1 6	____	7 6	____
4	____	____	2 3	____	7 8	____
5	____	____	2 7	____		
6	____	____	3 7	____		
7	____	____	4 3	____		
8	____	____	4 8	____		

2. Give an algorithm, based on the Floyd-Warshall algorithm with ***successors***, to determine for all pairs of starting and destination vertices the path that ***minimizes the sum*** of the intermediate vertex numbers. Please note the following as you give your answer: (15 points)
    a. Giving your solution as either a detailed transitivity diagram or C code is fine.
    b. If there is an input edge from vertex i to vertex j, then the sum would be zero. So, the corresponding matrix entry ***should not change***.
    c. If there were a path 10 → 20 → 15 → 12 →3, its sum would be 20 + 15 + 12 = 47. As usual vertex numbers start at 0, so the use of vertex 0 as an intermediate vertex is "free".
    d. A path may have no edges, i.e. a reflexive situation, so the sums on the diagonal will be zero.
3. Show the ***compressed*** adjacency list representation for this weighted graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.



4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 5 is the sink. 15 points.



S vertices:
T vertices:
Augmenting Paths and Contribution to Flow: