CSE 2320 Name _____

Test 1

Spring 2006 Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Suppose $f(x)$ is a monotonically increasing function. Which of the following approximates the summation?

   A. $\int_{m}^{n+1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m-1}^{n} f(x)dx$ B. $\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m+1}^{n+1} f(x)dx$

   C. $\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n+1} f(x)dx$ D. $\int_{m}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n} f(x)dx$

2. For which of the following sorts does the decision tree model not apply?
   A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT
3. Which of the following sorts is not stable?
   A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT
4. What is the definition of H_n?

   A. $\Theta\left(\sqrt{n}\right)$ B. $\sum_{k=1}^{n} k$ C. $\ln n$ D. $\sum_{k=1}^{n} \frac{1}{k}$

5. Which of the following functions is not in $\Omega(n)$?

   A. $\lg n$ B. $n \lg n$ C. $n^2 \lg n$ D. n^3

6. BUILD-MAX-HEAP is based on applying MAX-HEAPIFY in the following fashion:
   A. In ascending slot number order, for each slot that is a parent.
   B. In descending slot number order, for each slot that is a parent.
   C. $\frac{n}{2}$ times, each time from the root of the heap.
   D. n - 1 times, each time from the root of the heap.
7. Suppose that a binary search is to be performed on a table with 100 elements. The maximum number of elements that could be examined (probes) is:
   A. 4 B. 5 C. 6 D. 7
8. $f(n) = n \lg n$ is in all of the following sets, except

   A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega\left(\frac{1}{n}\right)$ D. $O\left(n^2\right)$

9. The expected time for insertion sort for n keys is in which set? (All $n!$ input permutations are equally likely.)

   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta\left(n^2\right)$

10. What is the value of $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$?

    A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 2

11. Suppose that you have correctly determined some c and n_o to prove that $f(n) \in \Omega(g(n))$. Which of the following is not necessarily true?

    A. c may be decreased B. c may be increased C. n_o may be increased D. $g(n) \in O(f(n))$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\lg n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

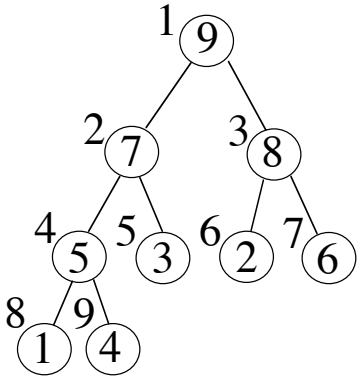
    A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

13. Which of the following is not true regarding a minheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 811.
14. Which of the following is not an application of a minheap?
    A. Choosing the pivots for QUICKSORT.
    B. Constructing sorted subfiles for external mergesort.
    C. Controlling multiway merges for external mergesort.
    D. Selecting the third smallest number from a set of one million numbers.
15. Which of the following will not be true regarding the decision tree for HEAPSORT for sorting *n* input values?
    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
    B. The height of the tree is $\Omega(n \log n)$.
    C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
    D. There will be *n*! leaves.

Long Answer

1. Array `a` contains `m ints` in ascending order and, similary, array `b` contains `n ints` in ascending order. Give code to merge the contents of these arrays (in ascending order) into `int` array `c`. Duplicate occurences of a value should be kept. 15 points
2. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points
3. Use the recursion-tree method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n$ is in $\Theta(n \log n)$. 10 points
4. Show the result after PARTITION manipulates the following subarray. 10 points

   7 0 2 4 3 1 6 9 8 5
5. Show the maxheap after performing HEAP-EXTRACT-MAX two times. 10 points



CSE 2320 Name _____

Test 2

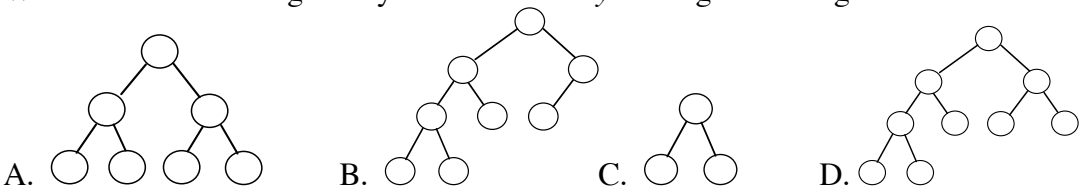
Spring 2006 Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which data structure operates in first-in-first-out fashion?

A. hashing with chaining B. hashing with open addressing C. queue D. stack

2. In a binary search tree, which element does not have a predecessor?
   A. any one of the leaves B. the maximum C. the minimum D. the root
3. When evaluating a prefix expression, the stack contains
   A. Both operands and operators
   B. Both parentheses and operators
   C. Operands only
   D. Operators only
4. What is the worst-case time to perform PREDECESSOR(L, x) for a sorted, doubly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. What is the worst-case time to perform PREDECESSOR(L, x) for an unsorted, singly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
6. Which of the following would not be used in implementing rat-in-a-maze in a depth-first fashion?
   A. Circular queue B. Recursion C. Stack D. 2-d array
7. What is the worst-case time to find the successor of a key in a red-black tree storing n keys? Assume that parent pointers are available.
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
8. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
   A. return stack[SP++] B. stack[SP++] = X C. stack[SP--] = X D. stack[++SP] = X
9. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



10. Circular linked lists are occasionally useful because
    A. some operations may be done in constant time.
    B. they are an alternative to red-black trees.
    C. they are useful for implementing circular queues.
    D. they avoid mallocs.
11. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(nodept x)
{
if (x==NULL)
  return;
traverse(x->left);
traverse(x->right);
// process x here
}
```

    A. inorder B. postorder C. preorder D. search for key `x`
12. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:
    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$
13. Assuming that each key stored in a double hash table with $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2 B. 2 C. 5 D. 10
14. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. $\frac{3}{2}\alpha$ D. 2α

15. Which of the following is not an issue in selecting a data structure for a dictionary?
    A. need for ordered retrieval
    B. need to delete keys
    C. number of keys to be stored
    D. size of the set of potential keys

Long Answer

1. Consider the following hash table whose keys were stored by linear probing using h(key, i) = (key + i) % 17.

```
0  -1
1  800
2  -1
3  700
4  -1
5  600
6  -1
7  500
8  -1
9  400
10 -1
11 -1
12 301
13 -1
14 201
15 -1
16 101
```

    a. Suppose 1000 is to be stored (using linear probing). Which slot will be used? (3 points)
    b. Suppose 1001 is to be stored (using linear probing) *after* 1000 has been stored. Which slot will be used? (4 points)

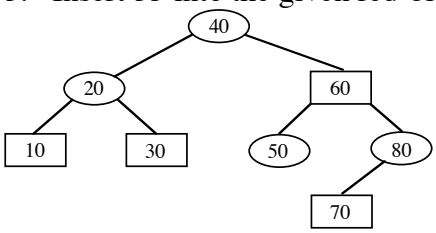
2. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 17$ and $h_2(key) = 1 + (key \% 16)$.

```
0  -1
1  800
2  -1
3  -1
4  701
5  -1
6  601
7  -1
8  501
9  -1
10 401
11 -1
12 301
13 -1
14 201
15 -1
16 101
```

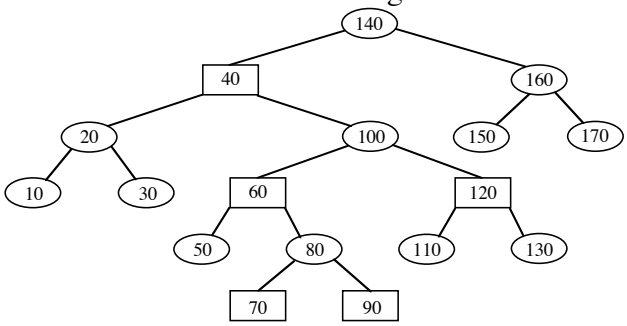
    a. Suppose 1000 is to be inserted (using double hashing). Which slot will be used? (4 points)

b. Suppose 1001 is to be inserted (using double hashing) *after* 1000 has been stored. Which slot will be used? (4 points)

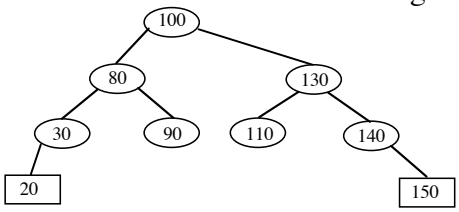
3. Insert 35 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



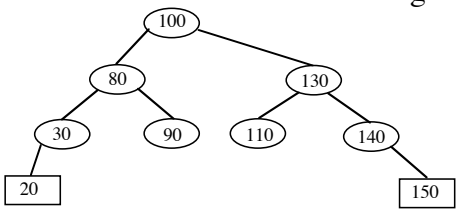
4. Insert 85 into the following red-black tree. Be sure to indicate the case(s) that you used. 10 points



5. Delete 30 from the following red-black tree. Be sure to indicate the case(s) that you used. 10 points



6. Delete 80 from the following red-black tree. Be sure to indicate the case(s) that you used. 10 points



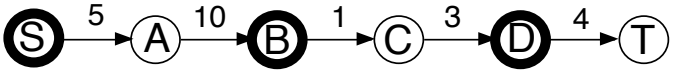
CSE 2320 Name _____

Test 3

Spring 2006 Last 4 Digits of Student ID # _____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. The capacity of the following cut is _____. (S vertices are bold.)



A. 1 B. 10 C. 13 D. 23

2. The fastest method for finding the diameter of a tree is to:
   A. Use breadth-first search.
   B. Use Dijkstra's algorithm.
   C. Use the Floyd-Warshall algorithm.
   D. Use the Ford-Fulkerson algorithm.
3. Suppose the compressed adjacency list representation is used for an undirected graph with n vertices and m edges. Assuming there are no self-loops, the number of entries in the two tables are:
   A. n for both
   B. m for both
   C. $n + 1$ and m

D. $n + 1$ and $2m$

4. Which of the following is a longest common subsequence for 0 1 0 2 1 2 and 0 0 1 1 2 2?
   A. 0 0 1 1 B. 0 0 1 1 2 C. 0 0 1 2 D. 0 1 2 0
5. When a graph is sparse, the best way to find a shortest path between every pair of vertices is:
   A. Dijkstra's algorithm using heap
   B. Dijkstra's algorithm using T-table
   C. Floyd-Warshall algorithm
   D. Warshall's algorithm
6. The worst-case time for Prim's algorithm implemented with a T-table is:
   A. $\Theta(V + E)$ B. $\Theta\left(V^2 + E\right)$ C. θ(V lg V) D. θ(V lg E)
7. Which of the following is true about KMP string search?
   A. Once the fail links have been constructed, the pattern is no longer needed.
   B. The fail links are constructed based on the pattern and may be applied to different texts.
   C. The fail links are constructed based on the text and may be applied to different patterns.
   D. The fail links are constructed for a particular pattern and a particular text.
8. Suppose that there is exactly one path from vertex 8 to vertex 10 in a directed graph: $8 \rightarrow 7 \rightarrow 3 \rightarrow 5 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?
   A. 3 B. 5 C. 7 D. 8
9. Which statement is not correct about depth-first search on a directed graph?
   A. Exploring an edge whose head is colored black will cause the edge to be a back edge.
   B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
   C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
   D. The run time is $\Theta(m + n)$, where m is the number of edges and n is the number of vertices.
10. Which of the following is not true regarding the Edmonds-Karp variant.
    A. An augmenting path is found using breadth-first search.
    B. An augmenting path may be used several times.
    C. An edge may go back-and-forth between being saturated and unsaturated.
    D. It solves the network flow problem in polynomial time.
11. Suppose that a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times is:
    A. finish(X) < finish(Y) B. finish(X) > finish(Y)
    C. finish(X) = finish(Y) D. could be either A. or B.
12. A fail link of `-1` requires the KMP matcher to take what action?
    A. Give up the search entirely, since the pattern cannot appear within the text.
    B. Move both pointers up one symbol.
    C. Move the pattern pointer to the next pattern symbol and set the text pointer to 0.
    D. Move the text pointer to the next text symbol and set the pattern pointer to 0.
13. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)
    A. Adjacency lists (ordered): $\Theta(\log V)$
    B. Adjacency lists (unordered): $\Theta(V)$
    C. Adjacency matrix: $\Theta(1)$
    D. Compressed adjacency lists (ordered): $\Theta(\log V)$
14. Suppose that an instance of bipartite matching has 5 vertices in the left column, 15 vertices in the right column, and 10 edges. The number of edges in the corresponding instance of network flow is:
    A. 25

B. 30
C. 55
D. 150

15. Which of the following is solved heuristically by a greedy method?
    A. Fractional knapsack
    B. Finding the shortest paths from a designated source vertex in a sparse graph.
    C. Minimum spanning tree
    D. 0/1 knapsack
16. Prim's algorithm, when implemented with a heap, is most suitable for:
    A. Finding the minimum spanning tree of a dense graph.
    B. Finding the minimum spanning tree of a sparse graph.
    C. Finding the shortest paths from a designated source vertex in a dense graph.
    D. Finding the shortest paths from a designated source vertex in a sparse graph.
17. What is the purpose of the first depth-first search when finding strongly connected components?
    A. To assure that two vertices that are in the same cycle will be output in the same component
    B. To assure that two vertices with no paths between them are not output in the same component
    C. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.
    D. To make sure that the input graph has no cycles.
18. Which of the following dynamic programming problems maximizes its cost function?
    A. Optimal Matrix Multiplication
    B. Parking Permits
    C. Shuttle-to-Airport
    D. Weighted Interval Scheduling
19. Suppose that the input for Huffman code tree construction is 10 symbols, each with probability 0.1 for occurring in a file to be compressed. What will be the expected bits per symbol when using the constructed tree?

    A. 3 B. 3.2 C. 3.4 D. 4

20. Before searching for a minimum cut in a network, it is useful to do the following:
    A. Determine the type of each edge using depth-first search.
    B. Find and record augmenting paths until none remains.
    C. Find one augmenting path.
    D. Perform a breadth-first search on the input network.

Long Answer

1. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
  p[0]=5    p[1]=4    p[2]=3    p[3]=4    p[4]=5    p[5]=6

        1         2         3         4         5
1    0   0   60   1  120   2  195   2  ???   ?

2  -------    0   0   48   2  120   2  222   2

3  -------  -------    0   0   60   3  150   4

4  -------  -------  -------    0   0  120   4

5  -------  -------  -------  -------    0   0
```

2. Fill in the KMP failure links. 10 points.

	0	1	2	3	4	5	6	7	8	9
1:										
pattern:	a	b	c	d	a	b	c	a	b	a
2:										

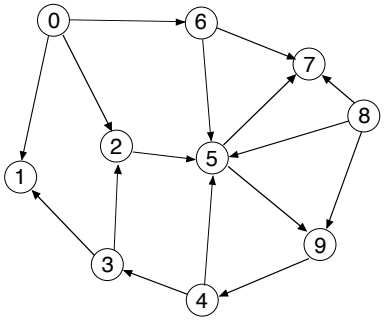
3. Solve the following instance of the parking permit problem by indicating the range of days covered for each of the permits used. 10 points.

Permit #	Duration	Cost
0	1	5
1	3	7
2	7	10

Permits are needed to cover the following days:

1
2
10
11
12
13
15
20
21
22
23
24
25

4. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answers in the tables below. 10 points

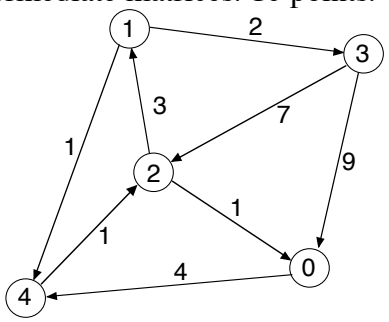


Vertex	Start	Finish
0	1	____
1	____	____
2	____	____
3	____	____
4	____	____
5	____	____
6	____	____
7	____	____
8	____	____
9	____	____

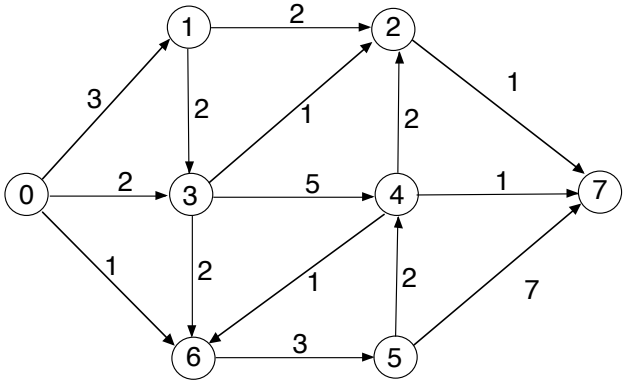
Edge	Type
0 1	____
0 2	____
0 6	____
2 5	____
3 1	____
3 2	____
4 3	____
4 5	____
5 7	____
5 9	____

Edge	Type
6 5	____
6 7	____
8 5	____
8 7	____
8 9	____
9 4	____

5. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following graph. The paths indicated in the final matrix must have **at least one** edge. You are not required to show the intermediate matrices. 10 points.



6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.



S vertices: 0
T vertices: 7
Augmenting Paths and Contribution to Flow: