

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to find the minimum of the n elements of an integer array is in:

- A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. The goal of the Huffman coding method is:

- A. Construct a max-heap for the symbols in an alphabet
 B. Find the symbols with high probability of occurring.
 C. Maximize the compression for every string.
 D. Minimize the expected bits per symbol.

3. Which of the following is true?

- A. $n^3 \in \Omega(n^2)$ B. $n \log n \in \Omega(n^2)$ C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $3^n \in O(2^n)$

4. Which of the following facts cannot be proven using one of the limit theorems?

- A. $n^2 \in O(n^3)$ B. $n^2 \in \Omega(n \log n)$ C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ D. $3^n \in \Omega(2^n)$

5. The function $2 \log n + \log n$ is in which set?

- A. $\Omega(n \log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

- A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega(n)$ D. $O(n^2)$

7. The number of calls to `mergeAB` while performing `mergesort` on n items is in:

- A. $\Theta(\log n)$ B. $\Theta(m + n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

8. What is required when calling `union(i, j)` for maintaining disjoint subsets?

- A. i is the ancestor of j in one of the trees B. i and j are in the same subset
 C. i and j are leaders for different subsets D. i and j are leaders for the same subset

9. Suppose that you have correctly determined some c and n_0 to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

- A. c may be decreased B. n_0 may be decreased C. n_0 may be increased D. $f(n) \in O(g(n))$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that

$T(n) \in \Omega(1)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

- A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

11. What is n , the number of elements, for the largest table that can be processed by binary search using no more than 6 probes?

- A. 31 B. 63 C. 64 D. 127

12. The time to determine whether an integer array, in ascending order, has any duplicate values is in which set?

- A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

13. Which of the following functions is not in $\Omega(n^2)$?

- A. n B. n^2 C. $n^2 \lg n$ D. n^3

14. What is the value of $\sum_{k=0}^t 2^k$?

- A. 2^k B. 2^t C. $2^{t+1} - 1$ D. $2^{t+1} + 1$

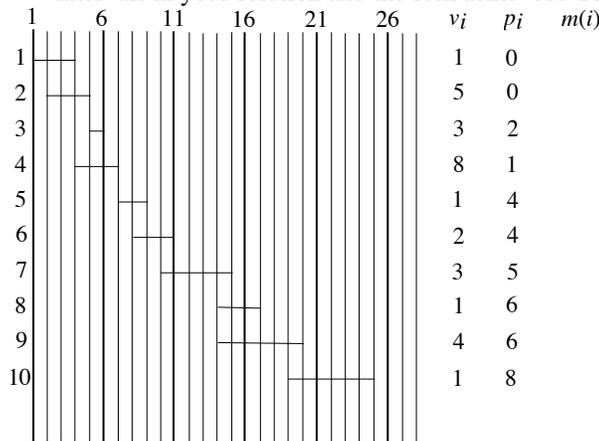
15. When solving the fractional knapsack problem, the items are processed in the following order.

- A. Ascending order of weight B. Ascending order of \$\$\$/lb

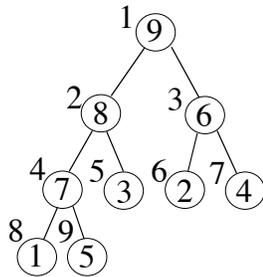
C. Descending order of weight D. Descending order of \$\$\$/lb

Long Answer

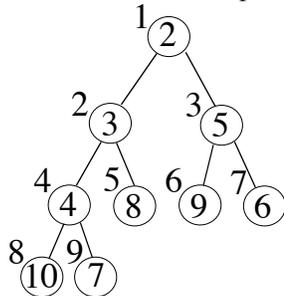
1. Prove that if $f(n) \in \Omega(g(n))$ then $\frac{1}{f(n)} \in O\left(\frac{1}{g(n)}\right)$. 5 points
2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



3. Give the greedy solution for the unweighted interval scheduling problem using the set of intervals for problem 2. You may simply give the indices for the intervals in the solution. 10 points
4. a. Show the maxheap after performing `getmax`. 5 points



- b. Show the minheap after changing the priority at subscript 4 to 1. 5 points



5. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta(\sqrt{n} \log n)$. 10 points
6. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta(\sqrt{n} \log n)$. 10 points

CSE 2320

Name _____

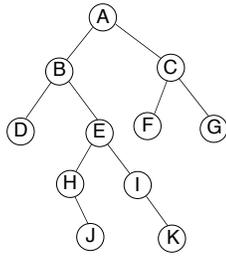
Test 2

Spring 2010

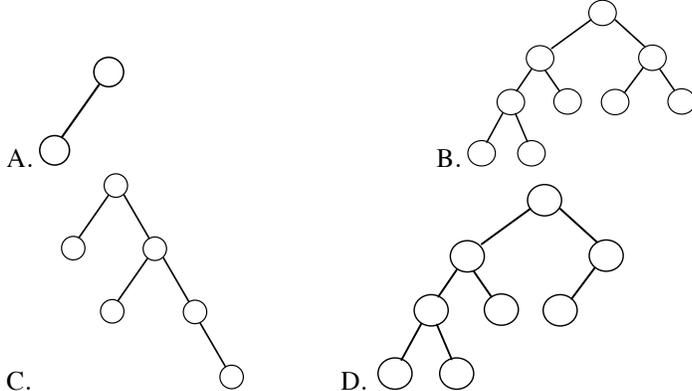
Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The queue for breadth-first rat-in-a-maze stores
 - A. all maze positions that have walls
 - B. maze positions that must be in the final path
 - C. maze positions that have been reached
 - D. the current path being explored
2. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at F?



- A. A
 - B. B
 - C. C
 - D. G
3. Which of the following is a longest common subsequence for 0 1 2 0 1 2 and 0 0 1 1 2 2?
 - A. 0 0 1 2 B. 0 1 2 0 C. 0 0 1 1 D. 0 0 1 1 2
 4. In the example of recycling the elements of a list in $O(1)$ time, which situation holds?
 - A. Both lists are circular
 - B. Both lists are not circular
 - C. The list to be recycled is circular, the garbage list is not
 - D. The garbage list is circular, the list to be recycled is not
 5. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with n nodes in ascending order is:
 - A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
 6. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing n keys? Assume that parent pointers are available.
 - A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
 7. The two mandatory pointers in a node for a rooted tree with linked siblings are:
 - A. Left child and right child B. Left child and parent
 - C. First child and right sibling D. Left sibling and right sibling
 8. In which situation will a sentinel be inappropriate?
 - A. Search for a key in an unordered table, to simplify and speed-up code
 - B. Search for a key in an unordered linked list, to simplify and speed-up code
 - C. Red-black tree, to simplify code
 - D. Binary search for a key in an ordered table, to simplify and speed-up code
 9. In a red-black tree holding n keys, what is the total number of left and right pointers that will be set to nil (the sentinel)?
 - A. $n - 1$ B. n C. $n + 1$ D. None of the above
 10. How should the successor of a node without a right child in an unbalanced binary search tree be found?
 - A. Examine the ancestors of the node
 - B. Go left, then proceed to the right
 - C. Go right, then proceed to the left
 - D. Preorder traversal
 11. Which of the following binary trees has *multiple* legal colorings as a red-black tree?



12. If POP is implemented as `return stack[--SP]`, then PUSH of element X is implemented as:
 - A. `return stack[SP++]` B. `stack[SP++] = X`
 - C. `stack[--SP] = X` D. `stack[++SP] = X`

13. An unsorted integer array with 1500 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
- If 1468 is returned from PARTITION, we must continue.
 - If 1469 is returned from PARTITION, we are done.
 - If 1470 is returned from PARTITION, we must continue.
 - If 1481 is returned from PARTITION, we must continue.
14. Which binary tree traversal corresponds to the following recursive code?
- ```
void traverse(noderef x)
{
 if (x==null)
 return;
 traverse(x.left);
 traverse(x.right);
 // process x here
}
```
- A. inorder B. postorder C. preorder D. search for key x
15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
- 700, 200, 600, 550, 500
  - 200, 700, 600, 300, 400, 500
  - 100, 1000, 200, 800, 300, 900, 500
  - 300, 400, 900, 800, 500

Long Answer

- List the four phases in a counting sort and give the asymptotic time needed for each phase. (5 points)
- Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

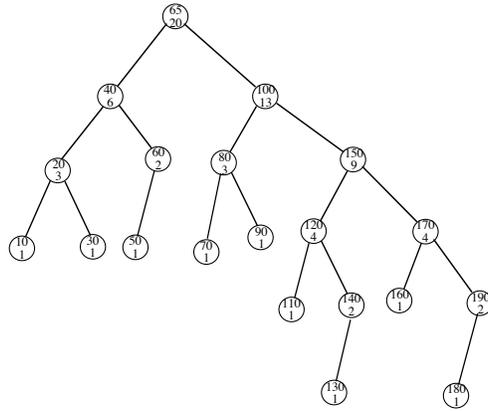
```
p[0]=6
p[1]=2
p[2]=4
p[3]=3
p[4]=2
```

|   | 1     | 2     | 3     | 4  |    |    |     |   |
|---|-------|-------|-------|----|----|----|-----|---|
| 1 | 0     | 0     | 48    | 1  | 60 | 1  | ??? | ? |
| 2 | ----- | 0     | 0     | 24 | 2  | 36 | 3   |   |
| 3 | ----- | ----- | 0     | 0  | 24 | 3  |     |   |
| 4 | ----- | ----- | ----- | 0  | 0  |    |     |   |

- Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

```
Version: 8 2 6 3 4 1 9 0 7 5
 1 2/Sedgewick
```

- Use dynamic programming to solve the following instance of the (monotone) longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. 10 points
- ```
5 10 15 20 25 7 10 15 22 25 5 10 22 26 27
```
- The following binary search tree includes keys and subtree sizes. Clearly circle those nodes that would be examined while (efficiently) determining the key with rank 13. (10 points)



6. A billion integers in the range $0 \dots 999,999$ are to be sorted by LSD radix sort. How much faster will this be done if radix $0 \dots 999$ is used rather than decimal ($0 \dots 9$) radix? Show your work. (10 points)

CSE 2320

Name _____

Test 3

Spring 2010

Last 4 Digits of Student ID # _____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Suppose a maximum bipartite matching with k edges is found using Edmonds-Karp. Which of the following does not hold?

A. The capacity of the minimum cut is k .	B. There will be $k + 1$ breadth-first searches.
C. All residual network capacities are zero or one.	D. Every augmenting path uses three edges.
2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z . If there is also an edge from Y to Z , then its type will be:

A. Back	B. Cross	C. Forward	D. Tree
---------	----------	------------	---------
3. Which edge is chosen in a phase of Kruskal's algorithm?

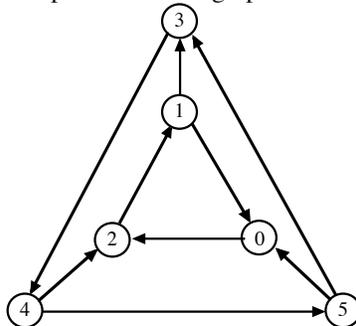
A. A minimum-weight edge that keeps the result free of cycles	B. A minimum-weight edge connecting T to S .
C. An edge that is on a shortest path from the source	D. An edge of maximum-weight in a cycle (to be excluded)
4. The fastest method for finding the diameter of a tree is to:

A. Use the Floyd-Warshall algorithm.	B. Use the Ford-Fulkerson algorithm.
C. Use breadth-first search.	D. Use Dijkstra's algorithm.
5. The capacity of any cut is:

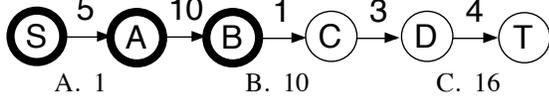
A. An upper bound on the maximum flow.	B. The same as the capacity of all other cuts.
C. The same as the maximum attainable flow.	D. A lower bound on the maximum flow.
6. Suppose a directed graph has a path from vertex X to vertex Y , but no path from vertex Y to vertex X . The relationship between the finish times for depth-first search is:

A. $finish(X) < finish(Y)$	B. $finish(X) > finish(Y)$
C. $finish(X) = finish(Y)$	D. could be either A. or B.
7. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:

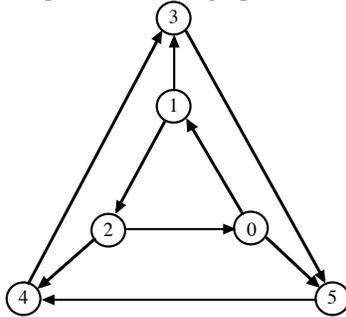
A. They are equal.
B. The amount of flow does not exceed the net flow.
C. The net flow does not exceed the amount of flow.
D. There is no relationship.
8. What is the number of strongly connected components in this graph?



9. The capacity of the following cut is _____. (S vertices are bold.)
- A. 1
B. 2
C. 3
D. 4

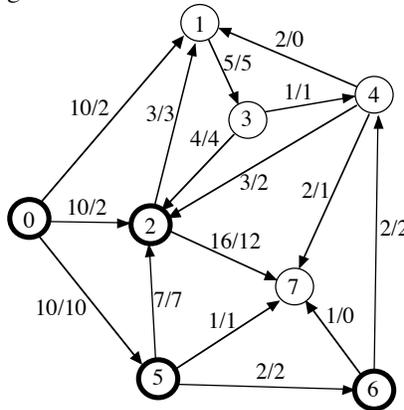


- A. 1
B. 10
C. 16
D. 23
10. When a graph is dense, the best way to find a minimum spanning tree is:
- A. Prim's algorithm using T-table
B. Floyd-Warshall algorithm
- C. Prim's algorithm using heap
D. Warshall's algorithm
11. The Edmonds-Karp variant is important because:
- A. It solves the bipartite matching problem.
- B. It solves the network flow problem in polynomial time.
- C. It solves the network flow problem using critical edges.
- D. It solves the network flow problem without using augmenting paths.
12. Which of the following cannot occur when additional edges are included in a directed graph?
- A. The graph acquires a cycle.
- B. The number of strong components may remain the same.
- C. The number of strong components may decrease.
- D. The number of strong components may increase.
13. The number of potential probe sequences when using linear probing with a table with m entries is:
- A. $O(\log m)$
B. m
C. $m(m-1)$
D. $m!$
14. What is the number of strongly connected components in this graph?



- A. 1
B. 2
C. 3
D. 4
15. The worst-case time for Prim's algorithm implemented with a minheap is:
- A. $\theta(V + E)$
B. $\theta(E \lg V)$
C. $\theta(V \lg V)$
D. $\theta(V \lg E)$

Problems 16, 17, and 18 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow.



16. The capacity of the indicated cut (S vertices are bold) is:
- A. 31
B. 32
C. 33
D. 34
17. The net flow across the given cut is:
- A. 14
B. 16
C. 18
D. 20
18. Suppose the flow is increased as much as possible using the augmenting path $0 \rightarrow 2 \rightarrow 4 \rightarrow 7$. Which is the critical edge?
- A. $0 \rightarrow 2$
B. $2 \rightarrow 4$
C. $4 \rightarrow 7$
D. Insufficient information
19. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers $0, 1, \dots, V-1$.)

- A. Adjacency lists (ordered): $\Theta(V)$ B. Adjacency lists (unordered): $\Theta(V)$
 C. Adjacency matrix: $\Theta(1)$ D. Compressed adjacency lists (ordered): $\Theta(V)$

20. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

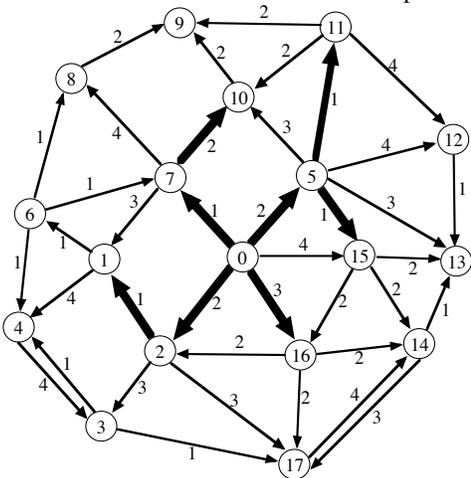
- A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. 2α

Long Answer

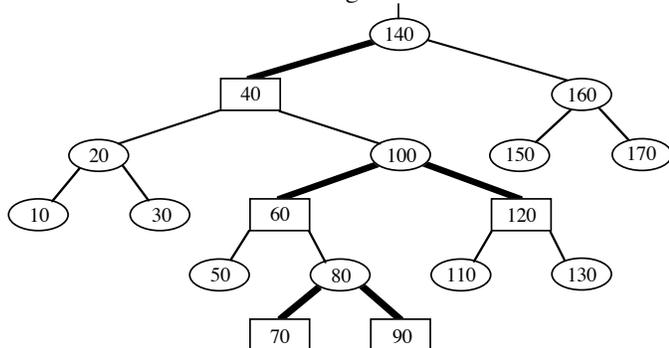
1. Consider the following hash table whose keys were stored by double hashing using $h_1(\text{key}) = \text{key} \% 17$ and $h_2(\text{key}) = 1 + (\text{key} \% 16)$.

0	-1
1	800
2	-1
3	-1
4	701
5	-1
6	601
7	-1
8	501
9	-1
10	401
11	-1
12	301
13	-1
14	201
15	-1
16	101

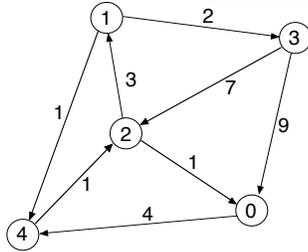
- a. Suppose 1000 is to be inserted (using double hashing). Which slot will be used? (5 points)
 b. Suppose 1001 is to be inserted (using double hashing) *after* 1000 has been stored. Which slot will be used? (5 points)
 2. What are the entries in the heap (for Dijkstra's algorithm) *before* and *after* moving the next vertex and edge into the shortest path tree? DO NOT COMPLETE THE ENTIRE TREE!!! Edges already in the shortest path tree are the thick ones. Vertex 0 is the source. 10 points.



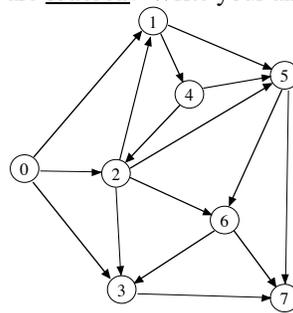
3. Insert 115 into the following red-black tree. Be sure to indicate the cases you used. (10 points)



4. Demonstrate the Floyd-Warshall algorithm, *with successors*, for the following graph. The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.

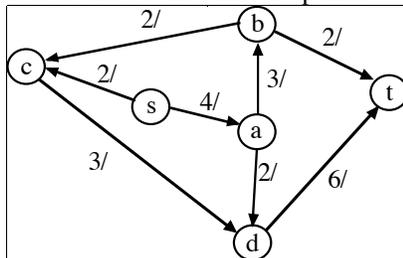


5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ordered. Write your answer in the tables below. 10 points



Vertex	Start	Finish	Edge	Type	Edge	Type
0	<u>1</u>	___	0 1	___	2 6	___
1	___	___	0 2	___	3 7	___
2	___	___	0 3	___	4 2	___
3	___	___	1 4	___	4 5	___
4	___	___	1 5	___	5 6	___
5	___	___	2 1	___	5 7	___
6	___	___	2 3	___	6 3	___
7	___	___	2 5	___	6 7	___

6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. *s* is the source and *t* is the sink. 10 points.



Minimum Cut:

S vertices: s

T vertices: t

Augmenting Paths and Contribution to Flow: