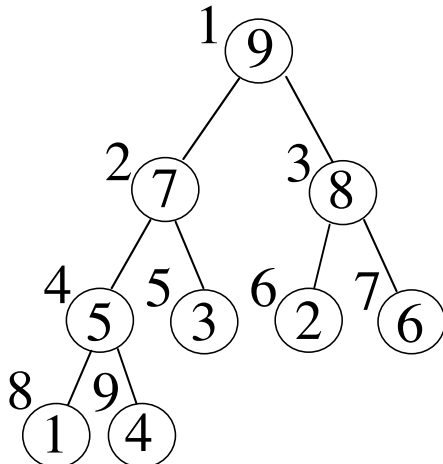Name _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to multiply two n x n matrices is:

   A. $\Theta(n)$ 　　 B. $\Theta(n\log n)$ 　　 C. $\Theta\left(n^2\right)$ 　　 D. $\Theta\left(n^3\right)$

2. For which of the following sorts does the decision tree model not apply?
   　　 A. Counting 　B. Insertion 　 C. MERGE-SORT 　　 D. QUICKSORT

3. Which of the following sorts is not stable?
   　　 A. HEAPSORT 　B. Insertion 　C. LSD Radix Sort 　　 D. MERGE-SORT

4. As n approaches ∞, $\dfrac{H_n}{H_{2n}}$ approaches?

   　　 A. 1 　　　 B. 2 　　　 C. $\ln n$ 　　 D. $n!$

5. The function $n + \log n$ is in which set?
   　　 A. $\Omega(n\log n)$ 　　　 B. $\Theta(\log n)$ 　　　 C. $\Theta(n)$ 　　 D. $\Theta(n\log n)$

6. Which of the following expressions corresponds to the time needed for LSD Radix Sort?
   　　 A. $\Theta(d + k + n)$ 　　 B. $\Theta(d(k + n))$ 　　 C. $\Theta(k + n)$ 　　　 D. $\Theta(dkn)$

7. Suppose a binary search is to be performed on a table with 50 elements. The maximum number of elements that could be examined (probes) is:
   　　 A. 4 　B. 5 　C. 6 　D. 7

8. $f(n) = n\lg n$ is in all of the following sets, except

   　　 A. $\Omega(\log n)$ 　 B. $\Theta(\log(n!))$ 　 C. $O\left(\dfrac{1}{n}\right)$ 　 D. $O\left(n^2\right)$

9. The expected time for QUICKSORT for n keys is in which set? (All $n!$ input permutations are equally likely.)

   　　 A. $\Theta(\log n)$ 　 B. $\Theta(n)$ 　 C. $\Theta(n\log n)$ 　 D. $\Theta\left(n^2\right)$

10. 　　 What is the value of $\displaystyle\sum_{k=0}^{t} 2^k$?

    A. 2^k 　 B. 2^t 　 C. $2^{t+1} - 1$ 　 D. $2^{t+1} + 1$

11. Suppose that you have correctly determined some c and n_o to prove that $f(n) \in O(g(n))$. Which of the following is not necessarily true?

    A. c may be decreased 　 B. c may be increased 　 C. n_o may be increased 　 D. $g(n) \in \Omega(f(n))$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that $T(n) \in \Omega(n)$ and $T(n) \in O\left(n^3\right)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(\lg n)$ 　　　 B. $T(n) \in O(n)$ 　　 C. $T(n) \in \Omega\left(n^2\right)$ 　　 D. $T(n) \in \Omega\left(n^3\right)$

13. Suppose the 201 priorities in a maxheap are unique. Which of the following subscripts may not store the minimum?
    　　 A. 100 　　　 B. 101 　　　 C. 199 　　　 D. 200

14. Suppose a minheap that can hold m records is used to produce sorted subfiles for an external mergesort. If the large input file is randomly ordered, the expected number of records in a subfile is:

    　　 A. m 　　　 B. $2m$ 　　　 C. $m\ln m$ 　　 D. m^2

15. Which of the following will not be true regarding the decision tree for insertion sort for sorting n input values?
    A. Every path from the root to a leaf will have $O(n\log n)$ decisions.
    B. The height of the tree is $\Omega(n\log n)$.
    C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
    D. There will be $n!$ leaves.

Long Answer
1. Suppose an `int` array `a` contains `m` zeroes followed by `n` ones, where `m` and `n` are unknown. The size of the array is given to you as `p`, i.e. `p==m+n`. Give C code for a binary search to determine `m` in $O(\log p)$ time. 15 points

2. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n^2$ is in $\Theta\left(n^2\right)$. 10 points

3. Use the recursion-tree method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n^2$ is in $\Theta\left(n^2\right)$. 10 points

4. Show the result after PARTITION manipulates the following subarray. 10 points

    | 3 | 1 | 6 | 9 | 8 | 5 | 7 | 0 | 2 | 4 |

5. Show the result after performing BUILD-MIN-HEAP. 10 points

CSE 2320   Name _____
Test 2
Summer 2006   Last 4 Digits of Student ID # _____
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each
1. Why is it common for a circular queue implementation to waste one table element?
    A. To avoid confusing an empty table with a full table
    B. To have a place to store the tail and head values
    C. To make sure the queue always has at least one element in use
    D. To perform some loops faster
2. In a binary search tree, which element does not have a successor?
    A. any one of the leaves   B. the maximum   C. the minimum D. the root
3. When evaluating a postfix expression, the stack contains
    A. Both operands and operators
    B. Both parentheses and operators
    C. Operands only
    D. Operators only
4. What is the worst-case time to perform MINIMUM(L) for an unsorted, doubly-linked list with n nodes?
    A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

5. What is the worst-case time to perform SUCCESSOR(L, x) for a sorted, singly-linked list with n nodes?
    A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

6. Recursion is often an alternative to using which data structure?
    A. Linked list   B. Queue   C. Stack   D. 2-d array

7. What is the worst-case time to find the successor of a key in an unbalanced binary search tree storing n keys? Assume that parent pointers are available.

    A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

8. If POP is implemented as return stack[SP--], then PUSH of element X is implemented as:
    A. return stack[SP++]   B. stack[SP++] = X   C. stack[SP--] = X   D. stack[++SP] = X

9. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

A.

B.

C.

D.
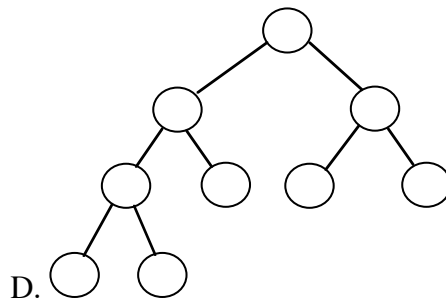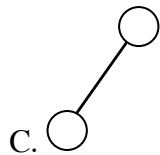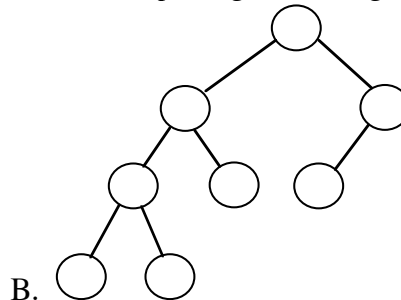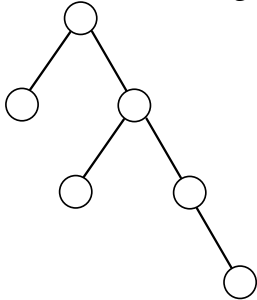
10. Primary clustering is a deficiency of which data structure?
    A. hashing with chaining
    B. double hashing
    C. linear probing
    D. red-black trees

11. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(nodept x)
{
if (x==NULL)
   return;
traverse(x->left);
// process x here
traverse(x->right);
}
```
    A. inorder   B. postorder   C. preorder   D. search for key x

12. The number of potential probe sequences when using linear probing with a table with m entries (m is prime) is:
    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

13. Assuming that each key stored in a double hash table with $\alpha = 0.5$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2   B. 2   C. 5   D. 10

14. The expected number of probes for an unsuccessful search in hashing by chaining with α as the load factor is:

A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. 2α

15. When a rooted tree with linked siblings is used, the two pointers in each node usually point to:
    A. first child and last child
    B. first child and right sibling
    C. left child and parent
    D. left child and right child

Long Answer

1. Consider the following hash table whose keys were stored by linear probing using
h(key, i) = (key + i) % 19. Show your work.

```
 0  -1
 1  400
 2  800
 3  -1
 4  -1
 5  -1
 6  101
 7  500
 8  -1
 9  -1
10  -1
11  201
12  600
13  -1
14  -1
15  -1
16  301
17  700
18  -1
```

   a. Suppose 1000 is to be stored (using linear probing). Which slot will be used? (3 points)
   b. Suppose 1001 is to be stored (using linear probing) *after* 1000 has been stored. Which slot will be used? (4 points)

2. Consider the following hash table whose keys were stored by double hashing using
h_1(key) = key % 19 and h_2(key) = 1 + (key % 18). Show your work.

```
 0  -1
 1  800
 2  401
 3  -1
 4  -1
 5  -1
 6  101
 7  501
 8  -1
 9  -1
10  -1
11  201
12  601
13  -1
14  -1
```

```
15 -1
16 301
17 701
18 -1
```

   a. Suppose 1000 is to be inserted (using double hashing). Which slot will be used? (4 points)

   b. Suppose 4001 is to be inserted (using double hashing) *after* 1000 has been stored. Which slot will be used? (4 points)
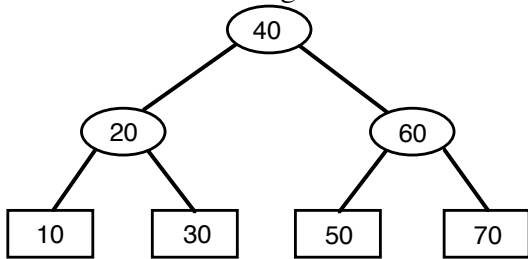
3. Insert 65 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)

4. Insert 125 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
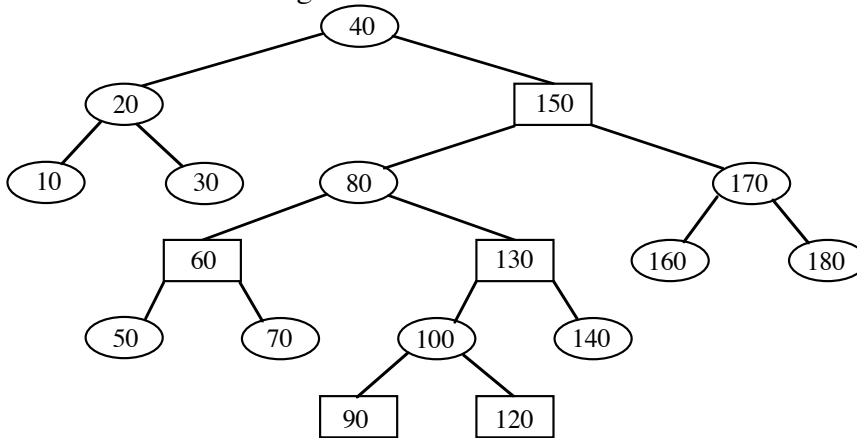
5. Delete 60 from the given red-black tree. Be sure to indicate the cases that you used. (10 points)
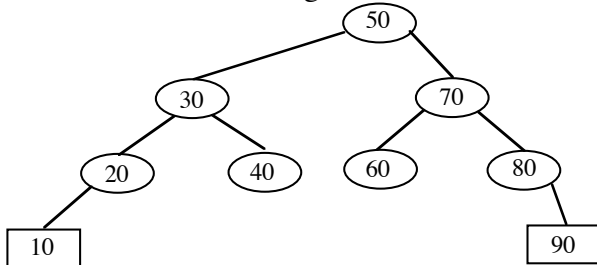
6. Delete 150 from the following red-black tree. Be sure to indicate the case(s) that you used. 10 points

Name _____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. An adjacency matrix is the most useful representation for which problem?
    A. Breadth-first search
    B. Finding strongly-connected components
    C. Maximum network flow
    D. Warshall's algorithm

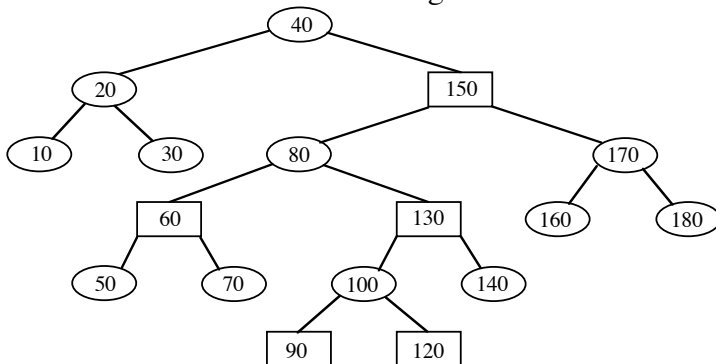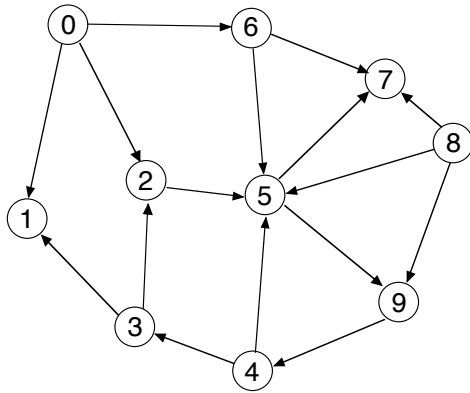2. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is:
    A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

3. Which of the following is true about any cut for an instance of the network flow problem?
    A. The capacity is a lower bound on the maximum flow.
    B. The capacity is an upper bound on the maximum flow.
    C. The capacity is equal to the maximum flow.
    D. The cut was found by a depth-first search on the original input network.

4. Suppose a depth-first search is performed on an undirected graph. The graph is a free (i.e. unrooted) tree if:
    A. all edges are tree edges
    B. both C and D
    C. there are no restarts
    D. there are no back edges

5. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. Assuming there are no self-loops, the number of entries in the two tables are:
    A. n for both
    B. m for both
    C. $n + 1$ and m
    D. $n + 1$ and $2m$

6. Which of the following is a longest common subsequence for 012012012 and 000111222?
    A.   012012   B.   000122   C.   001122   D.   01222

7. When a graph is dense, the best way to find a shortest path between every pair of vertices is:
    A. Dijkstra's algorithm using heap
    B. Dijkstra's algorithm using T-table
    C. Floyd-Warshall algorithm
    D. Warshall's algorithm

8. The worst-case time for Prim's algorithm implemented with a T-heap is:

    A. $\Theta(V + E)$ B. $\Theta\left(V^2 + E\right)$ C. $\theta(V \lg V)$ D. $\theta(V \lg E)$

9. Which of the following is true about KMP string search?
    A. Once the fail links have been constructed, the pattern is no longer needed.
    B. The fail links are constructed based on the pattern and may be applied to different texts.
    C. The fail links are constructed based on the text and may be applied to different patterns.
    D. The fail links are constructed for a particular pattern and a particular text.

10. Suppose that there is exactly one path from vertex 8 to vertex 10 in a directed graph: $5 \rightarrow 7 \rightarrow 8 \rightarrow 3 \rightarrow 2 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?
    A. 2   B. 5   C. 8   D. 10

11. Suppose that a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, then its type will be:
    A. Back   B. Cross   C. Forward   D. Tree

12. Which of the following is not true for the activity scheduling problem?
   A. The activities may have various durations.
   B. The greedy solution is a heuristic.
   C. There may be several optimal solutions.
   D. The goal is to maximize the number of activities.

13. Suppose that a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:
   A. finish(X) < finish(Y)   B. finish(X) > finish(Y)
   C. finish(X) = finish(Y)   D. could be either A. or B.

14. Which of the following is true about KMP string search?
   A. Once the fail links have been constructed, the pattern is no longer needed.
   B. The fail links are constructed based on the pattern and may be applied to different texts.
   C. The fail links are constructed based on the text and may be applied to different patterns.
   D. The fail links are constructed for a particular pattern and a particular text.

15. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)
   A. Adjacency lists (ordered): $\Theta(V)$
   B. Adjacency lists (unordered): $\Theta(V)$
   C. Adjacency matrix: $\Theta(1)$
   D. Compressed adjacency lists (ordered): $\Theta(V)$

16. Suppose that a maximum bipartite matching with k edges is found using Edmonds-Karp. Which of the following does not hold?
   A. All residual network capacities are zero or one.
   B. Every augmenting path uses three edges.
   C. The capacity of the minimum cut is k.
   D. There will be $k + 1$ breadth-first searches.

17. Which of the following is solved optimally by a greedy method with preprocessing by sorting?
   A. Fractional knapsack
   B. Finding the shortest paths from a designated source vertex in a sparse graph.
   C. Minimum spanning tree
   D. 0/1 knapsack

18. Dijkstra's algorithm, when implemented with a heap, is most suitable for:
   A. Finding the minimum spanning tree of a dense graph.
   B. Finding the minimum spanning tree of a sparse graph.
   C. Finding the shortest paths from a designated source vertex in a dense graph.
   D. Finding the shortest paths from a designated source vertex in a sparse graph.

19. Suppose that an instance of the fractional knapsack problem has more than one optimal solution by using the greedy approach. Each solution is distinguished by the amount of each available item that is placed in the knapsack. Which of the following must hold?
   A. All available items have the same $$$/lb ratio.
   B. Each available item has a different $$$/lb ratio.
   C. The amount chosen of the last item placed in the knapsack must be a fraction of the available amount, i.e. some of that item will be left out of the knapsack.
   D. The last two items chosen for the knapsack have the same $$$/lb ratio..

20. During a breadth-first search, the status of a gray vertex is:
   A. It has been completely processed.
   B. It is in the FIFO queue.
   C. It is in the priority queue.

D. It is undiscovered.

Long Answer

1. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=4   p[1]=3   p[2]=4   p[3]=6   p[4]=3   p[5]=6

	1		2		3		4		5	
1	0	0	48	1	144	1	144	1	???	?
2	-------		0	0	72	2	108	2	162	4
3	-------		-------		0	0	72	3	144	4
4	-------		-------		-------		0	0	108	4
5	-------		-------		-------		-------		0	0

2. Fill in the KMP failure links. 10 points.

	0	1	2	3	4	5	6	7	8	9
1:										
pattern:	a	b	c	a	b	c	d	a	b	c
2:										

3. What are the entries in the heap (for Prim's algorithm) before <u>and</u> after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do <u>not</u> need to show the binary tree for the heap ordering. 10 points.

4. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **<u>ordered</u>**. Write your answers in the tables below. 10 points

Vertex	Start	Finish
0	1	___
1	___	___
2	___	___
3	___	___
4	___	___
5	___	___
6	___	___
7	___	___
8	___	___
9	___	___

Edge	Type		Edge	Type
0 1	___		6 5	___
0 2	___		6 7	___
0 6	___		8 5	___
2 5	___		8 7	___
3 1	___		8 9	___
3 2	___		9 4	___
4 3	___			
4 5	___			
5 7	___			
5 9	___			

5. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following graph. The paths indicated in the final matrix must have **at least one** edge. You are not required to show the intermediate matrices. 10 points.
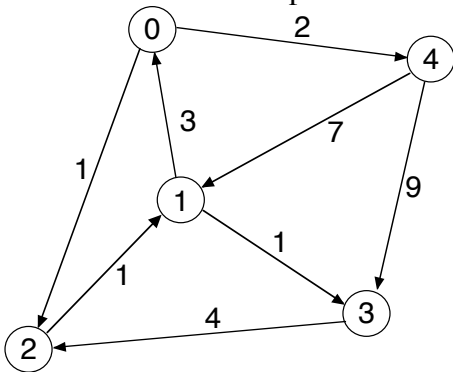
6. Give an optimal Huffman code tree for the provided symbols and probabilities. In addition, compute the expected number of bits per symbol. 10 points

```
A   .15
B   .2
C   .15
D   .13
E   .2
F   .07
G   .1
```