CSE 2320   Name _____
Test 1
Summer 2007   Last 4 Digits of Mav ID # _____
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each
1. The time to multiply two $n \times n$ matrices is:

      A. $\Theta(n)$ B. $\Theta(\max(m,n,p))$ C. $\Theta(n^3)$ D. $\Theta(mnp)$

2. The number of calls to `getmin` to build a Huffman code tree for n symbols is:
      A. $\theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$

3. Which sort has both its worst-case and expected times in $\Theta(n^2)$?

      A. heap   B. insertion   C. merge   D. selection

4. Suppose $H_n = \dfrac{11}{6}$. What is the value of n?

      A. 4   B. 5   C. 6   D. 7

5. The function $n^2 + 3n \log n$ is in which set?
      A. $\Omega(n \log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. Which of the following is not true?

A. $n^2 \in O(n^3)$

B. $n^2 \in \Omega(n \log n)$

C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$

D. $2^n \in \Omega(3^n)$

7. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?
      A. 31   B. 63   C. 64   D. 127

8. $f(n) = n \lg n$ is in all of the following sets, except

      A. $\Omega(\log n)$ B. $\Theta(\log(n!))$ C. $O(n)$ D. $O(n^2)$

9. The algorithm for the activity scheduling problem sorts the input by:
    A. Ascending order of finish time
    B. Ascending order of start time
    C. Descending order of finish time
    D. Descending order of start time

10. The subset sum problem takes n input values and attempts to find a combination of those values whose sum is m. The worst-case time for the dynamic programming solution is:
      A. $\Theta(\log m)$ B. $\Theta(n)$ C. $\Theta(m)$ D. $\Theta(mn)$

11. Suppose that you have correctly determined some c and n_o to prove $f(n) \in O(g(n))$. Which of the following is not necessarily true?
    A. c may be decreased   B. c may be increased   C. n_o may be increased   D. $g(n) \in \Omega(f(n))$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know

$T(n) \in \Omega(n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

      A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n \lg n)$

13. Which of the following initially processes its input by sorting?
    A. Bottom-up heap construction
    B. Huffman coding
    C. Optimal matrix multiplication
    D. Unweighted interval scheduling

14. Which of the following is not true regarding a maxheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 911.
15. Which of the following is a longest common subsequence for 0 1 2 0 1 2 0 1 2 and 0 0 0 1 1 1 2 2 2?
    A. 0 0 0 1 2   B. 0 1 2 1 2   C. 0 0 1 1 2 2   D. 0 1 2 0

Long Answer

1. Use dynamic programming to solve the following instance of (monotone) longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. 10 points
   11   12   13   14   15   2   1   4   3   6   5   8   7   10   9

2. Prove that if $g(n) \in \Omega(f(n))$ then $f(n) \in O(g(n))$. 5 points

3. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

4. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=2
p[1]=4
p[2]=7
p[3]=3
p[4]=2

	1		2		3		4	
1	0	0	56	1	98	2	---	–
2	-------		0	0	84	2	98	2
3	-------		-------		0	0	42	3
4	-------		-------		-------		0	0

6. Use the efficient construction to convert into a minHeap. 10 points
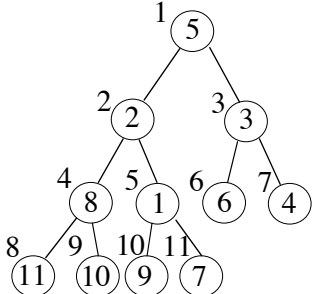
CSE 2320   Name _____
Test 2
Summer 2007   Last 4 Digits of Student ID # _____
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each
1. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?
    A. Like a circular queue, the maximum number of items is determined at initialization.
    B. One node is always wasted.
    C. The head points to the first element and the tail points to the last element.
    D. The tail points to the first element and the head points to the last element.
2. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at J?
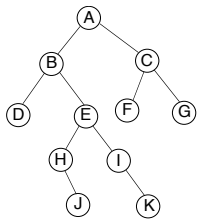
A. A
B. B
C. E
D. H

3. Which of the following sorts is not based on key comparisons?
    A. INSERTION-SORT   B.   LSD RADIX-SORT   C. MERGESORT   D. QUICKSORT
4. What is the worst-case time to perform MINIMUM(L) for a sorted, doubly-linked list with n nodes?
       A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. Doubly-linked lists are associated with which situation?
    A. List maintenance (INSERTION, DELETION) where each node is in several lists
    B. Ordered retrieval operations such as LOGICALSUCCESSOR and MAXIMUM
    C. SEARCHes where many hits will occur
    D. SEARCHes where many misses will occur
6. When evaluating a postfix expression, the stack contains
    A. Both operands and operators
    B. Both parentheses and operators
    C. Operands only
    D. Operators only
7. PARTITION is useful in solving all of the following problems, except
    A. Finding the k smallest elements   B. QUICKSORT
    C. Selection   D. Splitting an array for MERGESORT
8. Suppose a node x in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete x?
    A. Find the successor of x B. Inorder traversal
    C. Rotate x so it becomes a leaf   D. Splice the parent of x to either child of x
9. A sort is said to be stable when:
    A. Duplicate copies of a key will appear in the same order in the output as in the input.
    B. It removes duplicate copies of any key in the final output.
    C. It runs in $O(n \log n)$ time.
    D. The average time and the worst-case time are the same.
10. If POP is implemented as return stack[--SP], then the test for an empty stack is implemented as:
       A. return stack[SP++]   B. return SP==(-1)   C. return SP==0   D. stack[SP++] = X
11. Which of the following binary trees has more than one legal red-black tree coloring?

    A.   B.   C.   D.
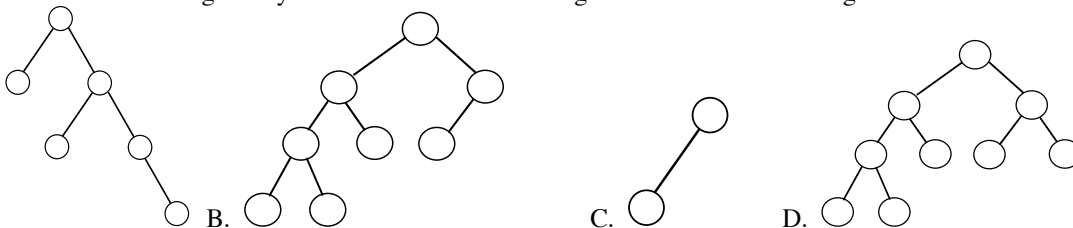
12. Which of the following will not be true regarding the decision tree for QUICKSORT for sorting n input values?
    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
    B. The height of the tree is $\Omega(n \log n)$.
    C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
    D. There will be $n!$ leaves.
13. In the example of concatenating two strings in $O(1)$ time, which situation holds?
    A. Both lists are not circular
    B. Both lists are circular
    C. The first list is circular, the list to be attached is not
    D. The first list is not circular, the list to be attached is circular
14. Which binary tree traversal corresponds to the following recursive code?
```
void traverse(noderef x)
```

```
{
if (x==null)
   return;
// process x here
traverse(x.left);
traverse(x.right);
}
```
   A. inorder   B. postorder   C. preorder   D. search for key x

15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
   A. 100, 1000, 200, 900, 300, 800, 400, 700, 500
   B. 200, 300, 400, 700, 600, 500
   C. 300, 600, 700, 650, 500
   D. 600, 100, 550, 540, 500

Long Answer

1. Give the unbalanced binary search tree that results when the keys 100, 50, 70, 60, 90, 80, 120 are inserted, in the given order, into an initially empty tree. (5 points)

2. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)
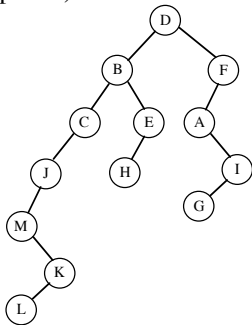
3. Show the result after PARTITION manipulates the following subarray. Be sure to indicate which version of PARTITION you applied. (10 points)

       7   8   2   5   3   1   4   9   0   6

4. A billion numbers in the range 0 . . . 9,999,999 are to be sorted by LSD radix sort. How much faster will this be done if a decimal radix is used rather than a binary radix? Show your work. (10 points)

5. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
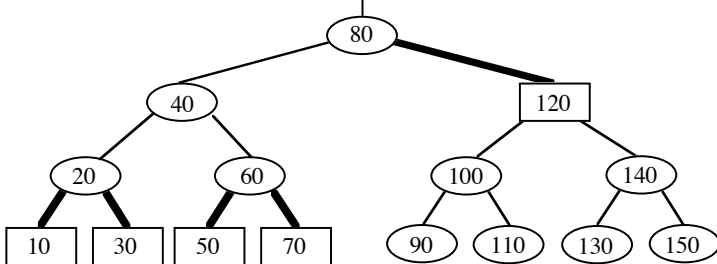
6. Insert 87 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)
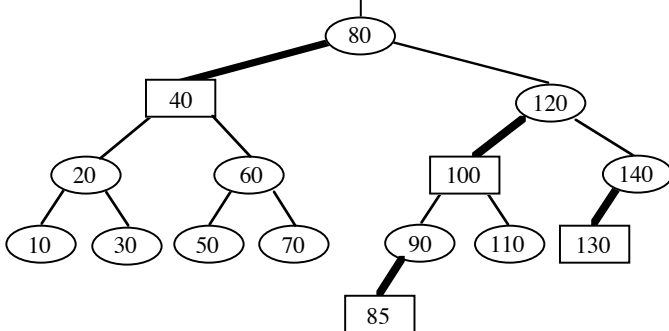
CSE 2320   Name _____
Test 3
Summer 2007   Last 4 Digits of Student ID # _____
Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. In Prim's algorithm, vertex x is included in the minimum spanning tree when:
   A. some vertex y moves from T to S and there is an edge from y to x.
   B. x has its entry extracted from the heap.
   C. x has its heap entry changed by a MIN-HEAP-DECREASE-KEY.
   D. x is read from the input file.

Problems 2, 3, and 4 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow.
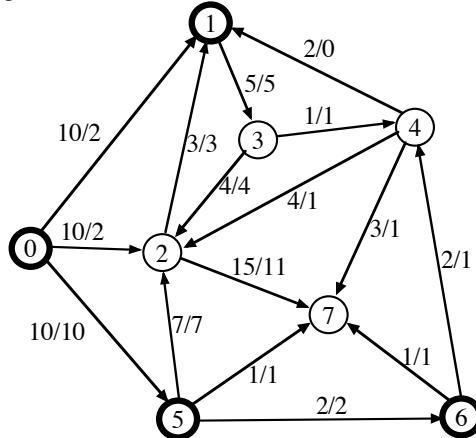
2. The capacity of the indicated cut (S vertices are bold) is:
   A. 20   B. 26   C. 31   D. 32
3. The net flow across the given cut is:
   A. 14   B. 16   C. 18   D. 20
4. Suppose the flow is increased as much as possible using the augmenting path $0 \rightarrow 2 \rightarrow 4 \rightarrow 7$. Which is the critical edge?
   A. $0 \rightarrow 2$ B. $2 \rightarrow 4$ C. $4 \rightarrow 7$ D. Insufficient information
5. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:
   A. finish(X) < finish(Y)   B. finish(X) > finish(Y)
   C. finish(X) = finish(Y)   D. could be either A. or B.
6. What is the number of strongly connected components in this graph?
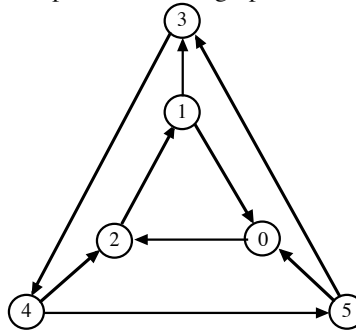

   A. 1   B. 2   C. 3   D. 4
7. Which of the following uses a max-heap?
   A. Breadth-first search
   B. Depth-first search
   C. Maximum capacity path
   D. Prim's algorithm
8. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers $0, 1, \ldots, V - 1$.)
   A. Adjacency lists (ordered): $\Theta(V)$
   B. Adjacency lists (unordered): $\Theta(V)$
   C. Adjacency matrix: $\Theta(1)$
   D. Compressed adjacency lists (ordered): $\Theta(V)$
9. The capacity of the following cut is _____. (S vertices are bold.)

$S \xrightarrow{5} A \xrightarrow{10} B \xrightarrow{1} C \xrightarrow{3} D \xrightarrow{4} T$

    A. 1   B. 10   C. 15   D. 23
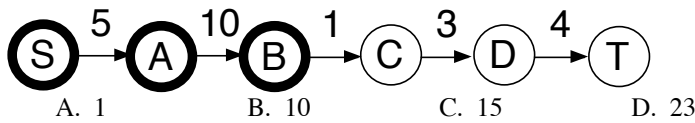
10. When a graph is dense, the best way to find a minimum spanning tree is:
   A. Floyd-Warshall algorithm
   B. Prim's algorithm using heap
   C. Prim's algorithm using T-table
   D. Warshall's algorithm

11. The Edmonds-Karp variant is important because:
   A. It solves the bipartite matching problem.
   B. It solves the network flow problem in polynomial time.
   C. It solves the network flow problem using critical edges.
   D. It solves the network flow problem without using augmenting paths.

12. During a breadth-first search, the status of a gray vertex is:
   A. It has been completely processed.
   B. It is in the FIFO queue.
   C. It is in the priority queue.
   D. It is undiscovered.

13. The fastest method for finding the diameter of a tree is to:
   A. Use breadth-first search.
   B. Use Dijkstra's algorithm.
   C. Use the Floyd-Warshall algorithm.
   D. Use the Ford-Fulkerson algorithm.

14. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?
   A. no edge can be a cross edge or a forward edge
   B. both C and D
   C. every edge is a tree edge
   D. there cannot be a back edge

15. For a double hash table with $\alpha = 0.5$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
       A. 1.2   B. 2   C. 5   D. 10

16. An adjacency matrix is the most useful representation for which problem?
   A. Breadth-first search
   B. Finding strongly-connected components
   C. Maximum network flow
   D. Warshall's algorithm

17. Suppose a maximum bipartite matching with k edges is found using Edmonds-Karp. Which of the following does not hold?
   A. All residual network capacities are zero or one.
   B. Every augmenting path uses three edges.
   C. The capacity of the minimum cut is k.
   D. There will be $k + 1$ breadth-first searches.

18. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:
   A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

19. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 0 to 4?

```
-1    3    3    3    3
-1    3    3    3    4
-1    1    1    1    4
-1    2    2    2    2
-1   -1   -1   -1   -1
```
       A. 0   B. 1   C. 2   D. 3

20. Which of the following is not true about probe sequences for an implementation of linear probing?
   A. All slots in the hash table appear in each probe sequence
   B. Every key has a probe sequence different from the probe sequences for other keys
   C. The elements of a probe sequence are subscripts in the hash table
   D. The probe sequence for a key cannot change

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.
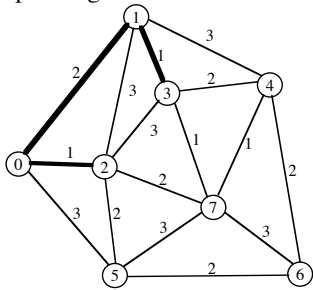

2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points
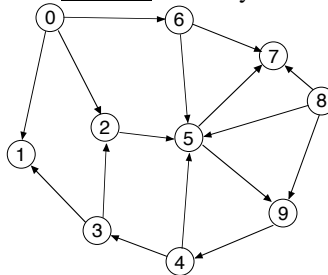

Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	___	0 1	___	6 5	___
1	___	___	0 2	___	6 7	___
2	___	___	0 6	___	8 5	___
3	___	___	2 5	___	8 7	___
4	___	___	3 1	___	8 9	___
5	___	___	3 2	___	9 4	___
6	___	___	4 3	___		
7	___	___	4 5	___		
8	___	___	5 7	___		
9	___	___	5 9	___		

3. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following graph. The paths indicated in the final matrix must have **at least one** edge. You are not required to show the intermediate matrices. 10 points.
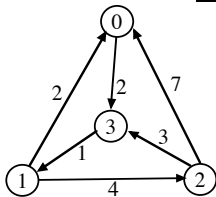
4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.
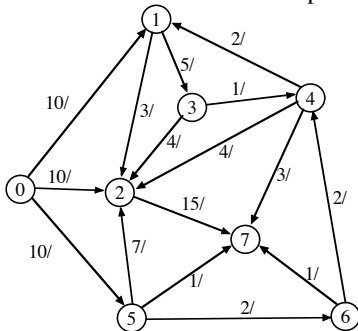

Minimum Cut:
    S vertices: 0
    T vertices: 7

Augmenting Paths and Contribution to Flow:

5. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 19$ and $h_2(key) = 1 + (key \% 18)$. ***Show your work***.

```
 0  -1
 1  -1
 2  800
 3  402
 4  -1
 5  -1
 6  101
 7  501
 8  -1
 9  -1
10  200
11  -1
12  601
13  -1
14  -1
15  -1
16  301
17  701
18  -1
```

   a. Suppose 2001 is to be inserted (using double hashing). Which slot will be used? (5 points)

   b. Suppose 2002 is to be inserted (using double hashing) *after* 2001 has been stored. Which slot will be used? (5 points)

6. Perform a breadth-first search on the following graph listing the BFS number, shortest path distance (hops) from the source (0), and the predecessor for each vertex. Assume that the adjacency lists are **ordered**. 10 points
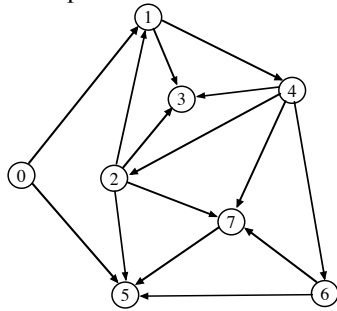
Vertex	BFS Number	Distance	Predecessor
0	_____	_____	_____
1	_____	_____	_____
2	_____	_____	_____
3	_____	_____	_____
4	_____	_____	_____
5	_____	_____	_____
6	_____	_____	_____
7	_____	_____	_____