CSE 2320 Name _____
Test 1
Summer 2009 Last 4 Digits of Mav ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to compute the sum of the n elements of an integer array is in:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. The number of calls to `getmin` to build a Huffman code tree for n symbols is:

   A. $\Theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$

3. Which of the following is not true?

   A. $n^2 \in O(n^3)$
   B. $n \log n \in \Omega(n^2)$
   C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$
   D. $3^n \in \Omega(2^n)$

4. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

   A. Ascending order of finish time
   B. Ascending order of start time
   C. Descending order of interval length
   D. Descending order of finish time

5. The function $n^2 + 3n \log n$ is in which set?

   A. $\Omega(n^2)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. $\log(n!)$ is in all of the following sets, except

   A. $\Omega(\log n)$ B. $\Theta(n \log n)$ C. $\Omega(n^2)$ D. $O(n^2)$

7. To sort a sub-array with n items using recursive (top-down) mergesort, how many calls to `mergeSort()` are needed?

   A. $n - 1$ B. n C. $2n - 1$ D. $2n - 2$

8. What is indicated when `find(i)==find(j)` while maintaining disjoint subsets?

   A. `i` and `j` are in the same subset
   B. `i` and `j` are leaders for different subsets
   C. `i` and `j` are leaders for the same subset
   D. `i` is the ancestor of `j` in one of the trees

9. Suppose that you have correctly determined some c and n_o to prove $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

   A. c may be decreased B. c may be increased C. n_o may be increased D. $f(n) \in O(g(n))$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^3)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

Short Answer. 3 points each

1. Give the definition of H_n.

2. Suppose a binary search is to be performed on a table with 60 elements. The maximum number of elements that could be examined (probes) is:

3. Give the subscripts for the parent, left child, and right child for the maxheap element stored at subscript 551. The heap is currently storing 1000 elements in a table with 2000 slots.

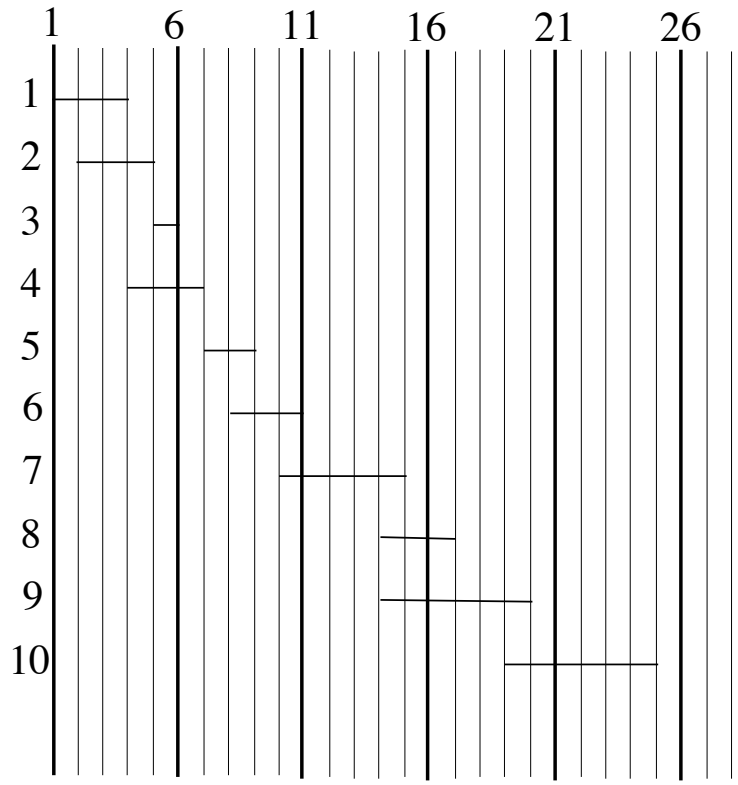
4. List the stable sorts we have studied so far.

5. Give a longest common subsequence for `aabb` and `abab`.

Long Answer

1. Prove that if $f(n) \in O(g(n))$ then $\frac{1}{f(n)} \in \Omega\left(\frac{1}{g(n)}\right)$. 5 points

2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



	v_i	p_i	$m(i)$
1	1	0	
2	5	0	
3	3	2	
4	8	1	
5	1	4	
6	2	4	
7	3	5	
8	1	6	
9	4	6	
10	1	8	

3. Use the recursion-tree method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2 \log n\right)$. 10 points

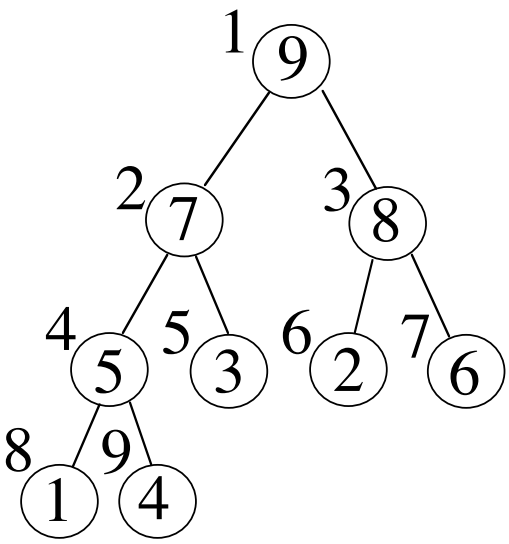
4. Use the substitution method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2 \log n\right)$. 10 points

5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=5 p[1]=4 p[2]=6 p[3]=2 p[4]=5 p[5]=6

	1		2		3		4		5	
1	0	0	120	1	88	1	138	3	**???**	**?**
2	-------		0	0	48	2	88	3	156	3
3	-------		-------		0	0	60	3	132	3
4	-------		-------		-------		0	0	60	4
5	-------		-------		-------		-------		0	0

6. Show the result after performing HEAP-EXTRACT-MAX twice on the following maxheap. 10 points



CSE 2320
Test 2
Summer 2009

Name _____

Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which version of rat-in-a-maze finds the shortest path?

   A. queue B. recursive C. red-black tree D. stack

2. How should the successor of a leaf in an unbalanced binary search tree be found?

   A. Examine the ancestors
   B. Go right, then proceed to the left
   C. Inorder traversal
   D. Preorder traversal

3. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:

   A. return stack[SP++]
   B. stack[SP++] = X
   C. stack[--SP] = X
   D. stack[++SP] = X

4. What is the worst-case time to perform MAXIMUM(L) for a circular, sorted, doubly-linked list with n nodes?

   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

5. For which of the following sorts does the decision tree model not apply?

   A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT

6. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:

   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(h)$

7. In the example of recycling the elements of a list in O(1) time, which situation holds?

   A. Both lists are circular
   B. Both lists are not circular
   C. The list to be recycled is circular, the garbage list is not
   D. The garbage list is circular, the list to be recycled is not

8. In which situation will a sentinel be inappropriate?

   A. Search for a key in an unordered table, to simplify and speed-up code
   B. Search for a key in an unordered linked list, to simplify and speed-up code
   C. Binary search for a key in an ordered table, to simplify and speed-up code
   D. Red-black tree, to simplify code

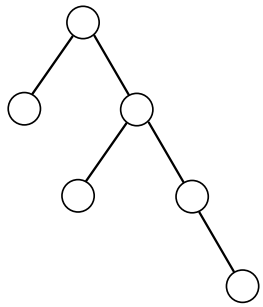
9. An unsorted integer array with 1500 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?

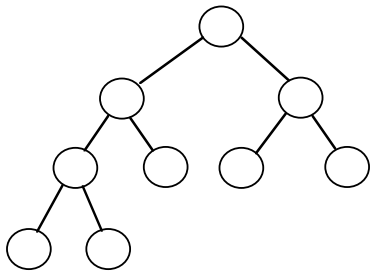
   A. If 1468 is returned from PARTITION, we must continue.
   B. If 1469 is returned from PARTITION, we are done.
   C. If 1470 is returned from PARTITION, we must continue.
   D. If 1481 is returned from PARTITION, we must continue.

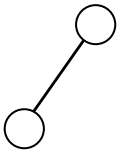
10. How should the successor of a node with a right child in an unbalanced binary search tree be found?

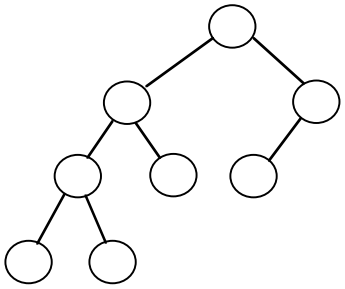
    A. Examine the ancestors
    B. Go left, then proceed to the right
    C. Go right, then proceed to the left
    D. Preorder traversal

11. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

A. 

B. 

C. 

D. 

12. Which of the following will not be true regarding the decision tree for MERGESORT for sorting n input values?

    A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
    B. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
    C. The height of the tree is $\Omega(n \log n)$.
    D. There will be $n!$ leaves.

13. The worst-case number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?

    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
if (x==null)
  return;
traverse(x.left);
traverse(x.right);
// process x here
}
```

    A. inorder B. postorder C. preorder D. search for key `x`

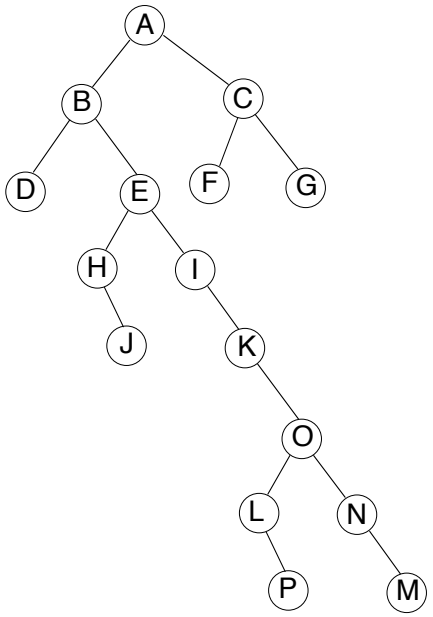
15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?

    A. 10, 200, 300, 100, 500
    B. 100, 1000, 200, 900, 300, 800, 400, 700, 500
    C. 200, 300, 400, 700, 600, 500
    D. 600, 100, 550, 540, 500

Long Answer

1. Give the unbalanced binary search tree that results when the keys 50, 70, 60, 90, 100, 80, 120 are inserted, in the given order, into an initially empty tree. (5 points)

2. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)



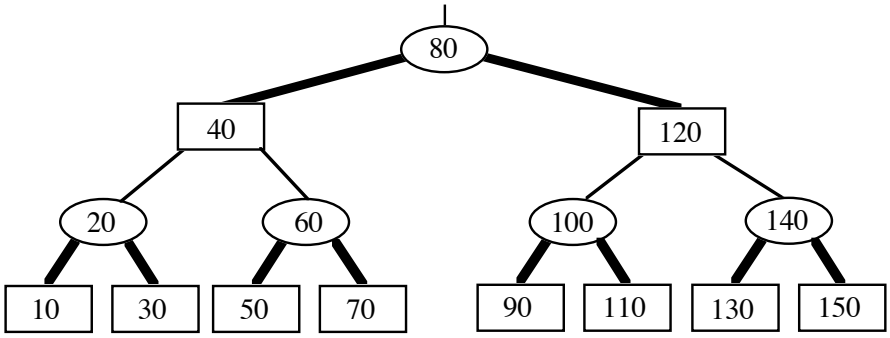
3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

8 2 5 3 6 1 9 0 7 4

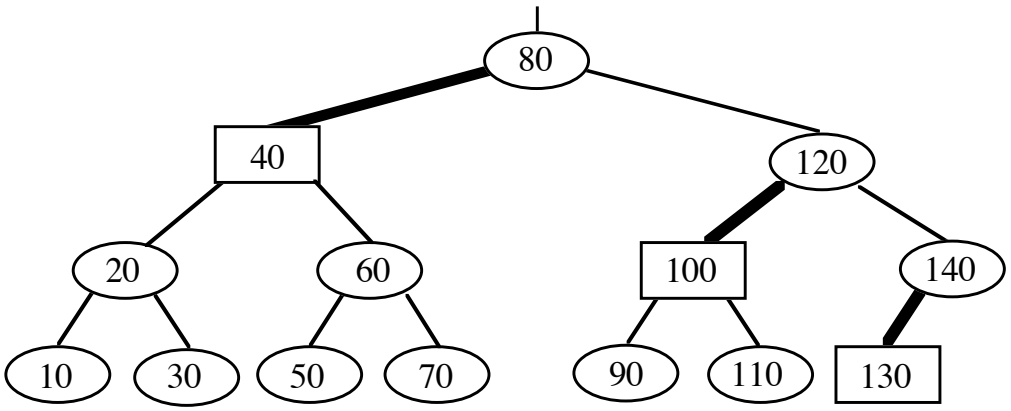
Version: 1 2/Sedgewick

4. A billion integers in the range 0 . . . 999,999 are to be sorted by LSD radix sort. How much faster will this be done if radix 0 . . . 999 is used rather than decimal (0 . . . 9) radix? Show your work. (10 points)

5. Insert 115 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



6. Insert 135 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320 Name _____
Test 3
Summer 2009 Last 4 Digits of Student ID # _____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. The worst-case time for Prim's algorithm implemented with a min-heap is:

   A. $\theta(V^2 + E)$ B. $\theta(E \log V)$ C. $\theta(V \log V)$ D. $\theta(V \log E)$

2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from Y to X, its type will be:

   A. Back B. Cross C. Forward D. Tree

3. The worst-case time for depth-first search is:

   A. $\theta(V \log E)$ B. $\theta(E \log V)$ C. $\theta(V \log V)$ D. $\theta(V + E)$

4. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?

   A. no edge can be a cross edge or a forward edge
   B. both C and D
   C. every edge is a tree edge
   D. there cannot be a back edge

5. Which statement is not correct about depth-first search on a directed graph?

   A. Exploring an edge whose head is colored black will cause the edge to be a back edge.
   B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
   C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
   D. The run time is $\Theta(m + n)$, where m is the number of edges and n is the number of vertices.

6. A topological ordering of a directed graph may be computed by:

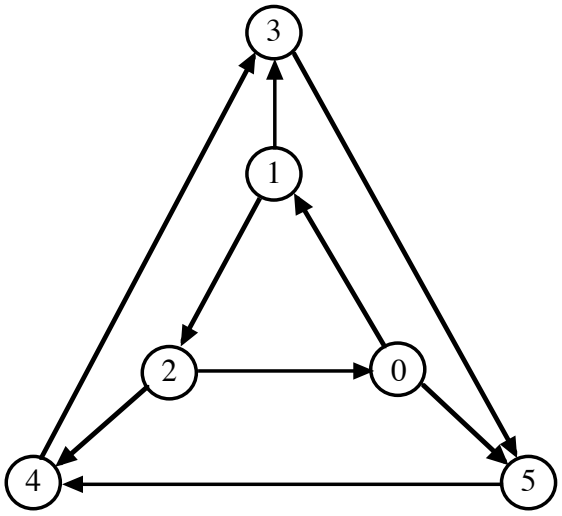
A. Ordering the vertices by descending finish time after DFS
B. Ordering the vertices by ascending discovery time after DFS
C. Ordering the vertices by ascending finish time after DFS
D. Ordering the vertices by descending discovery time after DFS

7. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?

A. hashing with chaining B. open addressing

C. ordered linked list D. unbalanced binary search tree

8. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

9. During a breadth-first search, the status of a white vertex is:

A. It has been completely processed.
B. It is in the FIFO queue.
C. It is in the priority queue.
D. It is undiscovered.

10. When a graph is sparse, the best way to find a minimum spanning tree is:

A. Floyd-Warshall algorithm
B. Prim's algorithm using heap
C. Prim's algorithm using T-table
D. Warshall's algorithm

11. Suppose a depth-first search is performed on an undirected graph. There are no cycles if:

A. no edge is a cross edge or forward edge
B. both C and D
C. there are no restarts
D. there are no back edges

12. When finding the strongly connected components, the number of components is indicated by:

A. The number of back edges found during the first depth-first search.
B. The number of cross edges found during the second depth-first search.
C. The number of restarts for the first depth-first search.
D. The number of restarts for the second depth-first search.

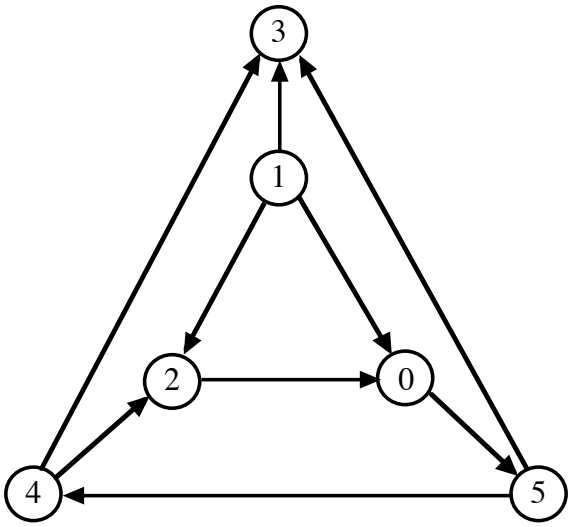
13. What is the purpose of the first depth-first search when finding strongly connected components?

A. To assure that two vertices that are in the same cycle will be output in the same component
B. To assure that the input graph has no cycles.
C. To assure that two vertices with no paths between them are not output in the same component
D. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.

14. The number of potential probe sequences when using linear probing with a table with *m* entries (*m* is prime) is:

A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

15. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

16. Suppose that there is exactly one path from vertex 8 to vertex 10 in a directed graph: $5 \rightarrow 7 \rightarrow 8 \rightarrow 3 \rightarrow 2 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?

A. 2 B. 5 C. 8 D. 10

17. In Dijkstra's algorithm, the final shortest path distance from the source s to a vertex x is known when:

A. some vertex y moves from T to S and there is an edge from y to x.
B. x has its entry extracted from the heap.

C. x is placed on the heap.
D. x is read from the input file.

18. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)

A. Adjacency lists (ordered): $\Theta(V)$
B. Adjacency lists (unordered): $\Theta(V)$
C. Adjacency matrix: $\Theta(1)$
D. Compressed adjacency lists (ordered): $\Theta(V)$

19. When using two breadth-first searches to find the diameter of a tree (in hops), the purpose of the first search is to find:

A. all vertices that could be an end of a diameter.
B. both ends of a diameter.
C. one end of a diameter.
D. the number of edges in the diameter.

20. Which of the following is not true about probe sequences for an implementation of double hashing?

A. The elements of a probe sequence are possible keys for the hash table
B. All slots in the hash table appear in each probe sequence
C. The probe sequence for a key cannot change
D. Two keys could have the same probe sequence

21. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. $\frac{3}{2}\alpha$ D. 2α

22. An adjacency matrix is the most useful representation for which problem?

A. Breadth-first search
B. Finding strongly-connected components
C. Maximum network flow
D. Warshall's algorithm

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

23. 265 would be inserted into which slot of the given table?

A. 0 B. 1 C. 2 D. 11

24. 133 would be inserted into which slot of the given table? (without previously inserting 265)

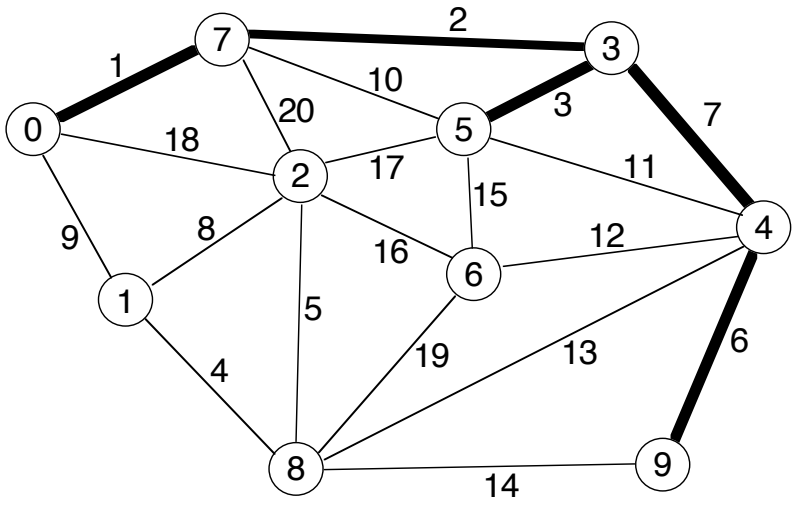
A. 0 B. 4 C. 6 D. 11

25. Which algorithm maintains multiple subtrees?

A. Dijkstra's.
B. Kruskal's
C. Prim's
D. Warshall's

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.



2. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 19$ and $h_2(key) = 1 + (key \% 18)$. Show your work.

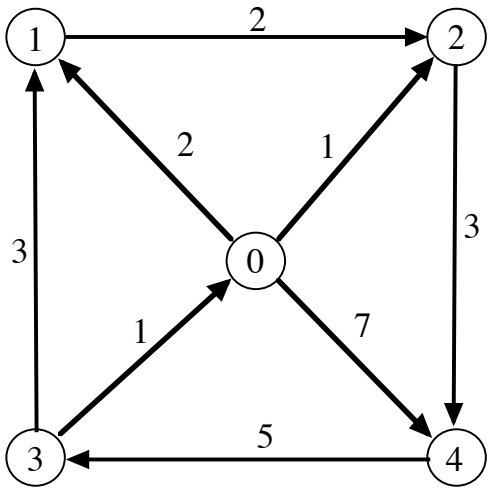
```
 0 -1
 1 -1
 2 800
 3 402
 4 -1
 5 -1
 6 101
 7 501
 8 -1
 9 -1
10 200
11 -1
12 601
13 -1
```

```
14 -1
15 -1
16 301
17 701
18 -1
```

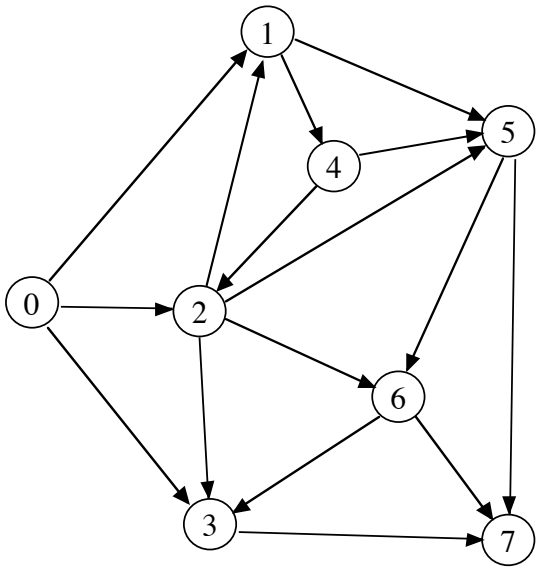
a. Suppose 2001 is to be inserted (using double hashing). Which slot will be used? (5 points)

b. Suppose 2002 is to be inserted (using double hashing) *after* 2001 has been stored. Which slot will be used? (5 points)

3. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

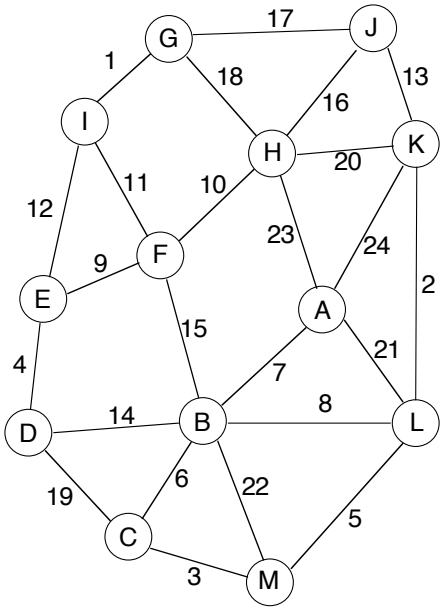


4. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points



Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	____	0 1	____	2 6	____
1	____	____	0 2	____	3 7	____
2	____	____	0 3	____	4 2	____
3	____	____	1 4	____	4 5	____
4	____	____	1 5	____	5 6	____
5	____	____	2 1	____	5 7	____
6	____	____	2 3	____	6 3	____
7	____	____	2 5	____	6 7	____

5. Demonstrate Kruskal's algorithm on this graph. 10 points



Extra Credit: Find a maximum flow (4 points) and a minimum cut (2 points) for this network using augmenting paths.