Name _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to multiply two $n \times n$ matrices is:

    A. $\Theta(n)$ B. $\Theta(\max(m,n,p))$ C. $\Theta(n^3)$ D. $\Theta(mnp)$

2. Which of the following is the best approximation for H_{mn}? (*m* and *n* are positive integers)

    A. $\dfrac{1}{mn}$ B. $\ln m + \ln n$ C. $\ln n$ D. 1

3. Which sort has both its worst-case and expected times in $\Theta(n^2)$?

    A. heap   B. insertion   C. merge   D. selection

4. Suppose $H_n = \dfrac{11}{6}$. What is the value of *n*?

    A. 4   B. 5   C. 6   D. 7

5. The function $3\log n + 2n$ is in which set?

    A. $\Omega(n\log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

6. Which of the following is not true?

    A. $2^n \in \Omega(3^n)$

    B. $n^2 \in O(n^3)$

    C. $n^2 \in \Omega(n\log n)$

    D. $g(n) \in \Omega(f(n)) \Leftrightarrow f(n) \in O(g(n))$

7. What is *n*, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?

    A. 31   B. 63   C. 64   D. 127

8. $f(n) = n\lg n$ is in all of the following sets, except

    A. $\Omega(\log n)$ B. $\Theta(\log(n!))$ C. $O(n)$ D. $O(n^2)$

9. Suppose there is a large, unordered table with *n* integers, possibly with repeated values. How much time is needed to determine the minimum value?

    A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n\log n)$

10. The recursion tree for mergesort has which property?
    A. each level has the same contribution
    B. it leads to a definite geometric sum
    C. it leads to a harmonic sum
    D. it leads to an indefinite geometric sum

11. Suppose that you have correctly determined some *c* and n_o to prove $f(n) \in O(g(n))$. Which of the following is not necessarily true?

    A. *c* may be decreased   B. *c* may be increased   C. n_o may be increased   D. $g(n) \in \Omega(f(n))$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know $T(n) \in \Omega(n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n\lg n)$

13. What is required when calling `union(i,j)` for maintaining disjoint subsets?
    A. `i` is the ancestor of `j` in one of the trees   B. `i` and `j` are in the same subset
    C. `i` and `j` are leaders for different subsets   D. `i` and `j` are leaders for the same subset

14. Which of the following is not true regarding a maxheap with 1000 elements?

A. Subscript 1 will store the maximum priority.
B. The parent for the node with subscript 500 is stored at subscript 250.
C. The left child for the node with subscript 200 is stored at subscript 400.
D. The right child for the node with subscript 405 is stored at subscript 911.

15. $\log(n!)$ is in all of the following sets, except

   A. $\Omega(\log n)$ B. $\Theta(n \log n)$ C. $O(n^2)$ D. $\Omega(n^2)$

Long Answer

1. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e. each table represents a set). Give Java code for a $\Theta(m + n)$ algorithm to find the **set difference** by producing a third array `C` (in ascending order) with the values that appear in `A`, but not `B`, **and** sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style **are unnecessary**.) 15 points
   Example: A = {1, 2, 4, 5, 7, 11}, B = {0, 1, 2, 6, 7, 13}, C = {4, 5, 11}, p = 3

2. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

3. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

4. Complete the function by writing the code to replace each `???` on the line to its right. 10 points

```
public static <T extends Comparable<? super T>> int
    binSearchFirst(T[] a,T key) throws ClassCastException
 {
 // Finds index of first slot with a key >= a given key
 // WARNING - Returns n if key>a[n-1]
   int low,high,mid,order;
   int n=a.length;
   low=0;
   high=???;              _____
   while (low???high)     _____
   {
     mid=???;             _____
     order=a[mid].compareTo(key);
     if (order???0)       _____
       low=mid+1;
     else
       high=mid-1;
   }
   return ???;            _____
 }
```

5. Use the efficient construction to convert into a minHeap. 5 points
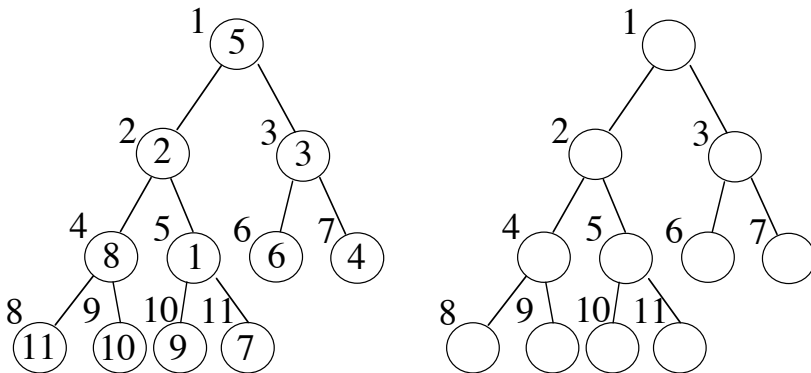
6. Show the maxheap after changing the priority at subscript 6 to 9. 5 points
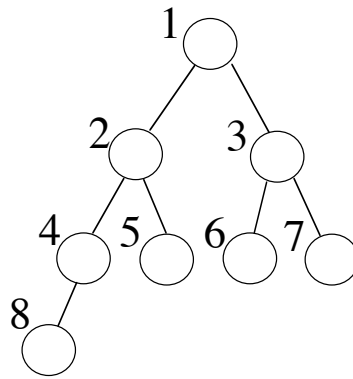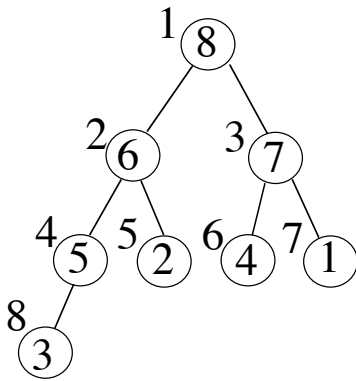
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Test 2

Summer 2011   Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1.   The queue for breadth-first rat-in-a-maze stores
     A. all maze positions that have walls
     B. maze positions that must be in the final path
     C. maze positions that have been reached
     D. the current path being explored

2. When solving the fractional knapsack problem, the items are processed in the following order.
     A. Ascending order of weight
     B. Ascending order of $$$/lb
     C. Descending order of weight
     D. Descending order of $$$/lb

3.   Which of the following is a longest common subsequence for 0 0 1 2 1 2 and 0 0 1 1 2 2?
           A. 0 0 1 1   B. 0 0 1 1 2   C. 0 0 1 2   D. 0 1 2 0

4.   What is the worst-case time to perform MINIMUM(L) for an unordered, doubly-linked list with n nodes?
           A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

5.   Given a pointer to a node, the worst-case time to delete the node from a doubly-linked list with n nodes in ascending order
     is:
           A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

6.   In the example of concatenating two strings in O(1) time, which situation holds?
     A. Both lists are circular
     B. The first list is not circular, the list to be attached is circular
     C. Both lists are not circular
     D. The first list is circular, the list to be attached is not

7.   Why is it common for a circular queue implementation to waste one table element?
     A. To avoid confusing an empty queue with a full queue
     B. To have a place to store the tail and head values
     C. To make sure the queue always has at least one element in use
     D. To perform some loops faster

8.   The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
     A. to assure that the final solution is free of duplicate values
     B. to determine the longest possible increasing subsequence terminated by a particular input value
     C. to search a table that will contain only the LIS elements at termination
     D. to sort the original input

9.   Which statement is correct regarding the unweighted and weighted activity scheduling problems?
     A. Both require dynamic programming
     B. Both are easily solved using a greedy technique
     C. Unweighted is solved using a greedy technique, weighted is solved by dynamic programming
     D. Weighted is solved using a greedy technique, unweighted is solved by dynamic programming

10. For which of the following sorts does the decision tree model not apply?
     A.   Insertion   B. LSD Radix Sort   C. MERGE-SORT   D. QUICKSORT

11. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
     A. return stack[SP++]   B. stack[SP++] = X   C. stack[SP--] = X   D. stack[++SP] = X

12. Recursion is often an alternative to using which data structure?

A. Linked list   B. Queue   C. Stack   D. 2-d array

13. The expected number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?

    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

14. Which of the following is solved heuristically by a greedy method?
    A. Unweighted interval scheduling
    B. 0/1 knapsack
    C. Fractional knapsack
    D. Huffman code

15. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?
    A. There will be $n!$ leaves.
    B. Every path from the root to a leaf will have $O(n \log n)$ decisions.
    C. The height of the tree is $\Omega(n \log n)$.
    D. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.

Long Answer
1. List the four phases in a counting sort and give the asymptotic time needed for each phase. (5 points)
2. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. (10 points)

```
p[0]=6
p[1]=2
p[2]=4
p[3]=3
p[4]=2
          1           2           3           4
  1     0    0     48    1     60    1    ???     ?

  2    -------        0    0     24    2     36    3

  3    -------     -------        0    0     24    3

  4    -------     -------     -------        0    0
```

3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

        8   2   5   3   4   1   9   0   7   6

    Version: 1   2/Sedgewick

4. Use dynamic programming to solve the following instance of the strictly longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. (No points for solving by inspection.) (10 points)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	10	15	20	25	7	10	15	22	25	5	10	22	26	27

5. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. (10 points)

	v_i	p_i	$m(i)$
1	3	0	
2	3	1	
3	1	2	
4	3	1	
5	2	2	
6	3	3	
7	5	5	
8	3	6	
9	6	6	
10	6	8	

6. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. (10 points)

```
A        0.25
B        0.02
C        0.5
D        0.04
E        0.12
F        0.07
```

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Test 3

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Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Which edge is chosen in a phase of Prim's algorithm?
    A. A minimum-weight edge connecting T to S.
    B. A minimum-weight edge that keeps the result free of cycles
    C. An edge of maximum-weight in a cycle (to be excluded)
    D. An edge that is on a shortest path from the source

2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, its type will be:
    A. Back   B. Cross   C. Forward   D. Tree

3. Which statement is incorrect regarding depth-first search on a directed graph?
    A. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
    B. The run time is $\Theta(m + n)$, where m is the number of edges and n is the number of vertices.
    C. Exploring an edge whose head is colored black will cause the edge to be a back edge.
    D. Exploring an edge whose head is colored gray will cause the edge to be a back edge.

4. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?
    A. no edge can be a cross edge or a forward edge
    B. both C and D
    C. every edge is a tree edge
    D. there cannot be a back edge

5. Suppose a depth-first search is performed on a directed graph. There are no cycles if:
    A. no edge is a cross edge or forward edge
    B. both C and D
    C. there are no restarts
    D. there are no back edges

6. A topological ordering of a directed graph may be computed by:
    A. Ordering the vertices by ascending finish time after DFS
    B. Ordering the vertices by descending discovery time after DFS
    C. Ordering the vertices by ascending discovery time after DFS
    D. Ordering the vertices by descending finish time after DFS

The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:
   A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$

8. What is the number of strongly connected components in this graph?
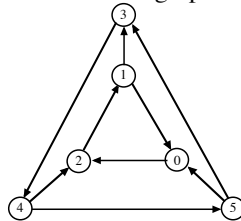
        A. 1   B. 2   C. 3   D. 4

9. During a breadth-first search, the status of a gray vertex is:
    A. It has been completely processed.
    B. It is in the FIFO queue.
    C. It is in the priority queue.
    D. It is undiscovered.

10. In a binary search tree, which element does not have a predecessor?
    A. any one of the leaves   B. the minimum   C. the maximum D. the root

11. Suppose a depth-first search is performed on an undirected graph. The graph is a free (i.e. unrooted) tree if:

A. all edges are tree edges
B. both C and D
C. there are no restarts
D. there are no back edges

12. Which of the following cannot occur when additional edges are included in a directed graph?
    A. The number of strong components may remain the same.
    B. The number of strong components may increase.
    C. The number of strong components may decrease.
    D. The graph acquires a cycle.

13. What is the purpose of the first depth-first search when finding strongly connected components?
    A. To assure that two vertices that are in the same cycle will be output in the same component
    B. To assure that two vertices with no paths between them are not output in the same component
    C. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.
    D. To make sure that the input graph has no cycles.

14. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:
    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

15. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)
    A. Adjacency lists (ordered): $\Theta(V)$
    B. Adjacency lists (unordered): $\Theta(V)$
    C. Adjacency matrix: $\Theta(1)$
    D. Compressed adjacency lists (ordered): $\Theta(V)$

16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

−1	3	3	3	3
−1	3	3	3	4
−1	1	1	1	4
−1	2	2	2	2
−1	−1	−1	−1	−1

    A. 0   B. 1   C. 2   D. 3

17. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:
    A. Use the Floyd-Warshall algorithm.
    B. Use breadth-first search.
    C. Use the Ford-Fulkerson algorithm.
    D. Use Dijkstra's algorithm.

18. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The number of entries in the two tables are:
    A. $n + 1$ and m
    B. n for both
    C. m for both
    D. n and m

19. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
    A. 600, 100, 550, 540, 500
    B. 10, 200, 300, 100, 500
    C. 100, 1000, 200, 900, 300, 800, 400, 700, 500
    D. 200, 300, 400, 700, 600, 500

20. Which of the following is not true about probe sequences for an implementation of double hashing?
    A. Two keys could have the same probe sequence
    B. All slots in the hash table appear in each probe sequence
    C. The elements of a probe sequence are possible keys for the hash table
    D. The probe sequence for a key cannot change

21. Which of the following binary trees can be legally colored as a red-black tree with its root colored red?

A.   B.   C.   D.
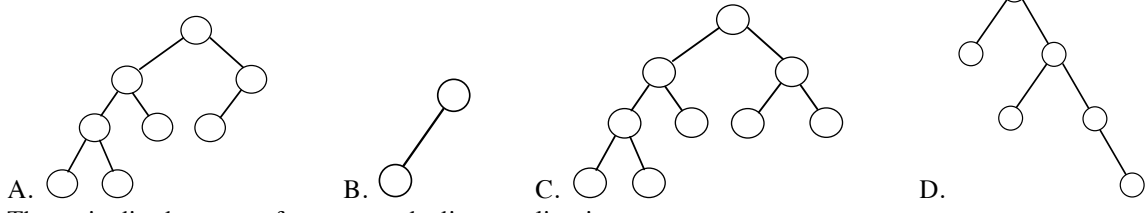
22. The main disadvantage of compressed adjacency lists is:
    A. They waste space
    B. Undirected graphs may not be represented
    C. Directed graphs may not be represented
    D. It is difficult to change the graph

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

23. 148 would be inserted into which slot of the given table?
    A. 0   B. 1   C. 2   D. 11

24. 133 would be inserted into which slot of the given table? (without previously inserting 148)
    A. 0   B. 4   C. 6   D. 11

25. Which edge is chosen in a phase of Kruskal's algorithm?
    A. A minimum-weight edge that keeps the result free of cycles
    B. An edge of maximum-weight in a cycle (to be excluded)
    C. An edge that is on a shortest path from the source
   D. A minimum-weight edge connecting T to S.

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before <u>and</u> after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do <u>not</u> need to show the binary tree for the heap ordering. 10 points.
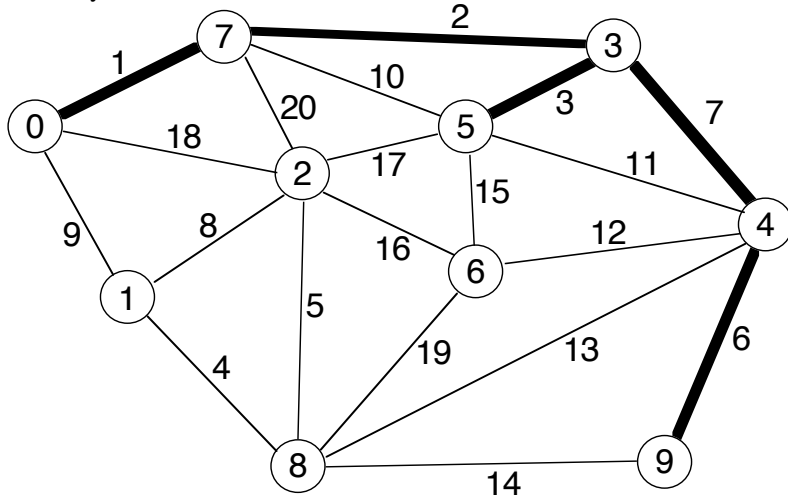

2. Demonstrate, for the graph below, the algorithm that uses <u>**two**</u> depth-first searches to determine the strongly-connected components. 10 points
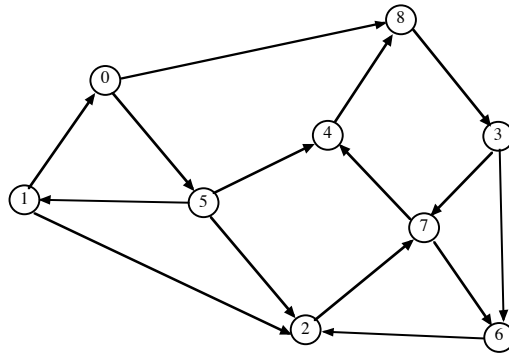
3. Demonstrate the Floyd-Warshall algorithm, **_with successors_**, for the following graph. The paths indicated in the final matrix must have **_at least one_** edge. You **_are not_** required to show the intermediate matrices. 10 points.
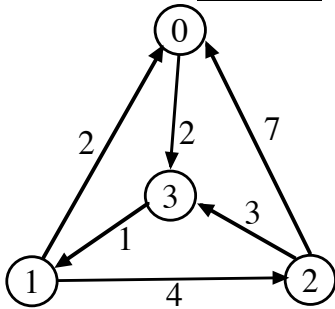
4. In lab 4, you implemented ranking for keys in red-black trees by storing the rank of the root within its subtree. In the following diagram for a right rotation, a, b, and c are keys, while α, β, and γ are root ranks before the rotation. Using only this information (and nothing from within the four subtrees A, B, C, and D), explain how to compute the new root ranks for the nodes of keys a, b, and c. 10 points
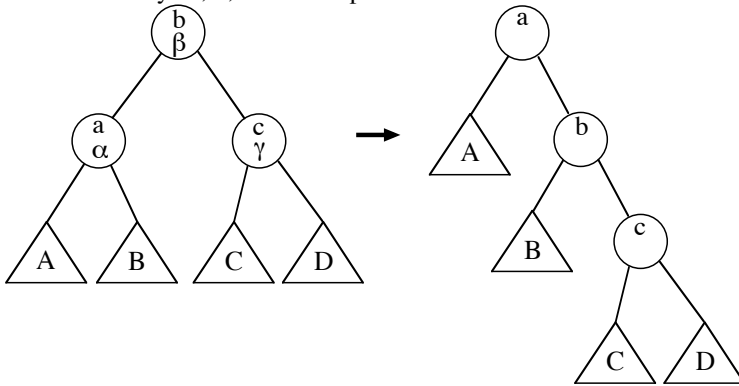
5. Insert 135 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)