CSE 2320 Name _____

Test 1

Summer 2013 Last 4 Digits of Mav ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to run the code below is in:

```
sum=1;
for (i=1; i<n; i=3*i)
  sum++;
```

A. $\Theta(\log n)$ B. $\Theta(\sqrt{n})$ C. $\Theta(n)$ D. $\Theta(n^2)$

2. The number of calls to `PQdelmin` to build a Huffman code tree for n symbols is:

A. $\Theta(\log n)$ B. n - 1 C. n D. 2n - 2

3. What is the value of $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$?

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 2

4. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.
   A. Ascending order of start time
   B. Descending order of interval length
   C. Ascending order of finish time
   D. Descending order of finish time

5. The function $n^2 + 3n\log n$ is in which set?

A. $\Theta(n)$ B. $\Theta(n\log n)$ C. $\Omega(n^2)$ D. $\Theta(\log n)$

6. The expected time for insertion sort for n keys is in which set? (All $n!$ input permutations are equally likely.)

A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n\log n)$ D. $\Theta(n^2)$

7. Bottom-up heap construction is based on applying `fixDown` in the following fashion:
   A. In ascending slot number order, for each slot that is a parent.
   B. In descending slot number order, for each slot that is a parent.
   C. $\frac{n}{2}$ times, each time from the root of the heap.
   D. n - 1 times, each time from the root of the heap.

8. What is indicated when `find(i)==find(j)` while maintaining disjoint subsets?
   A. `i` and `j` are leaders for the same subset
   B. `i` is the ancestor of `j` in one of the trees
   C. `i` and `j` are in the same subset
   D. `i` and `j` are leaders for different subsets

9. Suppose that you have correctly determined some c and n_o to prove $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

A. c may be decreased B. c may be increased C. n_o may be increased D. $f(n) \in O(g(n))$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A. $T(n) \in O(\log n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

11. Which of the following functions is not in $\Omega(n\log n)$?

A. n B. $n\lg n$ C. n^2 D. n^3

12. Suppose a binary search is to be performed on a table with 28 elements. The maximum number of elements that could be examined (probes) is:

A. 4 B. 5 C. 6 D. 7

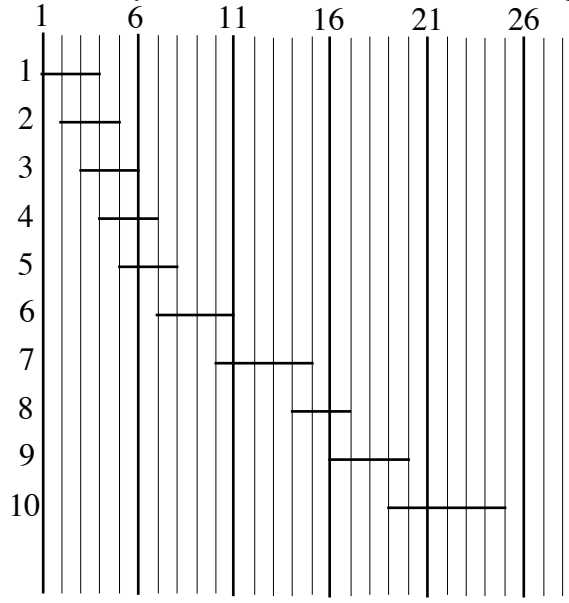
13. Suppose $H_n = \frac{11}{6}$. What is the value of *n*?

A. 3 B. 4 C. 5 D. 6

14. The recursion tree for mergesort has which property?
    A. each level has the same contribution
    B. it leads to a definite geometric sum
    C. it leads to a harmonic sum
    D. it leads to an indefinite geometric sum
15. Which of the following is not true regarding a maxheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 911.

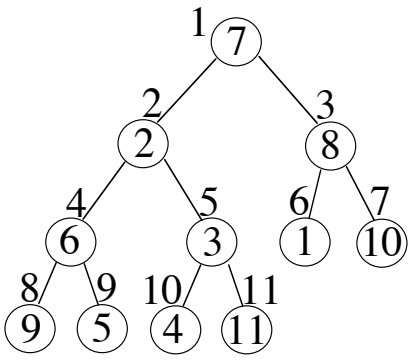
Long Answer

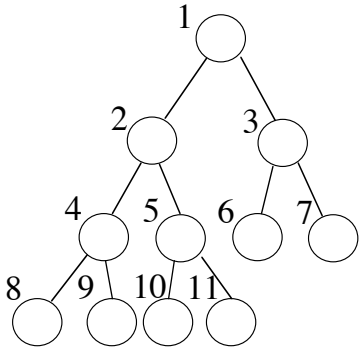
1. Prove that if $f(n) \in \mathrm{O}(g(n))$ then $\frac{1}{f(n)} \in \Omega\left(\frac{1}{g(n)}\right)$. 5 points
2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



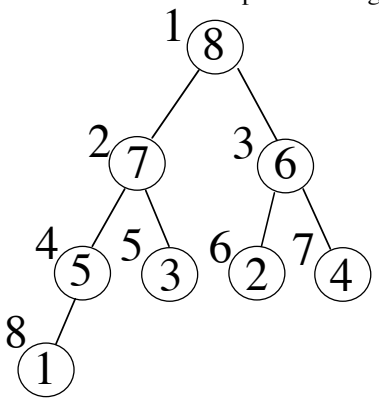
	v	*p*	*m*
1	5	0	
2	4	0	
3	3	0	
4	2	1	
5	1	2	
6	5	4	
7	4	5	
8	3	6	
9	2	7	
10	1	8	

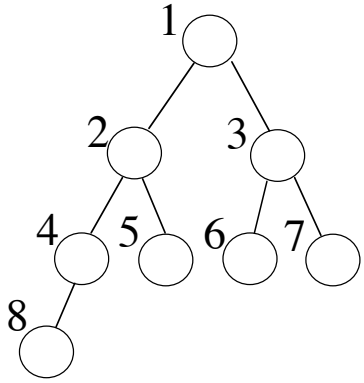
3. Use the recursion-tree method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2 \log n\right)$. 10 points
4. Use the substitution method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2 \log n\right)$. 10 points
5. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e. each table represents a set). Give C code for a $\Theta(m+n)$ algorithm to find the **set difference** by producing a third array `C` (in ascending order) with the values that appear in `A`, but not `B`, **and** sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style **are unnecessary**. Just give the control structures and assignment statements.) 10 points
   Example: `A` = {1, 2, 4, 5, 7, 11}, `B` = {0, 1, 2, 6, 7, 13}, `C` = {4, 5, 11}, `p` = 3
6. Use the efficient construction to convert into a minHeap. 5 points





7. Show the maxheap after changing the priority at subscript 6 to 9. 5 points





CSE 2320 Name _____

Test 2

Summer 2013 Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Recently, we considered an abstraction supporting the operations *allocate*, *allocateAny*, and *freeup* in constant time. Which of the following was not a feature of the implementation?
   A. a recycling list B. a circular, doubly-linked list C. a header D. arrays
2. What is the worst-case time to perform MINIMUM(L) for an unsorted, doubly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
3. Which of the following is a longest common subsequence for 0 1 2 0 1 2 and 0 0 1 1 2 2?
   A. 0 1 2 2 B. 0 0 1 1 C. 0 0 1 2 0 D. 0 0 2 2
4. In the example of recycling the elements of a list in O(1) time, which situation holds?
   A. Both lists are circular
   B. Both lists are not circular
   C. The list to be recycled is circular, the garbage list is not
   D. The garbage list is circular, the list to be recycled is not
5. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with n nodes in ascending order is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
6. Which of the following will not be true regarding the decision tree for MERGESORT for sorting n input values?
   A. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
   B. There will be $n!$ leaves.
   C. Every path from the root to a leaf will have $O(n^2)$ decisions.
   D. The height of the tree is $\Omega(n \log n)$.
7. Which sort treats keys as several digits and uses a counting sort for each position?
   A. counting B. insertion C. merge D. radix
8. Which data structure is associated with manipulating expressions?
   A. linked list B. stack C. queue D. decision tree
9. For which of the following sorts does the decision tree model not apply?
   A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT

10. The expected number of comparisons for finding the *k*th largest of *n* keys using PARTITION is in which asymptotic set?

A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

11. Memoization is associated with which technique?
A. top-down dynamic programming B. `fixDown`
C. greedy methods D. bottom-up dynamic programming

12. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
A. return stack[SP++] B. stack[SP++] = X C. stack[--SP] = X D. stack[++SP] = X

13. An array with 150 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
A. If 120 is returned from PARTITION, we must continue.
B. If 131 is returned from PARTITION, we must continue.
C. If 118 is returned from PARTITION, we must continue.
D. If 119 is returned from PARTITION, we are done.

14. Suppose a value *k* appears for *p* entries in the cost function table (*C*) for an instance of the longest monotonically increasing subsequence problem. Going left-to-right across the corresponding input sequence values (y_i), which statement is true?
(Stated formally: For $i_1 < i_2 < \cdots < i_p$, suppose $C_{i_1} = C_{i_2} = \cdots = C_{i_p} = k$. Which statement is true regarding $y_{i_1}, y_{i_2}, \cdots, y_{i_p}$?)
A. They are strictly decreasing
B. They are strictly increasing
C. They are monotonically decreasing
D. They are monotonically increasing

15. Recursion is often an alternative to using which data structure?
A. Linked list B. Queue C. Stack D. 2-d array

Long Answer

1. Give C code for a stable **<u>counting sort</u>** for an input array `A` with two million integer keys in the range 0 through 99. The output is to appear in an output array `B`. (Details of input/output, allocation, declarations, error checking, comments and style **<u>are unnecessary and will be penalized</u>**. Give **<u>only</u>** the control structures and assignment statements.) (15 points)
2. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
p[0]=6
p[1]=2
p[2]=4
p[3]=3
p[4]=2
```

	1		2		3		4	
1	0	0	48	1	60	1	???	?
2	-------		0	0	24	2	36	3
3	-------	-------		0	0	24		3
4	-------	-------	-------		0	0		

3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

8 2 6 3 4 1 9 0 7 5

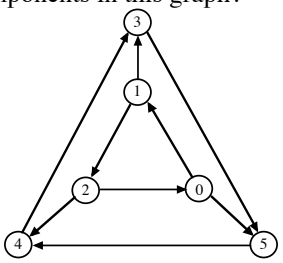
Version: 1 2/Sedgewick

4. Give diagrams showing how singly-linked lists may be used to implement the stack and queue abstractions. (10 points)
5. Use the dynamic programming solution for subset sums to determine a subset that sums to 15. (10 points)

i	0	1	2	3	4	5
S_i	0	2	3	5	7	11

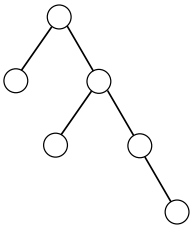
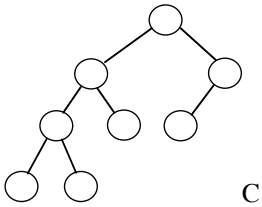
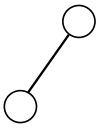
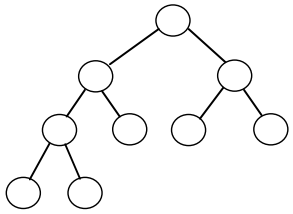
Test 3
Summer 2013 Last 4 Digits of Student ID # _____
Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. What is the number of strongly connected components in this graph?

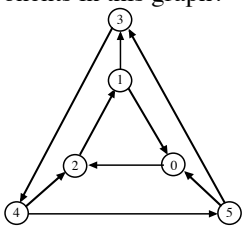


A. 1 B. 2 C. 3 D. 4

2. During a breadth-first search, the status of a black vertex is:
   A. It has been completely processed. B. It is in the FIFO queue.
   C. It is in the priority queue. D. It is undiscovered.
3. Which edge is chosen in a phase of Kruskal's algorithm?
   A. An edge of maximum-weight in a cycle (to be excluded)
   B. A minimum-weight edge that keeps the result free of cycles
   C. A minimum-weight edge connecting T to S.
   D. An edge that is on a shortest path from the source
4. When using two breadth-first searches to find the diameter of a tree, the purpose of the first search is to find:
   A. all vertices that could be an end of a diameter. B. both ends of a diameter.
   C. one end of a diameter. D. the number of edges in the diameter.
5. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., *V* - 1.)
   A. Adjacency lists (ordered): $\Theta(V)$ B. Adjacency lists (unordered): $\Theta(V)$
   C. Adjacency matrix: $\Theta(1)$ D. Compressed adjacency lists (ordered): $\Theta(V)$
6. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:
   A. finish(X) < finish(Y) B. finish(X) > finish(Y)
   C. finish(X) = finish(Y) D. could be either A. or B.
7. Which of the following cannot occur when edges are inserted into a directed graph?
   A. The graph acquires a cycle.
   B. The number of strong components may remain the same.
   C. The number of strong components may decrease.
   D. The number of strong components may increase.
8. Which of the following binary trees can be legally colored as a red-black tree <u>with its root colored red</u>?

A.  B.  C.  D. 

9. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

10. When a graph is dense, the best way to find a minimum spanning tree is:
    A. Prim's algorithm using T-table B. Floyd-Warshall algorithm
    C. Prim's algorithm using heap D. Warshall's algorithm

11. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. 2α

12. Suppose that there is only one path from vertex 5 to vertex 10 in a directed graph:
$5 \rightarrow 7 \rightarrow 8 \rightarrow 3 \rightarrow 2 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?

A. 2 B. 5 C. 8 D. 10

13. The number of potential probe sequences when using linear probing with a table with *m* entries is:

A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

14. The cycle property for minimum spanning trees may be used to find an MST by:
   A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
   B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
   C. Remove the maximum weight in any cycle until only a tree of edges remains.
   D. Remove the minimum weight in any cycle until only a tree of edges remains.

15. A topological ordering of a directed graph may be computed by:
   A. Ordering the vertices by ascending discovery time after DFS
   B. Ordering the vertices by ascending finish time after DFS
   C. Ordering the vertices by descending discovery time after DFS
   D. Ordering the vertices by descending finish time after DFS

16. In Dijkstra's algorithm, the final shortest path distance from the source s to a vertex x is known when:
   A. some vertex y moves from T to S and there is an edge from y to x.
   B. x has its entry extracted from the heap.
   C. x has its heap entry changed by a MIN-HEAP-DECREASE-KEY.
   D. x is read from the input file.

Problems 17 and 18 refer to the following hash table whose keys are stored by double hashing using $h_1(key) = key \% 13$ and $h_2(key) = 1 + (key \% 12)$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			120	186	187	162		122	110			194

17. 266 would be inserted into which slot of the given table?

A. 0 B. 1 C. 2 D. 7 E. 10 F. 11

18. 313 would be inserted into which slot of the given table? (266 has not been inserted)

A. 0 B. 1 C. 2 D. 7 E. 10 F. 11

Problems 19 and 20 refer to the following hash table whose keys are stored by linear probing using $h(key) = key \% 13$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

19. 148 would be inserted into which slot of the given table?

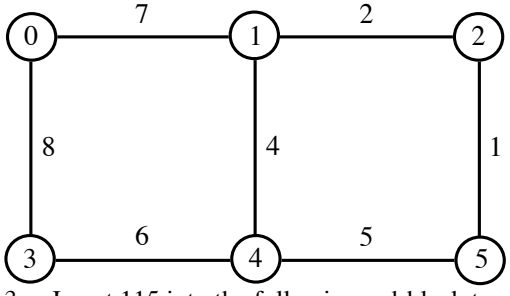
A. 0 B. 1 C. 2 D. 4 E. 11 F. 12

20. 133 would be inserted into which slot of the given table? (148 has not been inserted)

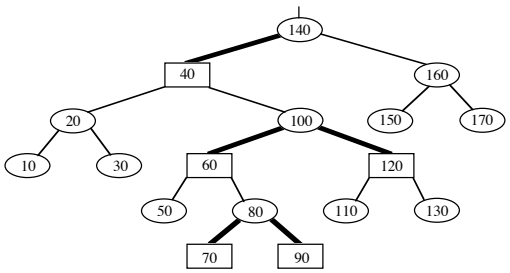
A. 0 B. 1 C. 2 D. 4 E. 11 F. 12

Long Answer

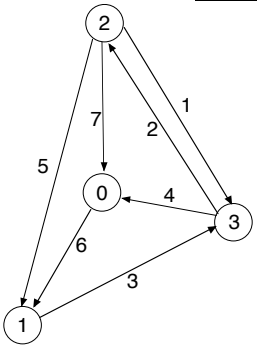
1. Give Kruskal's algorithm as pseudo-code. 10 points
2. Show the ***compressed*** adjacency list representation for this graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.



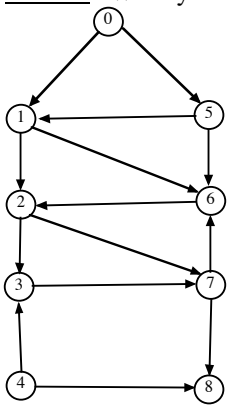
3. Insert 115 into the following red-black tree. Be sure to indicate the cases you used. 10 points



4. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points



5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ***ordered***. Write your answer in the tables below. 10 points



Vertex	Start	Finish
0	1	____
1	____	____
2	____	____
3	____	____
4	____	____
5	____	____
6	____	____
7	____	____
8	____	____

Edge	Type
0 1	____
0 5	____
1 2	____
1 6	____
2 3	____
2 7	____
3 7	____
4 3	____
4 8	____

Edge	Type
5 1	____
5 6	____
6 2	____
7 6	____
7 8	____

6. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.