

Multiple Choice:

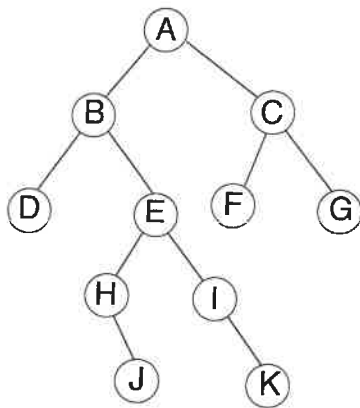
1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. In a binary search tree, which element does not have a predecessor?

C

A. any one of the leaves B. the maximum C. the minimum D. the root

2. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 8?



- A. A
- B. C
- C. F
- D. G

A

3. The time to fill in the dynamic programming matrix when computing the LCS for sequences of lengths m and n is:

D

A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

4. In the example of recycling the elements of a list in $O(1)$ time, which element becomes the first element of the garbage list?

B

- A. The first element of the circular list
- B. The second element of the circular list
- C. The last element of the circular list
- D. The second element of the original garbage list

5. The subset sum problem takes n input values and attempts to find a combination of those values whose sum is m . The worst-case time to extract the solution from the dynamic programming table is:

B

A. $\Theta(\log m)$ B. $\Theta(n)$ C. $\Theta(m)$ D. $\Theta(mn)$

6. Suppose that only numbers in $1 \dots 1000$ appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?

D

- A. 300, 400, 900, 800, 500
- B. 700, 200, 600, 550, 500
- C. 200, 700, 600, 300, 400, 500
- D. 100, 1000, 200, 800, 300, 900, 500

7. Which of the following would not be used in implementing rat-in-a-maze in a depth-first fashion?

A

- A. Circular queue
- B. Recursion
- C. Stack
- D. 2-d array

8. What does counting sort count?

D

- A. the number of bytes in the input array
- B. the number of different input values that have occurred
- C. the maximum length among all the strings being sorted
- D. the number of occurrences for each possible key value

9. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

A

- A. The head points to the first element and the tail points to the last element.
- B. The tail points to the first element and the head points to the last element.
- C. Like a circular queue, the maximum number of items is determined at initialization.
- D. One node is always wasted.

10. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?

D

- A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
- B. The height of the tree is $\Omega(n \log n)$.
- C. There will be a path from the root to a leaf with $\Omega(n \log n)$ decisions.
- D. There will be $n \log n$ leaves.

11. Suppose a value k appears for p entries in the cost function table (C) for an instance of the longest monotonically increasing subsequence problem. Going left-to-right across the corresponding input sequence values (y_i), which statement is true?

(Stated formally: For $i_1 < i_2 < \dots < i_p$, suppose $C_{i_1} = C_{i_2} = \dots = C_{i_p} = k$. Which statement is true regarding $y_{i_1}, y_{i_2}, \dots, y_{i_p}$?)

D

- A. They are monotonically decreasing
- B. They are strictly increasing
- C. They are monotonically increasing
- D. They are strictly decreasing

12. If POP is implemented as `return stack[SP--]`, then PUSH of element X is implemented as:

- D A. `return stack[SP++]` B. `stack[SP++] = X`
 C. `stack[--SP] = X` D. `stack[++SP] = X`

13. The worst-case number of comparisons for finding the k th largest of n keys using PARTITION is in which asymptotic set?

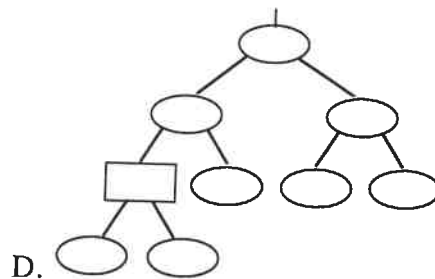
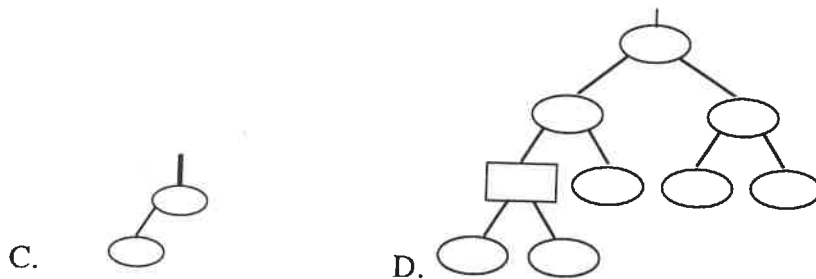
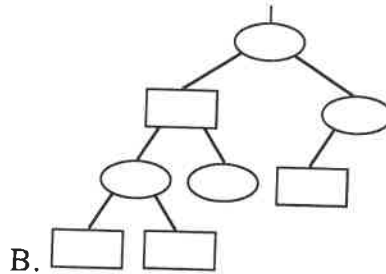
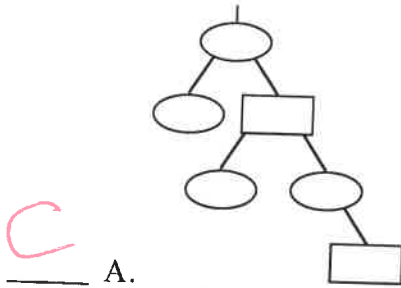
- D A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
  if (x==null)
    return;
  traverse(x.left);
  traverse(x.right);
  // process x here
}
```

- B A. inorder B. postorder C. preorder D. search for key x

15. Which of the following binary trees has an illegal red-black tree coloring?



2,

Compute $c[1][5]$

$k=1$ gives cost $276=c[1][1]+c[2][5]+p[0]*p[1]*p[5]$

$k=2$ gives cost $432=c[1][2]+c[3][5]+p[0]*p[2]*p[5]$

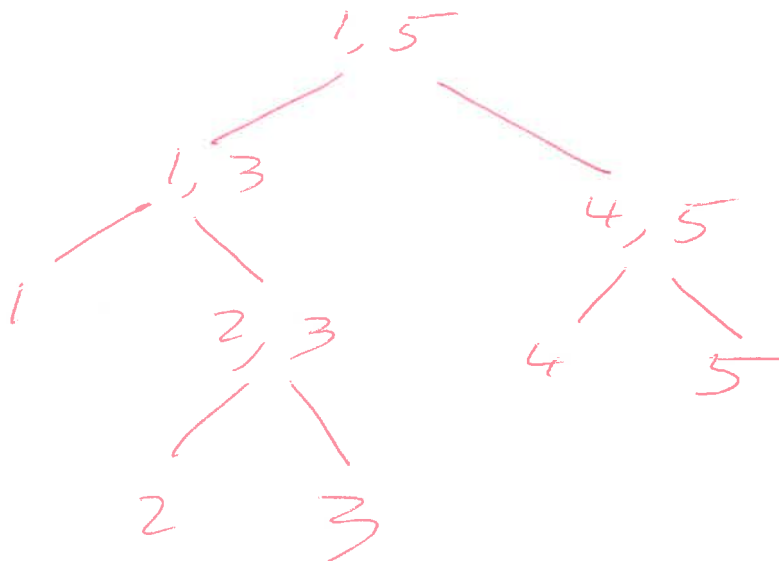
$k=3$ gives cost $208=c[1][3]+c[4][5]+p[0]*p[3]*p[5]$

$k=4$ gives cost $288=c[1][4]+c[5][5]+p[0]*p[4]*p[5]$

$c[1][5]=208, \text{trace}[1][5]=3$

	1	2	3	4	5
1	0	120	1	138	208
2		0	48	2	88
3			0	60	3
4				0	60
5					0

5
4 5
4
1 5
3
2 3
2
1 3
1

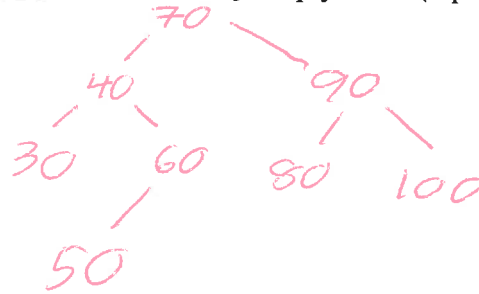


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3

Long Answer

1. Give the unbalanced binary search tree that results when the keys 70, 40, 60, 90, 50, 30, 80, 100 are inserted, in the given order, into an initially empty tree. (5 points)



2. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

	p[0]=5		p[1]=4		p[2]=6		p[3]=2		p[4]=5		p[5]=6	
	1		2		3		4		5			
1	0	0	120	1	88	1	138	3	???	?		
2	-----		0	0	48	2	88	3	156	3		
3	-----	-----			0	0	60	3	132	3		
4	-----	-----	-----				0	0	60	4		
5	-----	-----	-----	-----					0	0		

AB 8 2 6 3 5 1 9 0 7 4

A 8 B 2 6 3 5 1 9 0 7 4

2 A 8 B 6 3 5 1 9 0 7 4

2 A 8 6 B 3 5 1 9 0 7 4

2 3 A 6 8 B 5 1 9 0 7 4

2 3 A 6 8 5 B 1 9 0 7 4

2 3 1 A 8 5 6 B 9 0 7 4

2 3 1 A 8 5 6 9 B 0 7 4

2 3 1 0 A 5 6 9 8 B 7 4

2 3 1 0 A 5 6 9 8 7 B 4

2 3 1 0 < 4 > 6 9 8 7 5

3.

3. Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

8 2 6 3 5 1 9 0 7 4

4. Complete the following example of the efficient dynamic programming technique for finding a Longest Increasing Subsequence (monotone). Be sure to use binary searches and updates to the table below. Circle the inputs that are in the final LIS. (10 points)

	1	2	3	4	5	6	7	8	9	10	11	12
	10	20	30	40	15	45	17	25	35	47	37	27
C	1	2	3	4	2	5	3	4	5	6	6	5
j	0	1	2	3	1	4	5	7	8	9	9	8

1 10/1

2 20/2 15/5

3 30/3 17/7

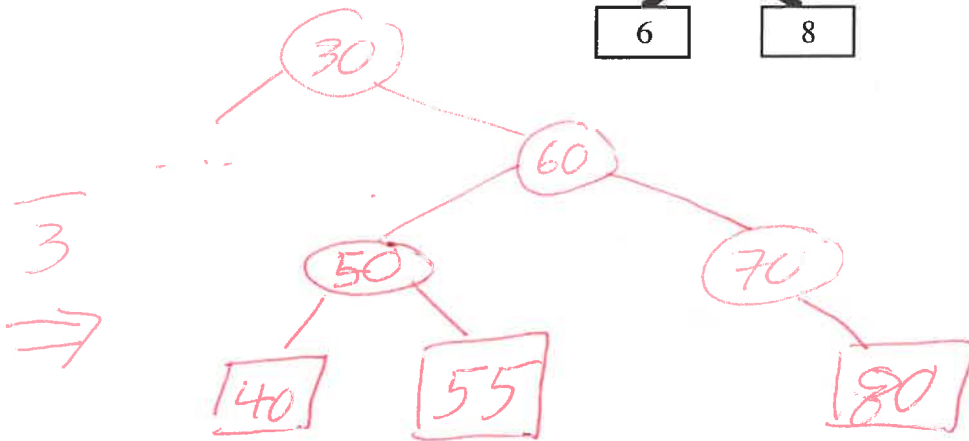
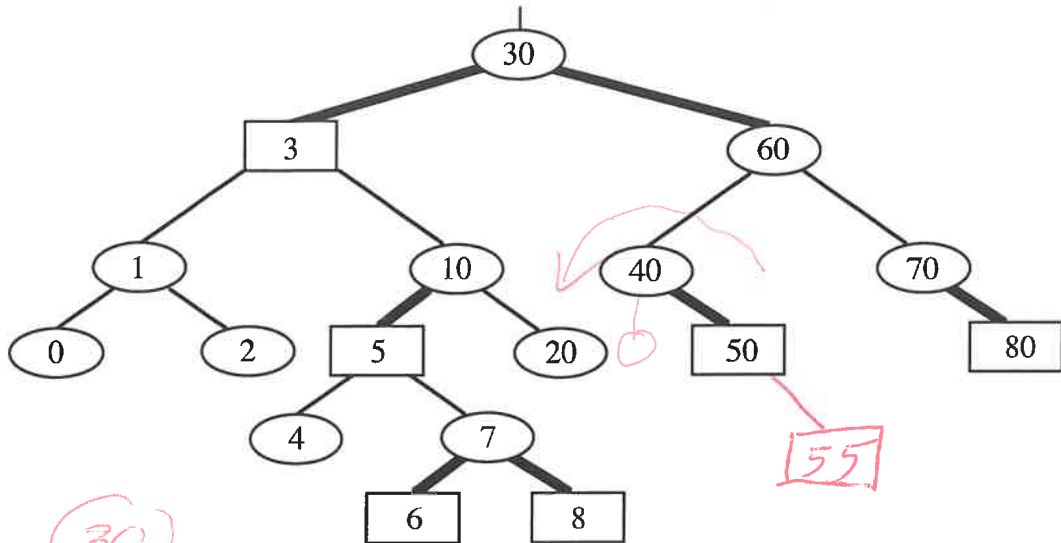
4 40/4 25/8

5 45/6 35/9 27/12

6 47/10 37/11

7

5. Insert 55 into the given red-black tree. Be sure to indicate the cases that are applied. (10 points)



6. A billion integers in the range $0 \dots 2^{30} - 1$ will be sorted by LSD radix sort. How much faster is this done using radix $0 \dots 2^{10} - 1$ rather than $0 \dots 2^6 - 1$? Show your work. (10 points)

2^{10}	2^6
$n = 1B$	$n = 1B$
$K = 2^{10}$	$K = 2^6$
$d = 3$	$d = 5$
$\Theta(d(n + K))$	$\Theta(d(n + K)) \leftarrow$
$\Theta(3(1B + 2^{10}))$	$\Theta(5(1B + 2^6))$
\uparrow	
$\frac{5}{3}$ faster	
(or runs in $\frac{3}{5}$ of the time)	