CSE 2320-004
Test 1
Fall 2018

Key (LA)
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The time for the following code is in which set?

```
for (i=0; i<n-5; i++)        n
  for (j=2; j<n; j+=j)       log n
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)      n
      c[i][j] += a[i][k]*b[k][j];
  }
```

C A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n^2 \log n)$ D. $\Theta(n^3)$

2. The recursion tree for mergesort has which property?

C A. it leads to a definite geometric sum B. it leads to a harmonic sum
C. each level has the same contribution D. it leads to an indefinite geometric sum

3. Which of the following is false?

C A. $n^2 \in O(n^3)$ B. $n \log n \in O(n^2)$
C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $3^n \in \Omega(2^n)$

4. Which of the following is not true regarding a maxheap with 1000 elements?

D A. Subscript 1 will store the maximum priority.
B. The parent for the node with subscript 500 is stored at subscript 250.
C. The left child for the node with subscript 200 is stored at subscript 400.
D. The right child for the node with subscript 405 is stored at subscript 911.

5. The function $2n \log n + \log n$ is in which set?

B A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Omega(n^2)$ D. $\Theta(\log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

A A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega(n)$ D. $O(n^2)$

7. Suppose the input to heapsort is always a table of n zeroes and ones. The worst-case time will be:

C A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n \log n)$ D. $\Theta(\log n)$

8. The time to run the code below is in:

```
for (i=5; i<=n-5; i++)
  for (j=2; j<n; j=2*j+1)
    sum+=i+j;
```

A A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

9. Suppose that you have correctly determined some c and n_0 to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

C A. c may be decreased B. n_0 may be increased C. n_0 may be decreased D. $f(n) \in O(g(n))$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A A. $T(n) \in \Omega(n^3)$ B. $T(n) \in O(\log n)$ C. $T(n) \in O(n)$ D. $T(n) \in \Omega(n^2)$

11. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 10 probes?

B A. 511 B. 1023 C. 2047 D. 4095

12. Heapsort may be viewed as being a faster version of which sort?

C A. insertion B. mergesort
C. selection D. `qsort`

13. Which of the following functions is not in $\Omega(n^2)$?

C A. $n^2 \lg n$ B. n^3 C. n D. n^2

14. $4^{\lg 7}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

D A. $\sqrt{7}$ B. 7 C. 25 D. 49

15. Which of the following is solved heuristically by a greedy method?

D A. Fractional knapsack
B. This semester's first lab assignment
C. Unweighted interval scheduling
D. 0/1 knapsack

16. What is the value of $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$?

$\frac{1}{1-\frac{2}{3}}$

D A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 3

17. Suppose there is a large table with n integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurences of a particular value?

B A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

18. The number of calls to `merge()` while performing `mergesort` on n items is in:

C A. $\Theta(\log n)$ B. $\Theta(1)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

19. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

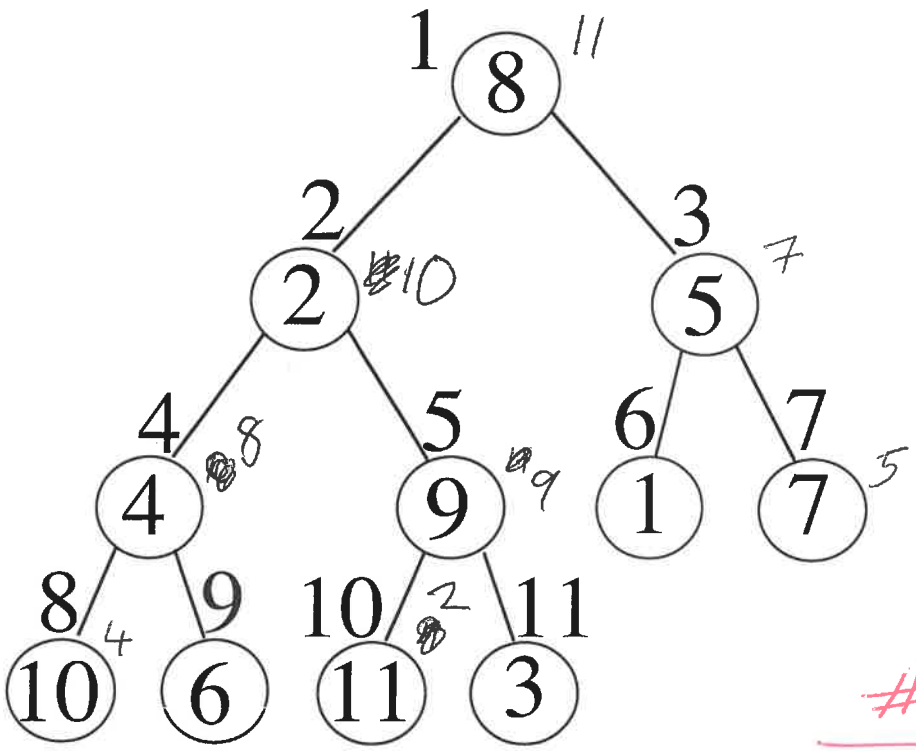
D A. Descending order of start time
B. Ascending order of interval length
C. Descending order of finish time
D. Ascending order of finish time

20. The goal of the Huffman coding method is:

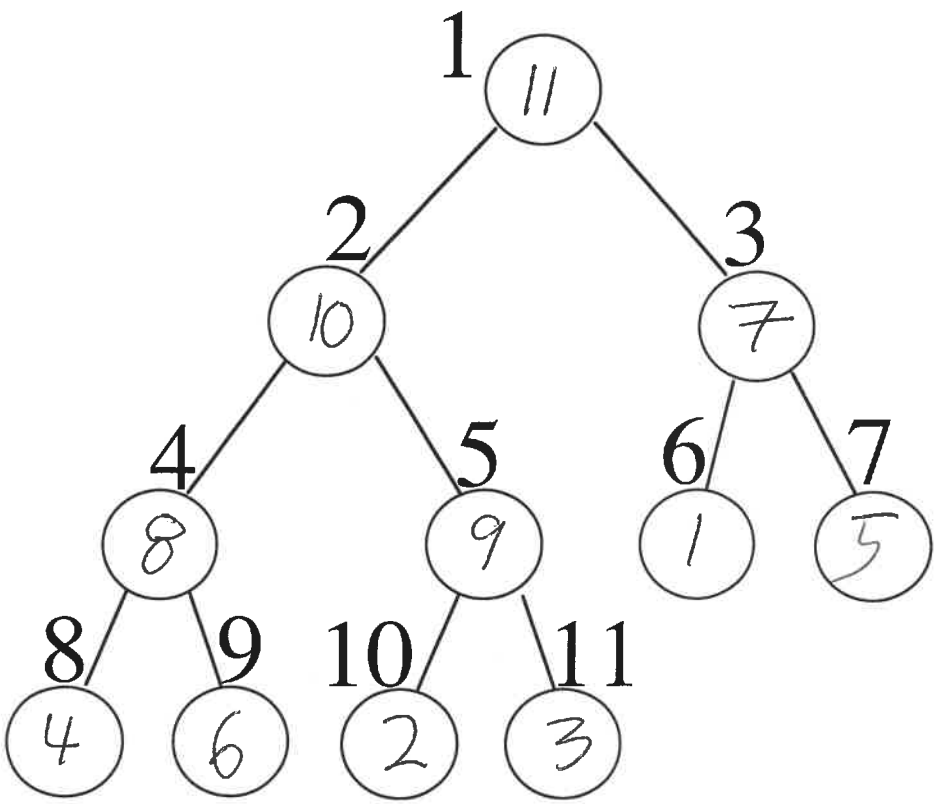
B A. Construct a tree that is order preserving
B. Minimize the expected bits per symbol.
C. Find the symbols with high probability of occuring.
D. Maximize the compression for every string.

Long Answer

1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points



# Incorrect	Points
1	1
2	3
3	6
4	10



$$n^{\lg 3} + n^2 \sum_{k=0}^{\lg n - 1} \left(\frac{3}{4}\right)^k$$

$$= n^{\lg 3} + n^2 \frac{\left(\frac{3}{4}\right)^{\lg n} - 1}{\frac{3}{4} - 1}$$

$$= n^{\lg 3} + n^2 \frac{n^{\lg \frac{3}{4}} - 1}{-\frac{1}{4}}$$

$$= n^{\lg 3} + n^2 \frac{1 - n^{\lg \frac{3}{4}}}{\frac{1}{4}}$$

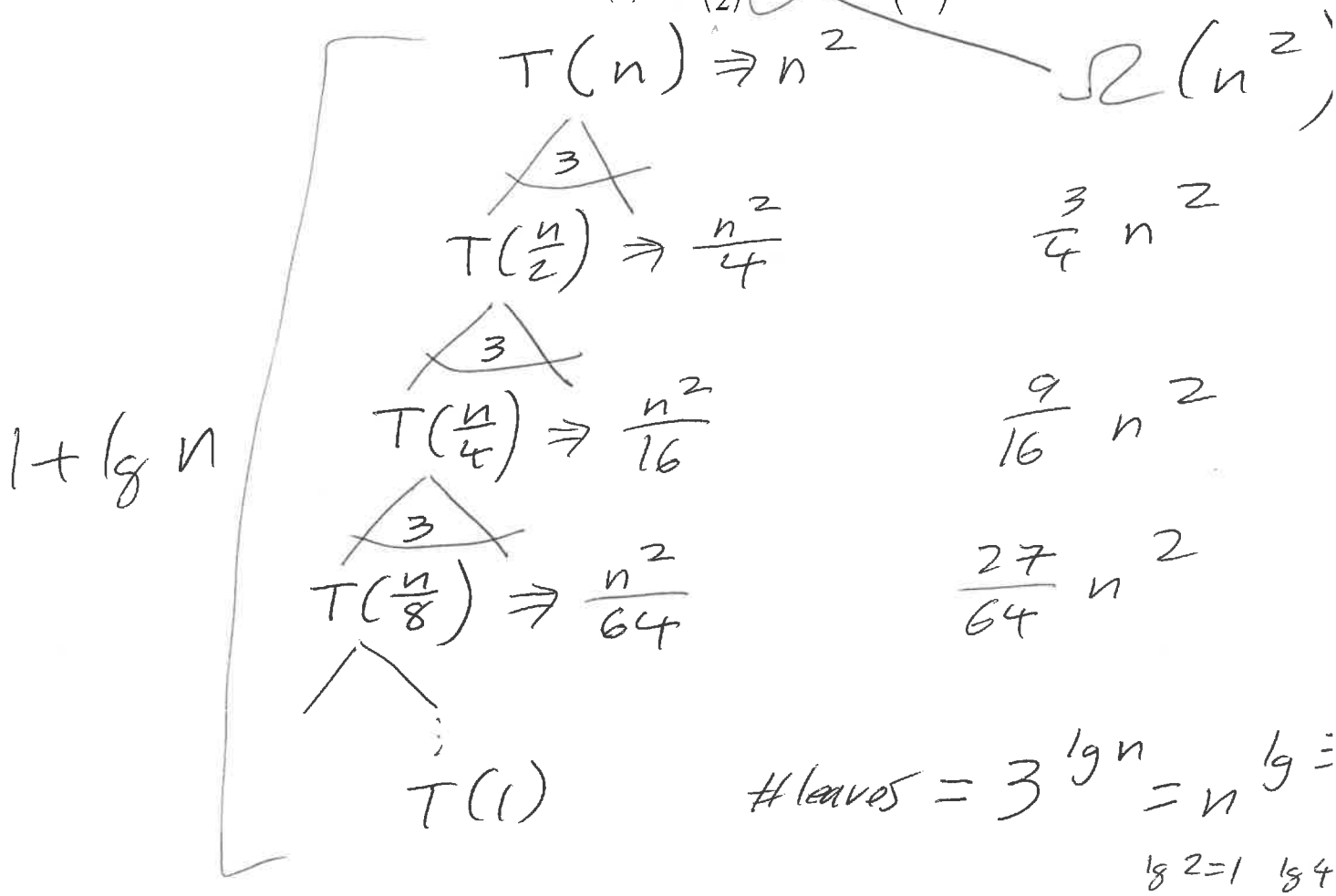
$$= n^{\lg 3} + 4n^2 \left(1 - n^{\lg \frac{3}{4}}\right)$$

$$= n^{\lg 3} + 4n^2 - 4n^2 \, n^{\lg \frac{3}{4}}$$

$$= n^{\lg 3} + 4n^2 - 4n^2 \cdot n^{-.4}$$

$$= \Theta(n^2)$$

5. Use the recursion-tree method to show that $T(n) = 3T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2\right)$. 10 points



$$n^{\lg 3} + n^2 \sum_{K=0}^{\lg n - 1} \left(\frac{3}{4}\right)^K$$

$$\leq n^{\lg 3} + n^2 \sum_{K=0}^{\infty} \left(\frac{3}{4}\right)^K$$

$$= n^{\lg 3} + n^2 \frac{1}{1 - \frac{3}{4}}$$

$$= n^{\lg 3} + 4n^2 = O(n^2)$$

6. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{2}\right) + n^2$ is in $O(n^2)$. (You do not need to show that $T(n)$ is in $\Omega(n^2)$.) 10 points

Assume $T(k) \leq ck^2$ for $k < n$

$$T\left(\frac{n}{2}\right) \leq c\frac{n^2}{4}$$

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$\leq 3c\frac{n^2}{4} + n^2$$

$$= cn^2 - \frac{1}{4}cn^2 + n^2$$

$$\leq cn^2 \quad \text{for} \quad c \geq 4$$