

Multiple Choice:

1. Write the letter or value for your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

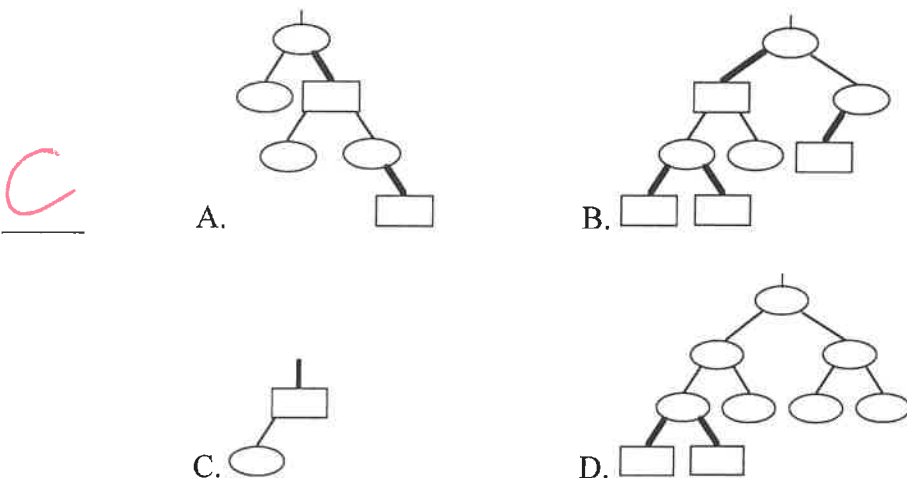
1. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The number of entries in the two tables are:

- D A. n for both B. m for both C. n and m D. $n + 1$ and m

2. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

- C A. α B. 2α C. $\frac{\alpha}{2}$ D. $\frac{2}{3}\alpha$

3. Which of the following binary trees has an illegal red-black tree coloring?

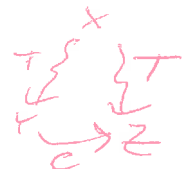


4. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z . If there is also an edge from Y to Z , then its type will be:

- B A. Back B. Cross C. Forward D. Tree

5. Which edge is chosen in a phase of Kruskal's algorithm?

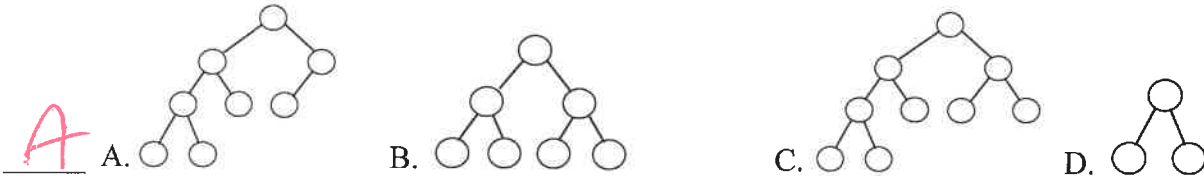
- C A. An edge that is on a shortest path from the source
 B. An edge of maximum-weight in a cycle (to be excluded)
 C. A minimum-weight edge that keeps the result free of cycles
 D. A minimum-weight edge connecting T to S .



6. When using two breadth-first searches to find the diameter of a tree, the purpose of the first search is to find:

- C A. all vertices that could be an end of a diameter. B. both ends of a diameter.
 C. one end of a diameter. D. the number of edges in the diameter.

7. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



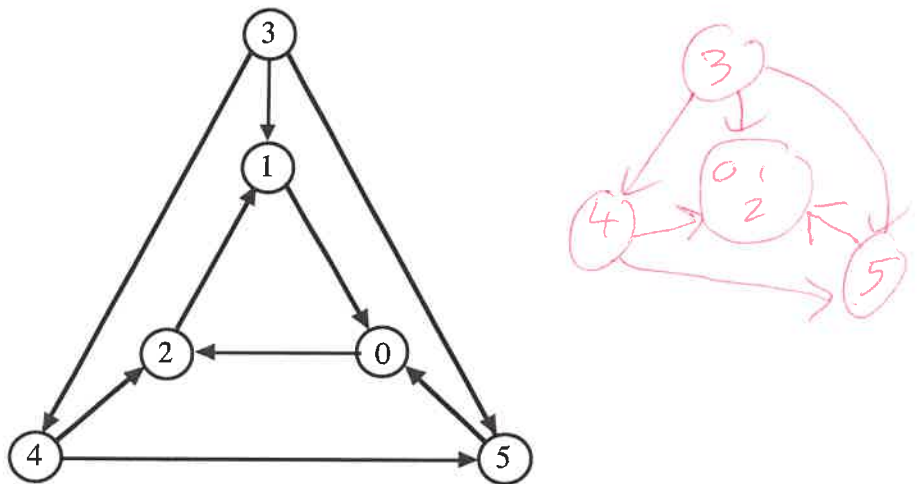
8. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:

- A A. $finish(X) > finish(Y)$ B. $finish(X) < finish(Y)$
 C. $finish(X) = finish(Y)$ D. could be either A. or B.

9. Which algorithm maintains multiple subtrees?

- D A. Prim's B. Warshall's C. Dijkstra's. D. Kruskal's

10. What is the number of strongly connected components in this graph?



4

11. Suppose that there is only one path from vertex 5 to vertex 10 in a directed graph:

$5 \rightarrow 7 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?

7

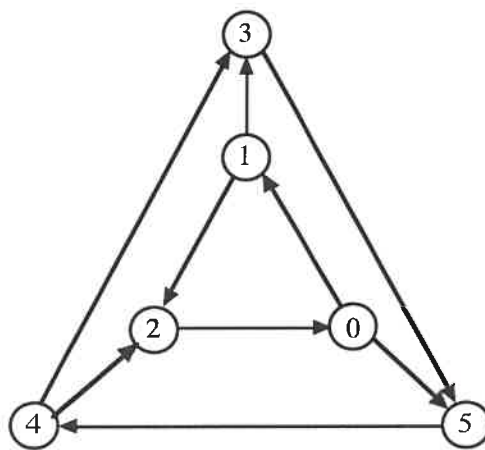
12. A topological ordering of a directed graph may be computed by:

- A
- A. Ordering the vertices by descending finish time after DFS
 - B. Ordering the vertices by ascending discovery time after DFS
 - C. Ordering the vertices by ascending finish time after DFS
 - D. Ordering the vertices by descending discovery time after DFS

13. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

- C
- A. $O(\log m)$
 - B. m
 - C. $m(m-1)$
 - D. $m!$

14. What is the number of strongly connected components in this graph?



1

15. The worst-case time for Prim's algorithm implemented with a minheap is:

- B
- A. $\theta(V + E)$
 - B. $\theta(E \log V)$
 - C. $\theta(V \log V)$
 - D. $\theta(V \log E)$

16. Which of the following cannot occur when additional edges are included in a directed graph?

- D
- A. The graph acquires a cycle.
 - B. The number of strong components may remain the same.
 - C. The number of strong components may decrease.
 - D. The number of strong components may increase.

17. What is required when calling `union(i, j)` for maintaining disjoint subsets?

- C
- A. i and j are leaders for the same subset
 - B. i and j are in the same subset
 - C. i and j are leaders for different subsets
 - D. i is the ancestor of j in one of the trees

18. The maximum number of rotations while inserting a key into a red-black tree is:

- B
- A. 1
 - B. 2
 - C. 3
 - D. the black-height

19. When finding the strongly connected components, the number of components is indicated by:

C

- A. The number of cross edges found during the second depth-first search.
- B. The number of back edges found during the first depth-first search.
- C. The number of restarts for the second depth-first search.
- D. The number of restarts for the first depth-first search.

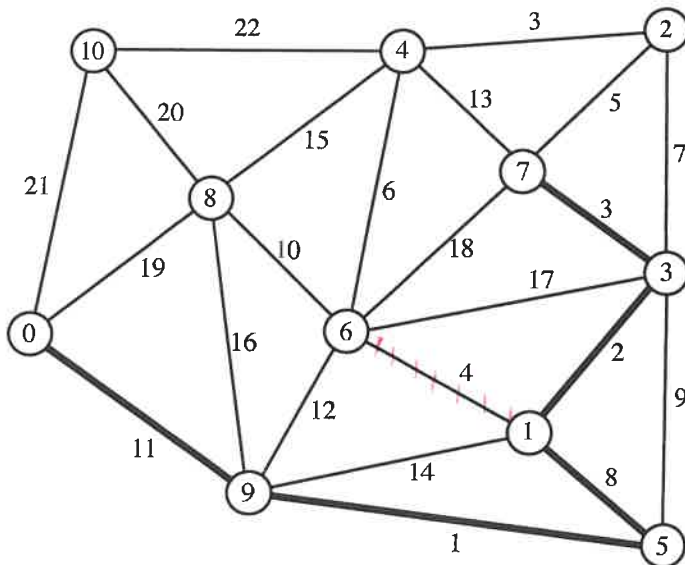
20. In Dijkstra's algorithm, the final shortest path distance from the source s to a vertex x is known when:

B

- A. x is placed on the heap.
- B. x has its entry extracted from the heap.
- C. x is read from the input file.
- D. some vertex y moves from T to S and there is an edge from y to x .

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



Before

2 5(7)

4 13(7)

6 4(1)

8 16(9)

10 21(0)

After

2 5(7)

4 6(6)

8 10(6)

10 21(0)

2. Consider the following hash table whose keys were stored by double hashing using $h_1(\text{key}) = \text{key} \% 11$ and $h_2(\text{key}) = 1 + (\text{key} \% 10)$. Show your work.

0	22
1	
2	142
3	17
4	4
5	15
6	28
7	
8	
9	130
10	10

a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)

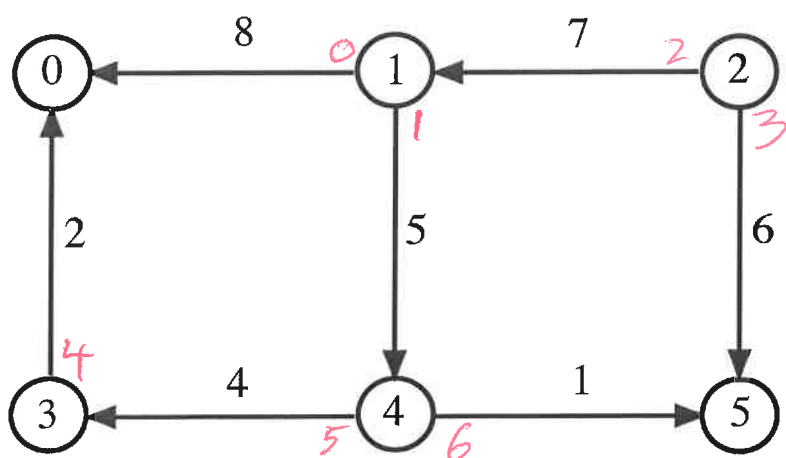
$$\begin{array}{r} 12 \\ 11 \overline{)142} \\ \underline{11} \\ 32 \\ \underline{22} \\ 10 = h_1 \end{array} \qquad \begin{array}{r} 14 \\ 10 \overline{)142} \\ \underline{10} \\ 42 \\ \underline{40} \\ 2 + 1 = 3 = h_2 \end{array}$$

10, 2

b. Suppose 130 is to be inserted (using double hashing) after 142 has been stored. Which slot will be used? (5 points)

$$\begin{array}{r} 11 \\ 11 \overline{)130} \\ \underline{11} \\ 20 \\ \underline{11} \\ 9 = h_1 \end{array}$$

3. Show the *compressed* adjacency list representation this weighted graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.



tail Tab	
0	0
1	0
2	2
3	4
4	5
5	7
6	7

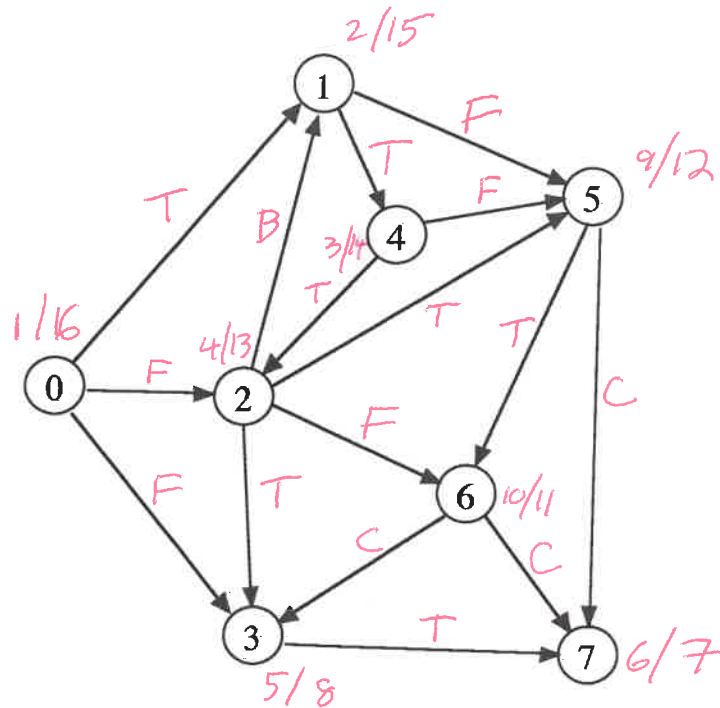
head Tab		
	v_x	wt
0	0	8
1	4	5
2	1	7
3	5	6
4	0	2
5	3	4
6	5	1

	0	1	2	3	4
0	11 2	9 2	3 2	4 3	13 2
1	17 4	15 4	9 4	11 3	4 4
2	8 0	6 1	11 0	5 3	10 1
3	32 1	15 1	24 1	26 1	19 1
4	13 2	11 2	5 2	10 2	15 2

4. Demonstrate the Floyd-Warshall algorithm, *with successors*, for the following input adjacency matrix. (∞ represents infinity) The paths indicated in the final matrix must have *at least one* edge. You *are not* required to show the intermediate matrices. 10 points.

	0	1	2	3	4
0	∞	∞	3	4	∞
1	∞	∞	∞	11	4
2	8	6	∞	5	∞
3	∞	15	∞	∞	20
4	∞	∞	5	∞	∞

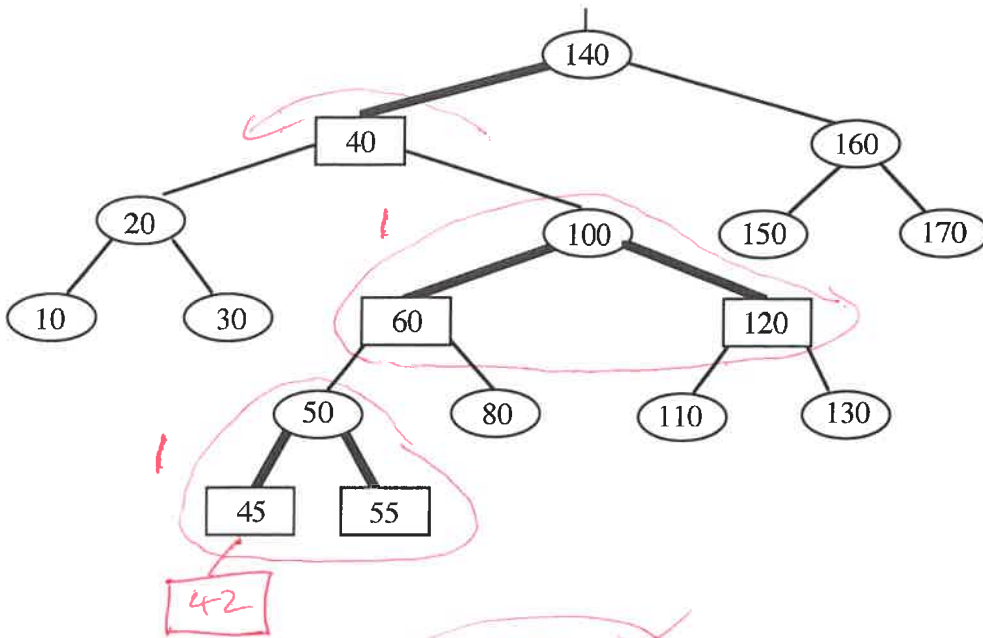
5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ordered. Write your answer in the tables below. 10 points



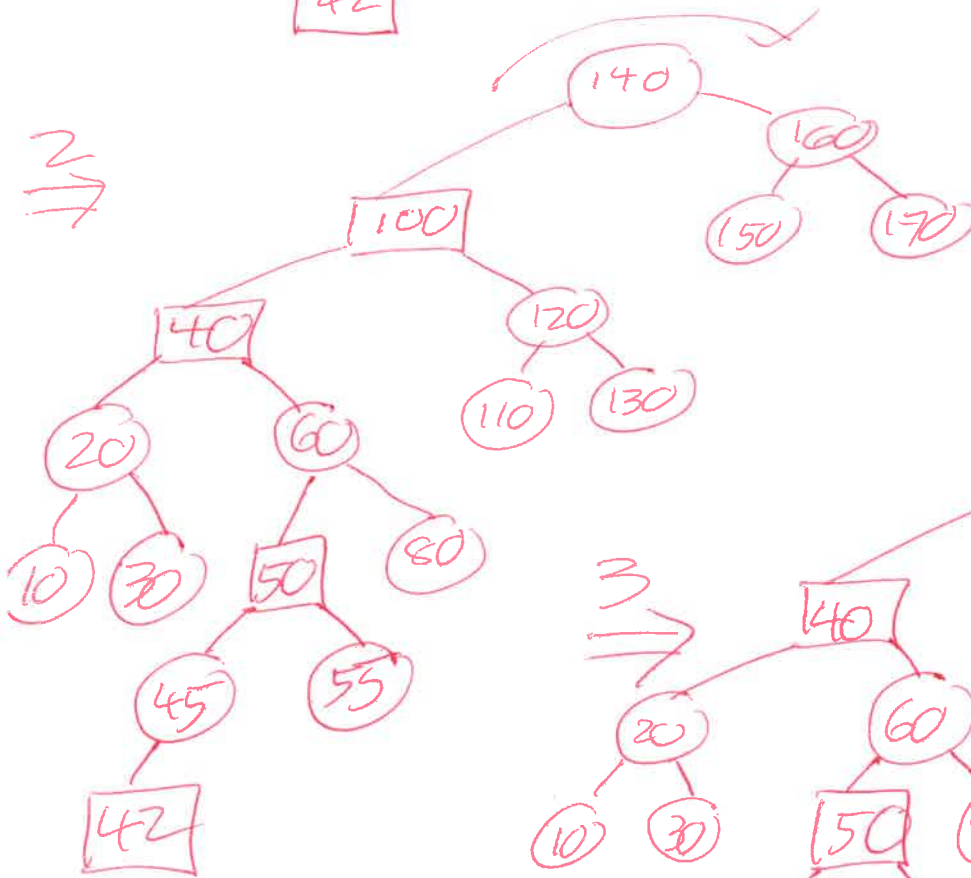
Vertex	Start	Finish	Edge	Type	Edge	Type
0	<u>1</u>	<u>16</u>	0 1	<u>T</u>	2 6	<u>F</u>
1	<u>2</u>	<u>15</u>	0 2	<u>F</u>	3 7	<u>T</u>
2	<u>4</u>	<u>13</u>	0 3	<u>F</u>	4 2	<u>T</u>
3	<u>5</u>	<u>8</u>	1 4	<u>T</u>	4 5	<u>F</u>
4	<u>3</u>	<u>14</u>	1 5	<u>F</u>	5 6	<u>T</u>
5	<u>9</u>	<u>12</u>	2 1	<u>B</u>	5 7	<u>C</u>
6	<u>10</u>	<u>11</u>	2 3	<u>T</u>	6 3	<u>C</u>
7	<u>6</u>	<u>7</u>	2 5	<u>T</u>	6 7	<u>C</u>

32 lines \Rightarrow 2 errors lose 1 point

6. Insert 42 into the given red-black tree. Be sure to indicate the cases that you used. 10 points.



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