CSE 2320-004
Test 1
Fall 2019

Key
____
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter or value of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. Suppose there is a large table with n integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurences of a particular value?

__B__ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

2. A sort is said to be stable when:

__B__ A. The expected time and the worst-case time are the same.
B. Items with the same key will appear in the same order in the output as in the input.
C. It removes duplicate copies of any key in the final output.
D. It runs in $O(n \log n)$ time.

3. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

__D__ A. Descending order of start time B. Ascending order of interval length
C. Descending order of finish time D. Ascending order of finish time

4. The number of calls to `heapExtractMin` to build a Huffman code tree for n symbols is:

__D__ A. $\Theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$

5. The time for the following code is in which set?

```
for (i=0; i<n-5; i++)
  for (j=2; j<n; j+=5)
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)
      c[i][j] += a[i][k]*b[k][j];
  }
```

__D__ A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n^2 \log n)$ D. $\Theta(n^3)$

6. The number of calls to `merge()` while performing `mergesort` on n items is in:

__C__ A. $\Theta(\log n)$ B. $\Theta(1)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

7. Which of the following is true?

A ___ A. $n^3 \in \Omega(n^2)$ B. $n\log n \in \Omega(n^2)$

C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $3^n \in O(2^n)$

8. Which of the following is not true regarding a maxheap with 1000 elements?

A ___ A. Subscript 1 will store the minimum priority.
B. The parent for the node with subscript 500 is stored at subscript 250.
C. The left child for the node with subscript 200 is stored at subscript 400.
D. The right child for the node with subscript 455 is stored at subscript 911.

9. The function $2n\log n + \log n$ is in which set?

B ___ A. $\Theta(n)$ B. $\Theta(n\log n)$ C. $\Omega(n^2)$ D. $\Theta(\log n)$

10. $f(n) = \lg n$ is in all of the following sets, except

C ___ A. $O(\log n)$ B. $O(\log(n!))$ C. $\Omega(n)$ D. $o(n^2)$

11. Suppose the input to heapsort is always a table of n zeroes and ones. The worst-case time will be:

C ___ A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n \log n)$ D. $\Theta(\log n)$

12. The time to run the code below is in:

```
for (i=n-1; i>=0; i-=2)
  for (j=5; j<n; j*=3)
    sum+=i+j;
```

A ___ A. $\Theta(n\log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

13. Suppose that you have correctly determined some c and n_0 to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

C ___ A. c may be decreased B. n_0 may be increased C. n_0 may be decreased D. $f(n) \in O(g(n))$

14. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A A. $T(n) \in \Omega(n^3)$ B. $T(n) \in O(\log n)$ C. $T(n) \in O(n)$ D. $T(n) \in \Omega(n^2)$

15. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 10 probes?

B A. 511 B. 1023 C. 2047 D. 4095

16. Bottom-up maxheap construction is based on applying `maxHeapify` in the following fashion:

B
A. In ascending slot number order, for each slot that is a parent.
B. In descending slot number order, for each slot that is a parent.
C. $\frac{n}{2}$ times, each time from subscript 1.
D. In descending slot number order, for each slot that is a leaf.

17. Which of the following functions is not in $\Omega(n^2)$?

C A. $n^2 \lg n$ B. n^3 C. n D. n^2

18. Evaluate $4^{\lg 5}$. (Recall that $\lg x = \log_2 x$.)

25

19. Which of the following is solved heuristically by a greedy method?

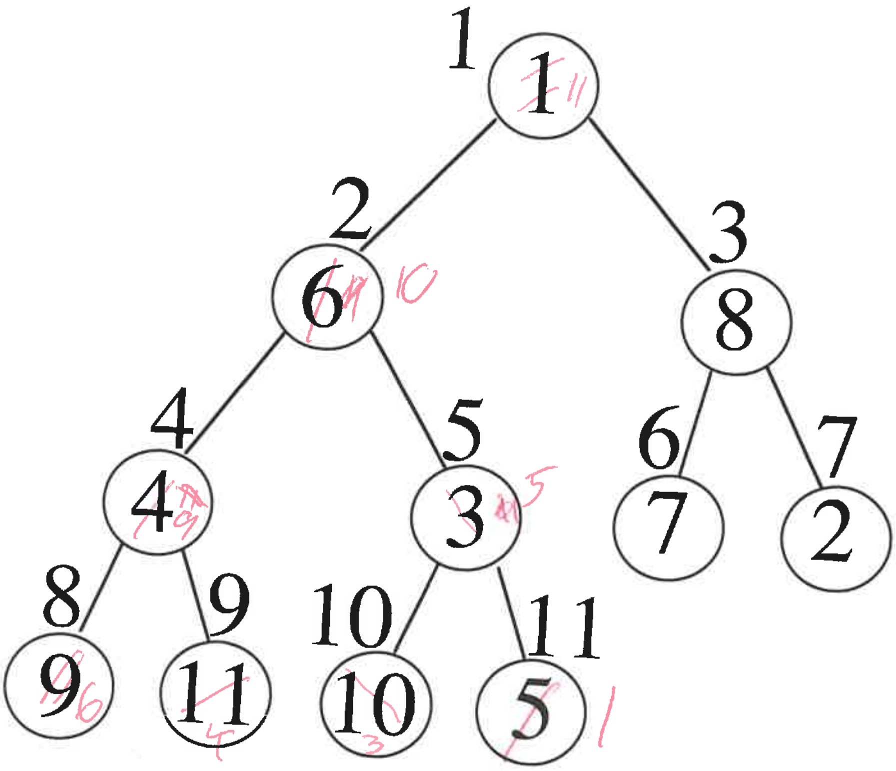
D
A. Fractional knapsack
B. This semester's first lab assignment
C. Unweighted interval scheduling
D. 0/1 knapsack

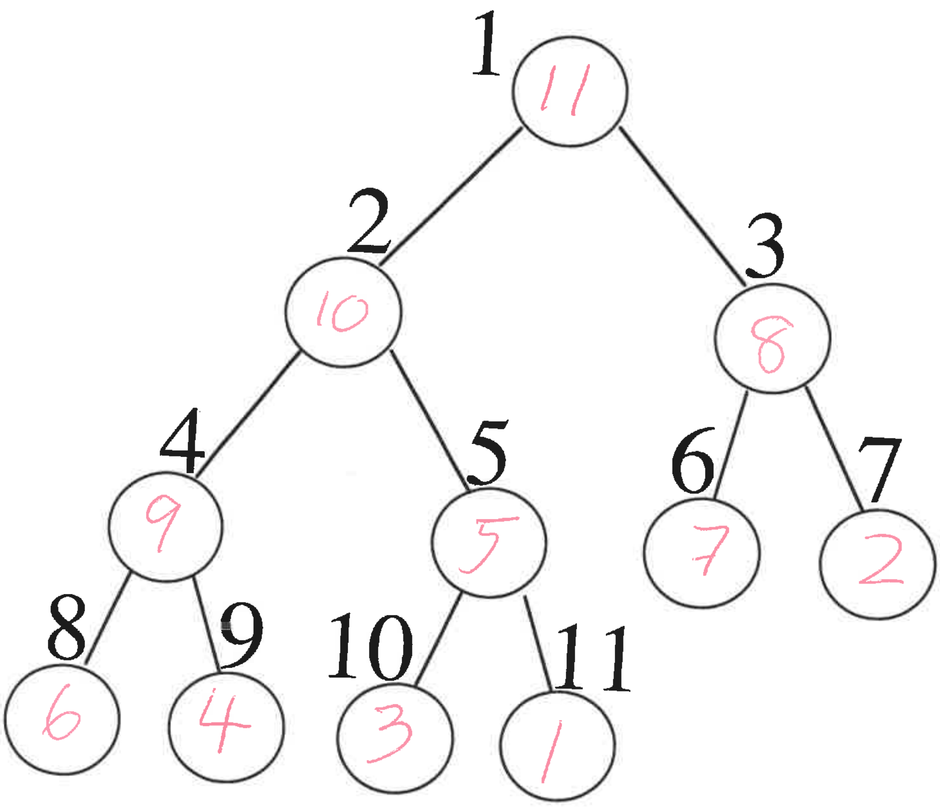
20. What is the value of $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$?

2

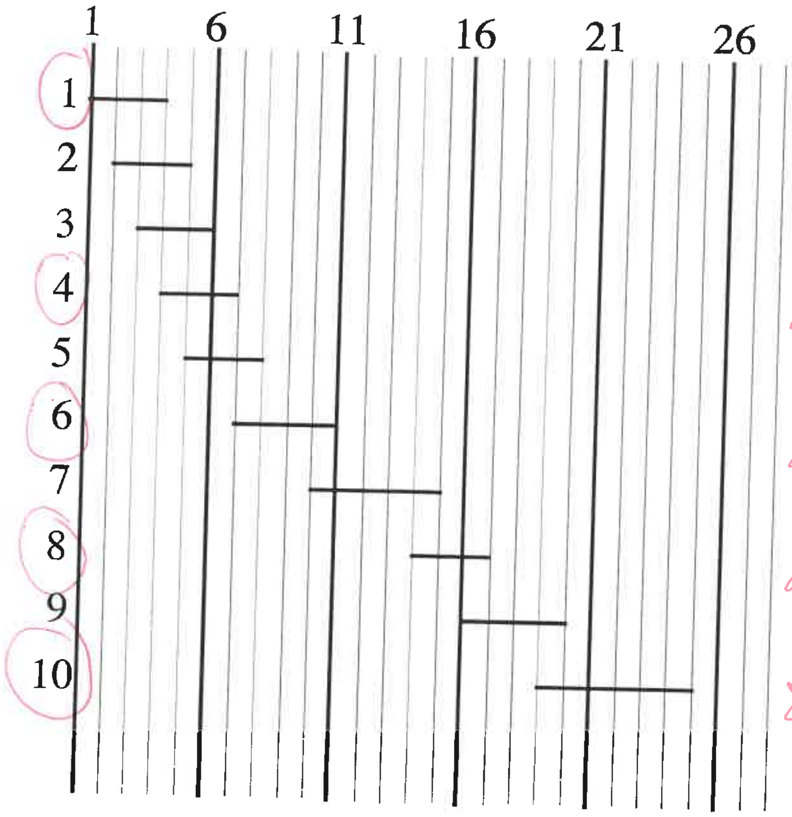
Long Answer

1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points





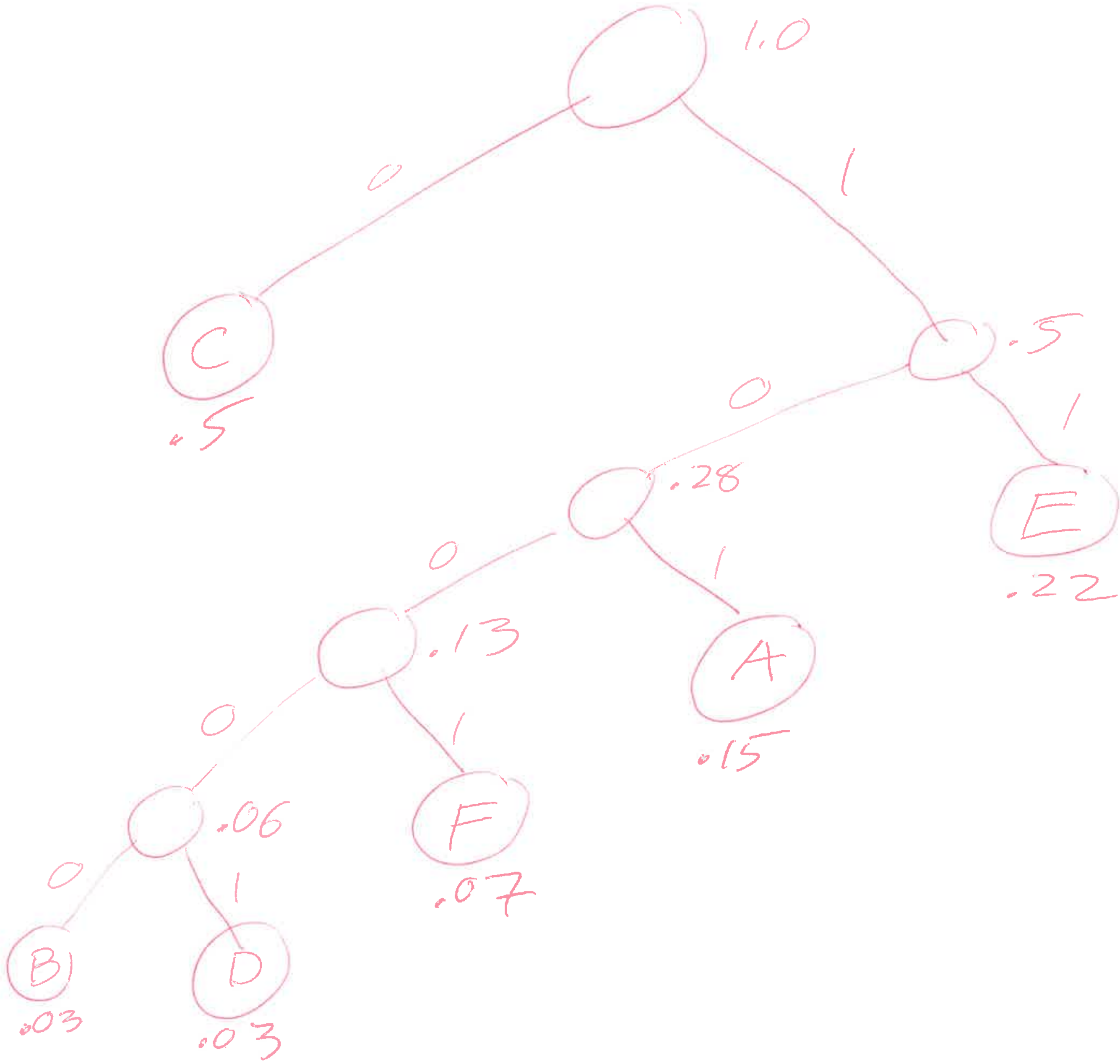
2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



	v	p	m
1	5	0	5
2	4	0	5
3	3	0	~~4~~ 5
4	2	1	7
5	1	2	7
6	5	4	12
7	4	5	12
8	3	6	15
9	2	7	15
10	1	8	16

3. Give the greedy solution for the unweighted interval scheduling problem using the set of intervals for problem 2. You may simply give the indices for the intervals in the solution. 5 points

1, 4, 6, 8, 10


1.0
0
1
C
.5
.5
0
1
.28
E
.22
0
1
.13
A
.15
0
1
.06
F
.07
0
1
B
.03
D
.03

4. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

A	0.15	3	.45
B	0.03 ✓	5	.15
C	0.5	1	.50
D	0.03 ✓	5	.15
E	0.22	2	.44
F	0.07 ✓	4	.28
			1.97

A

B

C

D

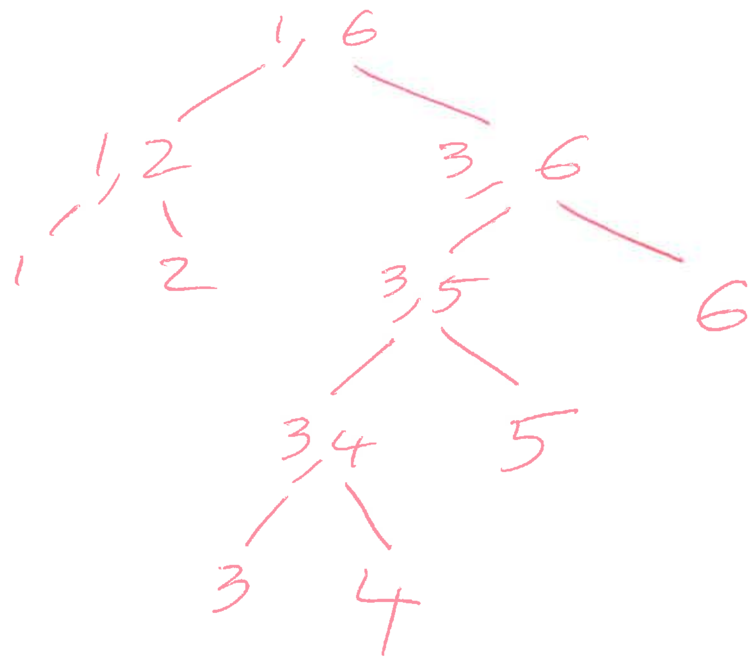
E

F

5. Give the tree corresponding to the following instance of optimal matrix multiplication. 5 points

6
4 3 2 5 4 7 5

	1		2		3		4		5		6	
1	0	0	24	1	64	2	96	2	176	2	230	2
2	-------		0	0	30	2	64	2	138	2	196	2
3	-------		-------		0	0	40	3	96	4	166	5
4	-------		-------		-------		0	0	140	4	240	4
5	-------		-------		-------		-------		0	0	140	5
6	-------		-------		-------		-------		-------		0	0



00 4

$T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta(\sqrt{n}\log n)$

$1 + \log_4 n$

$T(n) \Rightarrow \sqrt{n}$

2

$T\left(\frac{n}{4}\right) \Rightarrow \sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2} \quad \cdot 2 \quad \sqrt{n}$

2

$T\left(\frac{n}{16}\right) \Rightarrow \sqrt{\frac{n}{16}} = \frac{\sqrt{n}}{4} \quad \cdot 4 \quad \sqrt{n}$

2

$T\left(\frac{n}{64}\right) \Rightarrow \sqrt{\frac{n}{64}} = \frac{\sqrt{n}}{8} \quad \cdot 8 \quad \sqrt{n}$

$\vdots$

$T(1)$

$\#\text{leaves} = 2^{\log_4 n}$

$= n^{\log_4 2}$

$= \sqrt{n}$

$\sqrt{n}(\log_4 n + 1) = \Theta(\sqrt{n}\log n)$

6. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta\left(\sqrt{n}\log n\right)$. 10 points

0.0.4

$T(n) = 2T(\frac{n}{4}) + \sqrt{n}$ is in $O(\sqrt{n} \log n)$

Assume $T(k) \le c\sqrt{k} \log_4 k$ for $k < n$

$$T(\tfrac{n}{4}) \le c\sqrt{\tfrac{n}{4}} \log_4 \tfrac{n}{4}$$

$$= c\frac{\sqrt{n}}{2} \log_4 n - c\frac{\sqrt{n}}{2}$$

$$T(n) = 2T(\tfrac{n}{4}) + \sqrt{n}$$

$$\le 2\left[c\frac{\sqrt{n}}{2} \log_4 n - c\frac{\sqrt{n}}{2}\right] + \sqrt{n}$$

$$= c\sqrt{n} \log_4 n - c\sqrt{n} + \sqrt{n}$$

$$\le c\sqrt{n} \log_4 n \text{ for } c \ge 1$$

7. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $O\left(\sqrt{n}\log n\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(\sqrt{n}\log n\right)$.) 10 points