

Multiple Choice:

1. Write the letter or value for your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. For which graph representation is querying for the presence of an edge supported by binary search?

D

- A. Adjacency lists (ordered) B. Adjacency lists (unordered)
C. Adjacency matrix D. Compressed adjacency lists (ordered)

2. For a double hash table with $\alpha = 0.9$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

10

3. Which disjoint subset implementation implements the find operation in constant time?

A

- A. Implementation 1 B. Implementation 2
C. Implementation 3 D. All three implementations

4. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Z to X, then its type will be:

A

- A. Back B. Cross C. Forward D. Tree



5. Which algorithm maintains multiple subtrees?

B

- A. Dijkstra's B. Kruskal's C. Prim's D. Warshall's

6. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

C

- A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

7. Which of the following cannot occur when additional edges are included in a directed graph?

C

- A. The number of strong components may remain the same.
B. The number of strong components may decrease.
C. The number of strong components may increase.
D. The graph acquires a cycle.

8. Suppose an instance of bipartite matching has 4 vertices in the left column, 8 vertices in the right column, and 20 edges. The number of edges in the corresponding instance of network flow is:

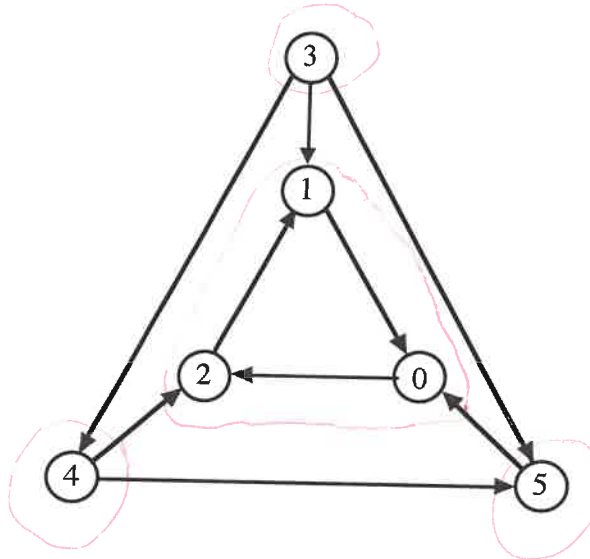
32

9. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:

B

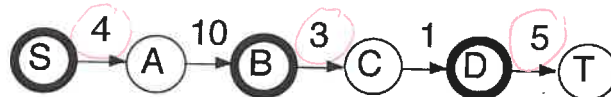
- A. The amount of flow does not exceed the net flow.
 B. They are equal.
 C. The net flow does not exceed the amount of flow.
 D. There is no relationship.

10. What is the number of strongly connected components in this graph?



4

11. The capacity of the following cut is _____. (S vertices are bold.)



12

12. A topological ordering of a directed graph may be computed by:

B

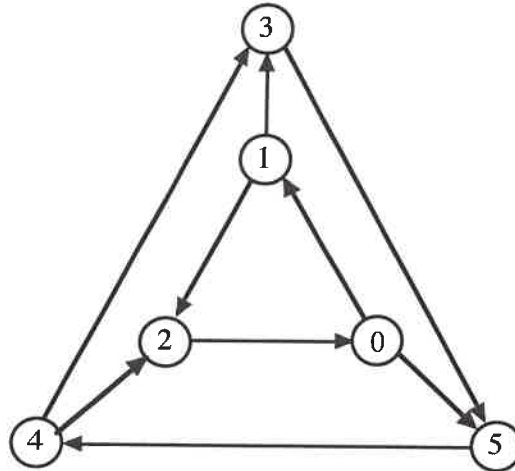
- A. Ordering the vertices by descending discovery time after DFS
 B. Ordering the vertices by descending finish time after DFS
 C. Ordering the vertices by ascending finish time after DFS
 D. Ordering the vertices by ascending discovery time after DFS

13. During a breadth-first search, the status of a white vertex is:

C

- A. It is in the FIFO queue.
 B. It has been completely processed.
 C. It is undiscovered.
 D. It is in the priority queue.

14. What is the number of strongly connected components in this graph?



1

15. The worst-case time for Prim's algorithm implemented with a minheap is:

- A A. $\theta(E \log V)$ B. $\theta(V^2 + E)$ C. $\theta(V \log E)$ D. $\theta(V \log V)$

16. During depth-first search on an undirected graph, a cycle is indicated by which edge type?

- A A. Back B. Cross C. Forward D. Tree

17. The worst-case time for the memoryless version of Dijkstra's algorithm is:

- C A. $\theta(V^2 + E)$ B. $\theta(E \log V)$ C. $\theta(EV)$ D. $\theta(V^2 \log V)$

18. Before searching for a minimum cut in a network, it is useful to do the following:

- B A. Determine the type of each edge using depth-first search.
 B. Find and record augmenting paths until none remain.
 C. Find one augmenting path to determine the source-side vertices.
 D. Perform a breadth-first search on the input network.

19. What is the expected number of probes for an unsuccessful search in hashing by chaining when there are 1000 items stored in a structure with 100 linked lists?

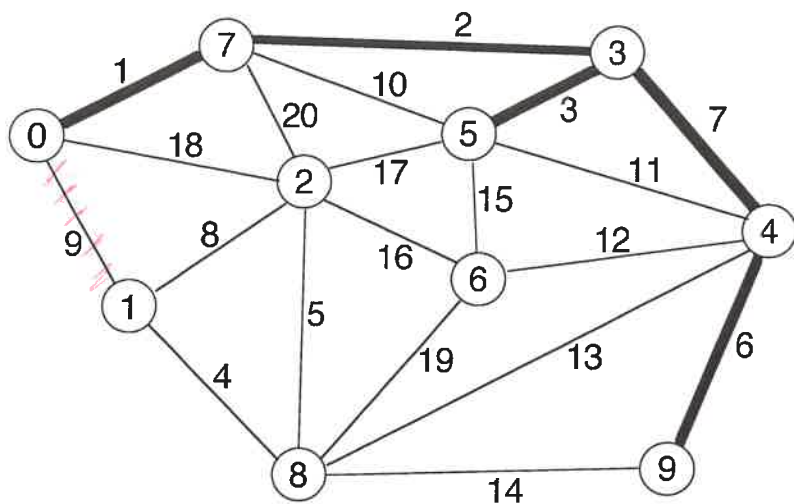
10

20. What is the Edmonds-Karp variant?

- C A. Searching a residual network for an augmenting path of maximum capacity.
 B. Using the capacity of cuts to bound the amount of flow.
 C. Using BFS to search a residual network for an augmenting path.
 D. Using DFS to search a residual network for an augmenting path.

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges not currently in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



Before

| | |
|---|-------|
| 1 | 9(0) |
| 2 | 17(5) |
| 6 | 12(4) |
| 8 | 13(4) |

After

| | |
|---|-------|
| 2 | 8(1) |
| 6 | 12(4) |
| 8 | 4(1) |

2. Consider the following hash table whose keys were stored by double hashing using $h_1(\text{key}) = \text{key} \% 11$ and $h_2(\text{key}) = 1 + (\text{key} \% 10)$. **Show your work.**

| | |
|----|------------|
| 0 | 220 |
| 1 | 660 |
| 2 | 442 |
| 3 | 333 |
| 4 | ————— 2002 |
| 5 | 555 |
| 6 | ————— 222 |
| 7 | 777 |
| 8 | 882 |
| 9 | 999 |
| 10 | |

a. Suppose 222 is to be inserted (using double hashing). Which slot will be used? (5 points)

$$\begin{array}{r} 20 \\ 11 \overline{) 222} \\ \underline{22} \\ 2 = h_1 \end{array} \qquad \begin{array}{r} 22 \\ 10 \overline{) 222} \\ \underline{20} \\ 22 \\ \underline{20} \\ 2 + 1 = 3 \end{array}$$

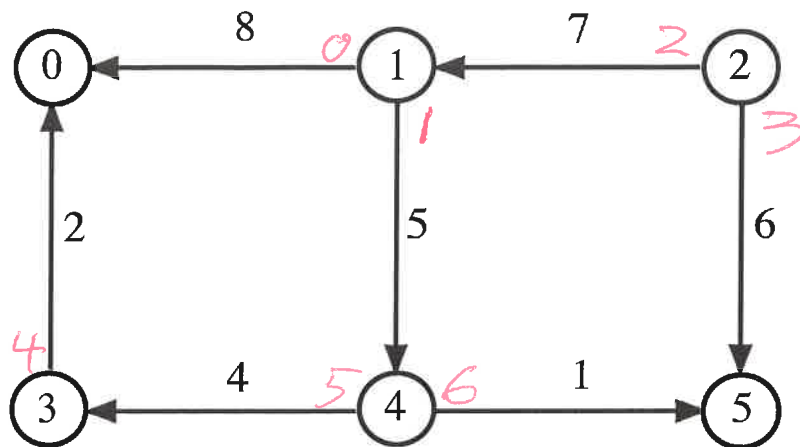
2, 5, 8, 0, 3, (6)

b. Suppose 2002 is to be inserted (using double hashing) **after** 222 has been stored. Which slot will be used? (5 points)

$$\begin{array}{r} 182 \\ 11 \overline{) 2002} \\ \underline{11} \\ 90 \\ \underline{88} \\ 22 \\ \underline{22} \\ 0 = h_1 \end{array} \qquad \begin{array}{r} 2 \\ 10 \overline{) 2002} \\ \underline{20} \\ 2 + 1 = 3 = h_2 \end{array}$$

0, 3, 6, 9, 1, (4)

3. Show the *compressed* adjacency list representation for this weighted, directed graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.



tail Tab

| | |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 2 | 2 |
| 3 | 4 |
| 4 | 5 |
| 5 | 7 |
| 6 | 7 |

head Tab

| | head | wt |
|---|------|----|
| 0 | 0 | 8 |
| 1 | 4 | 5 |
| 2 | 1 | 7 |
| 3 | 5 | 6 |
| 4 | 0 | 2 |
| 5 | 3 | 4 |
| 6 | 5 | 1 |
| 7 | | |

| | | | | | |
|---|----|----|----|----|----|
| | 0 | 1 | 2 | 3 | 4 |
| 0 | 11 | 2 | 3 | 4 | 5 |
| 1 | 17 | 4 | 5 | 6 | 7 |
| 2 | 23 | 7 | 8 | 9 | 10 |
| 3 | 30 | 10 | 11 | 12 | 13 |
| 4 | 37 | 13 | 14 | 15 | 16 |

If either distance or successor is wrong then the entire entry is wrong.

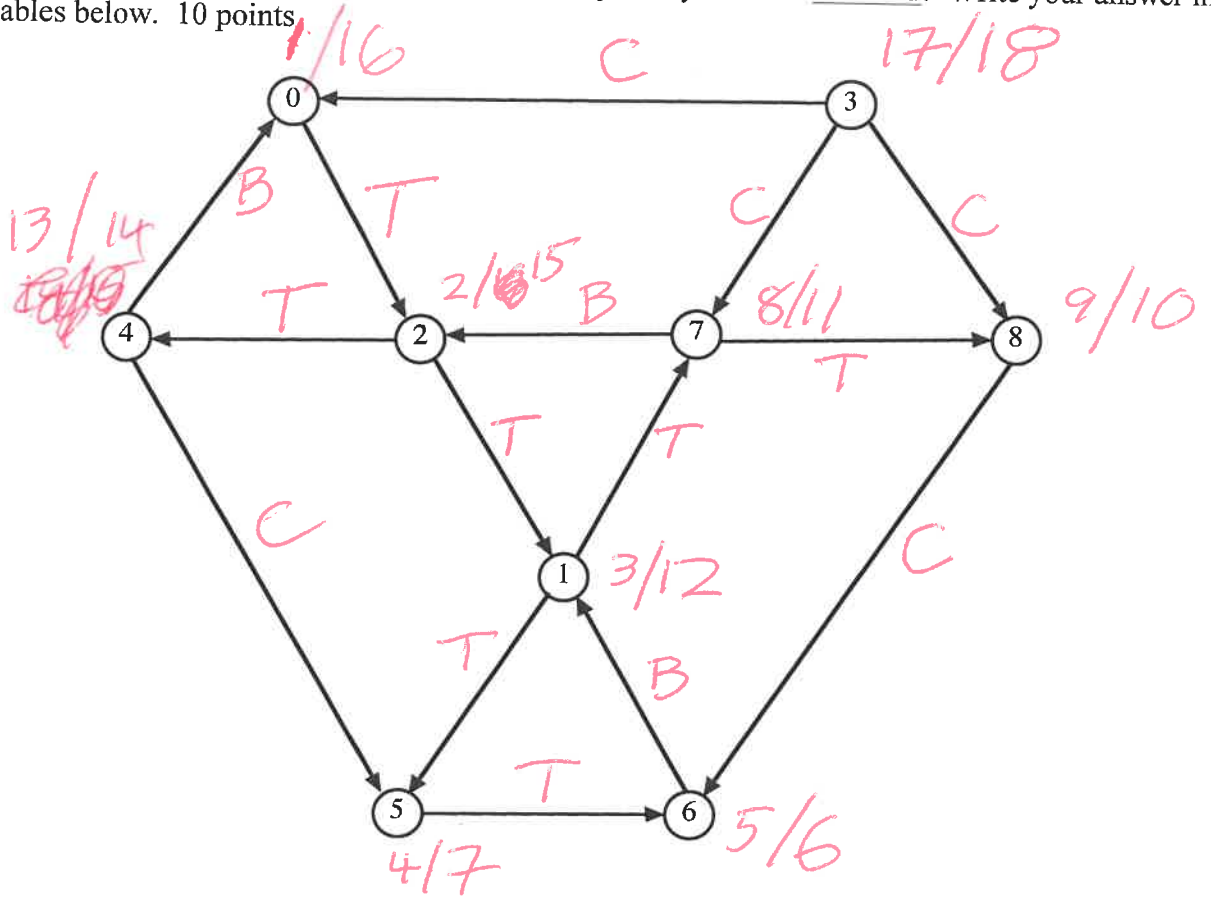
| # entries wrong | lost points |
|----------------------------|------------------------|
| < 1 | 0 |
| < 2 | 1 |
| < 4 | 2 |
| < 5 | 3 |

entries wrong = lost points

4. Demonstrate the Floyd-Warshall algorithm, *with successors*, for the following input adjacency matrix. (∞ represents infinity) The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.

| | 0 | 1 | 2 | 3 | 4 |
|---|----------|----------|----------|----------|----------|
| 0 | ∞ | ∞ | 3 | ∞ | 4 |
| 1 | ∞ | ∞ | 11 | 12 | 4 |
| 2 | 8 | 6 | ∞ | 5 | ∞ |
| 3 | 15 | ∞ | 20 | ∞ | ∞ |
| 4 | ∞ | ∞ | 5 | ∞ | ∞ |

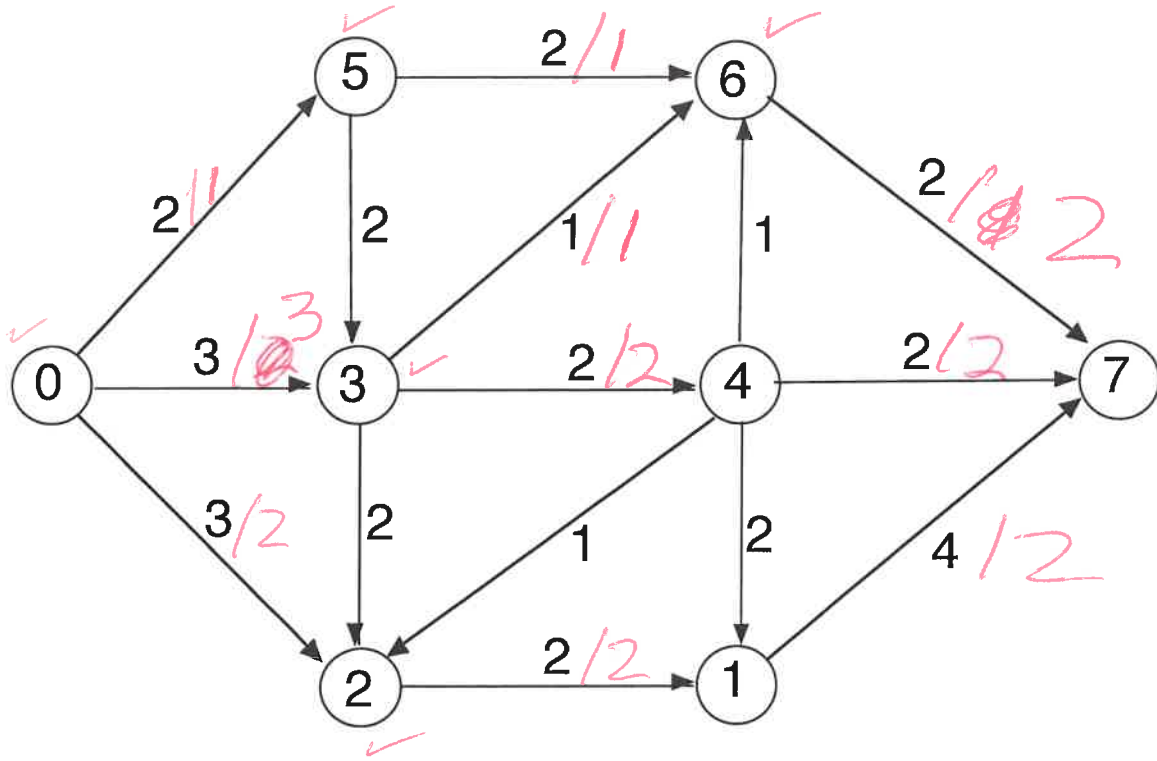
5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points



| Vertex | Start | Finish | Edge | Type | Edge | Type |
|--------|-----------|-----------|------|----------|------|----------|
| 0 | <u>1</u> | <u>16</u> | 0 2 | <u>T</u> | 4 5 | <u>C</u> |
| 1 | <u>3</u> | <u>12</u> | 1 5 | <u>T</u> | 5 6 | <u>T</u> |
| 2 | <u>2</u> | <u>15</u> | 1 7 | <u>T</u> | 6 1 | <u>B</u> |
| 3 | <u>17</u> | <u>18</u> | 2 1 | <u>T</u> | 7 2 | <u>B</u> |
| 4 | <u>13</u> | <u>14</u> | 2 4 | <u>T</u> | 7 8 | <u>T</u> |
| 5 | <u>4</u> | <u>7</u> | 3 0 | <u>C</u> | 8 6 | <u>C</u> |
| 6 | <u>5</u> | <u>6</u> | 3 7 | <u>C</u> | | |
| 7 | <u>8</u> | <u>11</u> | 3 8 | <u>C</u> | | |
| 8 | <u>9</u> | <u>10</u> | 4 0 | <u>B</u> | | |

33 slots / 2 errors = 1 point

6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.



Minimum Cut:

S vertices: 0, 2, 3, 5, 6

T vertices: 7, 1, 4

Augmenting Paths and Contribution to Flow:

0, 2, 1, 7 / 2

0, 3, 4, 7 / 2

0, ~~3~~, 6, 7 / 1

0, 5, 6, 7 / 1

Max flow = 6