CSE 3318-004
Test 1
Fall 2020

KEY
Your name EXACTLY as it appears on your UTA ID Card

Stop at 12:20. No calculators.

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. What is the value of $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k$?

$\frac{3}{2}$

2. Suppose there is a large table with n integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurences of a particular value?

B A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

3. The number of calls to `merge()` while performing `mergesort` on n items is in:

C A. $\Theta(\log n)$ B. $\Theta(1)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

4. Heapsort may be viewed as being a faster version of which sort?

C A. insertion B. mergesort C. selection D. `qsort`

5. Suppose a Huffman code tree is constructed for an alphabet with four symbols where each symbol has a probability of 0.25 of occuring. Give the expected bits per symbol.

2

6. The time to run the code below is in:

```
for (i=n-1; i>=0; i-=2)
  for (j=5; j<n; j*=3)
    sum+=i+j;
```

A A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

7. A sort is said to be stable when:

C A. The expected time and the worst-case time are the same.
B. It runs in $O(n \log n)$ time.
C. Items with the same key will appear in the same order in the output as in the input.
D. It removes duplicate copies of any key in the final output.

8. Which of the following is true?

D A. $3^n \in o(2^n)$ B. $n \log n \in \Omega(n^2)$

C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $n^3 \in \Omega(n^2)$

9. Which of the following is not true regarding a maxheap with 1000 elements?

B A. Subscript 1 will store the maximum priority.
B. The right child for the node with subscript 405 is stored at subscript 911.
C. The parent for the node with subscript 500 is stored at subscript 250.
D. The left child for the node with subscript 200 is stored at subscript 400.

10. The function $n^2 + 3n \log n$ is in which set?

A A. $\Omega(n^2)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

11. $f(n) = \lg n$ is in all of the following sets, except

C A. $O(\log n)$ B. $O(\log(n!))$ C. $\Omega(n)$ D. $o(n^2)$

12. Which of the following facts can be proven using one of the limit theorems?

D A. $n^2 \in \Omega(n^3)$ B. $n^2 \in O(n \log n)$

C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ D. $3^n \in \Omega(2^n)$

13. The time to run the code below is in:

```
for (i=n-5; i>=5; i--)
  for (j=2; j<n; j+=2)
    sum+=i+j;
```

B A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

14. Suppose the input to heapsort is always a table of n ones. The worst-case time will be:

A A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n \log n)$ D. $\Theta(\log n)$

15. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know $T(n) \in \Omega(n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n \lg n)$

16. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 10 probes?

B A. 511 B. 1023 C. 2047 D. 4095

17. What is the definition of H_n?

D A. $\Theta(\sqrt{n})$ B. $\sum_{k=1}^{n} k$ C. $\ln n$ D. $\sum_{k=1}^{n} \frac{1}{k}$

18. Which of the following functions is not in $\Omega(n^2)$?

C A. $n^2 \lg n$ B. n^3 C. n D. n^2

19. $4^{\lg 5}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

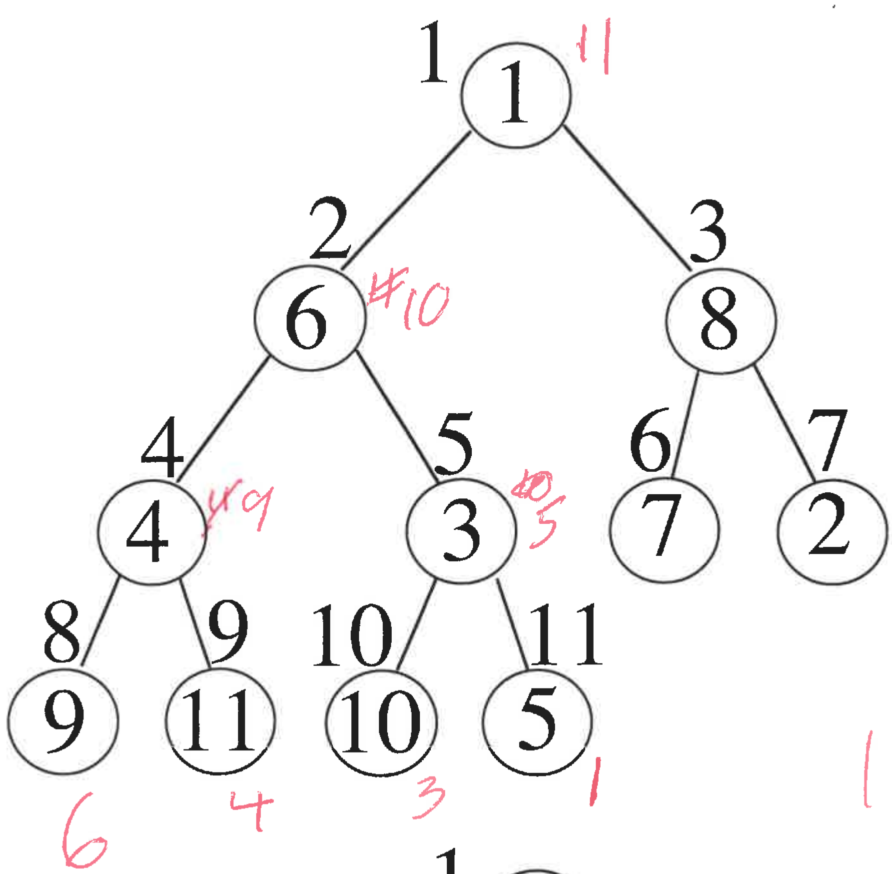
C A. $\sqrt{7}$ B. 7 C. 25 D. 49

20. The number of calls to `heapExtractMin` to build a Huffman code tree for n symbols is:

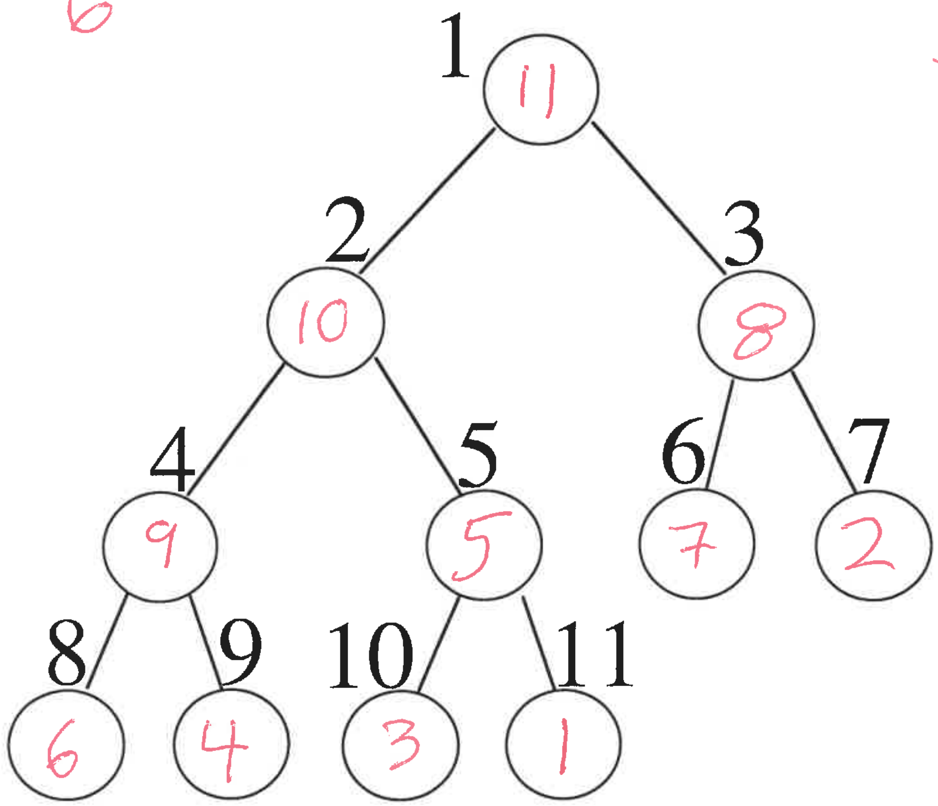
D A. $\Theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$

Long Answer

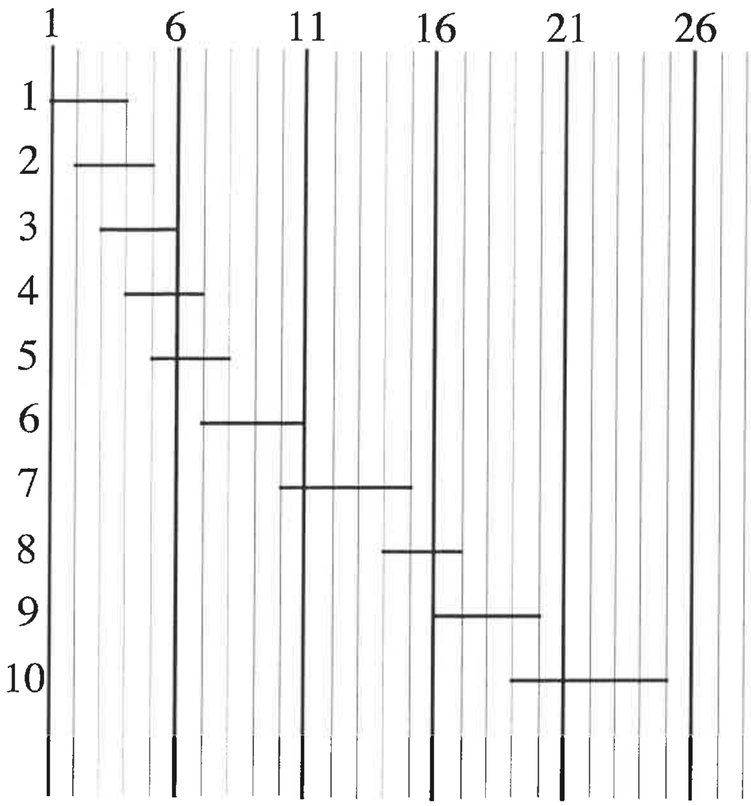
1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points



1 off per error



2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



	v	p	m
1	5	0	5
2	4	0	5
3	3	0	5
4	2	1	7
5	1	2	7
6	5	4	12
7	4	5	12
8	3	6	15
9	2	7	15
10	1	8	16

3. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication. Give C code for a $\Theta(m+n)$ algorithm to find the ***set intersection*** by producing a third array `C` with the values that are common to both `A` and `B` (in ascending order) and sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style ***are unnecessary***.) 10 points

```
i=0;
j=0;
p=0;
while (i<m && j<n)
    if (A[i]==B[j])
    {
        C[p]=A[i];
        i++;
        j++;
        p++;
    }
    else if (A[i]<B[j])
        i++;
    else
        j++;
```

4. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=6 p[1]=3 p[2]=2 p[3]=2 p[4]=4 p[5]=6

	1		2		3		4		5	
1	0	0	36	1	48	1	96	3	???	?
2	-------		0	0	12	2	36	3	96	3
3	-------		-------		0	0	16	3	64	4
4	-------		-------		-------		0	0	48	4
5	-------		-------		-------		-------		0	0

168 3

c(1,5)

c(1,1) + c(2,5) + p[0] p[1] p[5]
0 96 6 3 6 108 204

c(1,2) + c(3,5) + p[0] p[2] p[5]
36 64 6 2 6 172

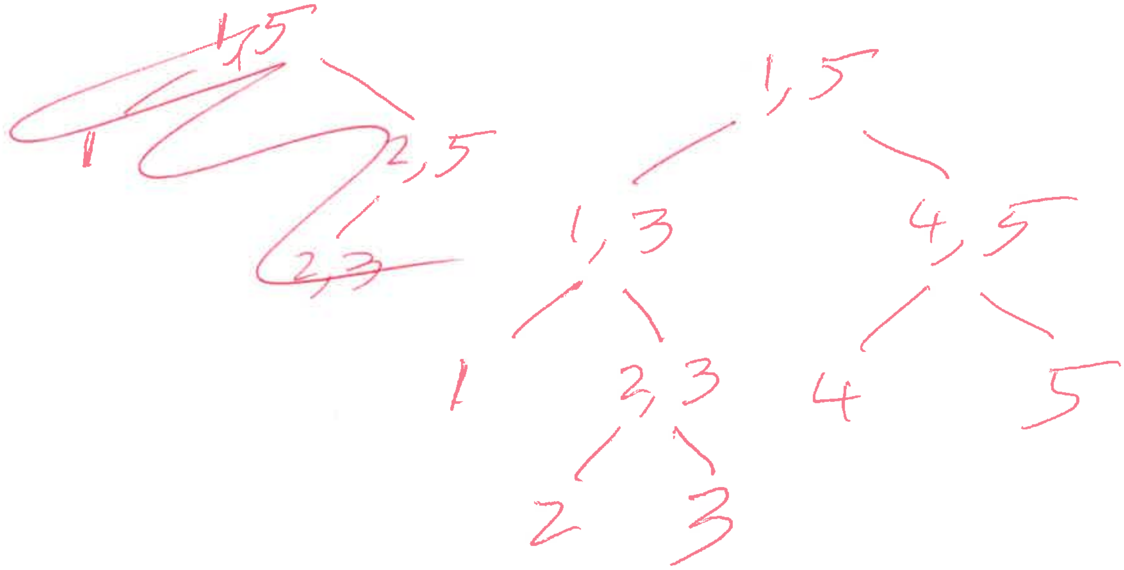
c(1,3) + c(4,5) + p[0] p[3] p[5]
48 48 6 2 6 $\boxed{16}$

c(1,4) + c(5,5) + p[0] p[4] p[5]
96 0 6 4 6 240

72
36
64
172

72
96
168

144
96
240



5. Use the recursion-tree method to show that $T(n) = T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2\right)$. 10 points

$\Omega(n^2)$

$1 + \lg n$	$T(n)$	n^2
	$T\left(\frac{n}{2}\right)$	$\frac{n^2}{4}$
	$T\left(\frac{n}{4}\right)$	$\frac{n^2}{16}$
	$T\left(\frac{n}{8}\right)$	$\frac{n^2}{64}$
	$\vdots$	
	$T(1)$	1

$$n^2 \sum_{K=0}^{\lg n - 1} \left(\frac{1}{4}\right)^K + 1 \le n^2 \sum_{K=0}^{\infty} \left(\frac{1}{4}\right)^K + 1$$

$$= n^2 \frac{1}{1 - \frac{1}{4}} + 1$$

$$= \frac{4}{3} n^2 + 1$$

$$= O(n^2)$$

6. Use the substitution method to show that $T(n) = T\left(\frac{n}{2}\right) + n^2$ is in $O\left(n^2\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^2\right)$.) 10 points

Suppose $T(k) \leq ck^2$ for $k<n$

$T\left(\frac{n}{2}\right) \leq c\frac{n^2}{4}$

$T(n) = T\left(\frac{n}{2}\right) + n^2$

$\leq c\frac{n^2}{4} + n^2$

$= cn^2 - \frac{3}{4}cn^2 + n^2$

$\leq cn^2$ for $c \geq \frac{4}{3}$

or

$= \left(\frac{c}{4}+1\right)n^2$

$\leq cn^2$ for $c \geq \frac{4}{3}$

$\frac{c}{4} + 1 \leq c$

$1 \leq \frac{3}{4}c$

$\frac{4}{3} \leq c$