CSE 2320
Test 1
Spring 2018

Key 1

Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter of your answer on the line (______) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. The time to run the code below is in:

```
for (i=n-1; i>=0; i-=2)
  for (j=1; j<n; j+=j)
    sum+=i+j;
```

A A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

2. A sort is said to be stable when:

A A. Items with the same key will appear in the same order in the output as in the input.
B. It removes duplicate copies of any key in the final output.
C. It runs in $O(n \log n)$ time.
D. The expected time and the worst-case time are the same.

3. Which of the following is not true?

D A. $n^3 \in \Omega(n^2)$ B. $n^2 \in \Omega(n \log n)$
C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $\log\log n \in \Omega(\log n)$

4. The number of calls to `merge()` while performing `mergesort` on n items is in:

C A. $\Theta(\log n)$ B. $\Theta(1)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

5. Which of the following facts can be proven using one of the limit theorems?

B A. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ B. $3^n \in \Omega(2^n)$
C. $n^2 \in \Omega(n^3)$ D. $n^2 \in O(n \log n)$

6. Which of the following best approximates $H_m - H_n$? $(m > n)$

C A. H_{m-n} B. $1/(m-n)$ C. $\ln(m/n)$ D. $\ln(m-n)$

7. Johnson's rule is an example of:

C ____ A. a dynamic programming technique that gives an optimal solution
B. a dynamic programming technique that gives an approximate solution
C. a greedy technique that gives an optimal solution
D. a greedy technique that gives an approximate solution

8. The time to run the code below is in:

```
for (i=n-1; i>=0; i--)
  for (j=15; j<n; j+=2)
    sum+=i+j;
```

B ____ A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

9. Suppose you have correctly determined some c and n_0 to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

C ____ A. c may be decreased C. n_0 may be decreased B. n_0 may be increased D. $f(n) \in O(g(n))$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

C ____ A. $T(n) \in O(n)$ B. $T(n) \in \Omega(n^2)$ C. $T(n) \in \Omega(n^3)$ D. $T(n) \in O(\log n)$

11. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?

C ____ A. 63 B. 64 C. 127 D. 255

12. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

C ____ A. Ascending order of start time B. Descending order of interval length
C. Ascending order of finish time D. Descending order of finish time

13. Which of the following functions is not in $\Omega(n^2)$?

C ____ A. $n^2 \lg n$ B. n^3 C. n D. n^2

14. What is the value of $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$?

D A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 3

15. When solving the fractional knapsack problem, the items are processed in the following order.

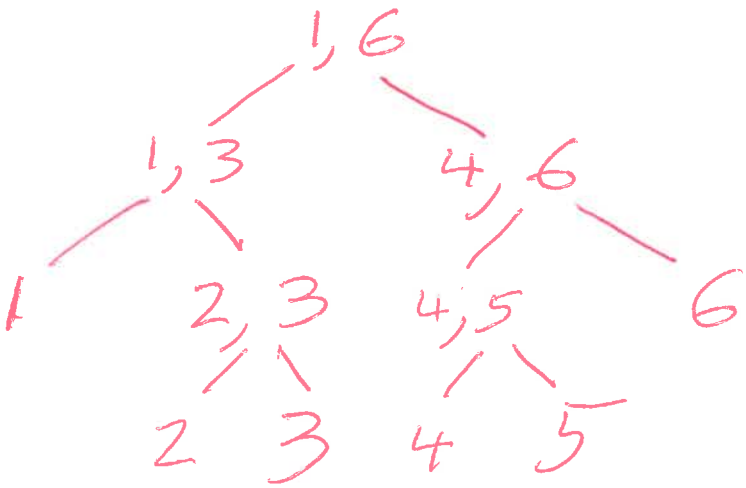
D A. Ascending order of weight B. Ascending order of $$$/lb
C. Descending order of weight D. Descending order of $$$/lb

Long Answer

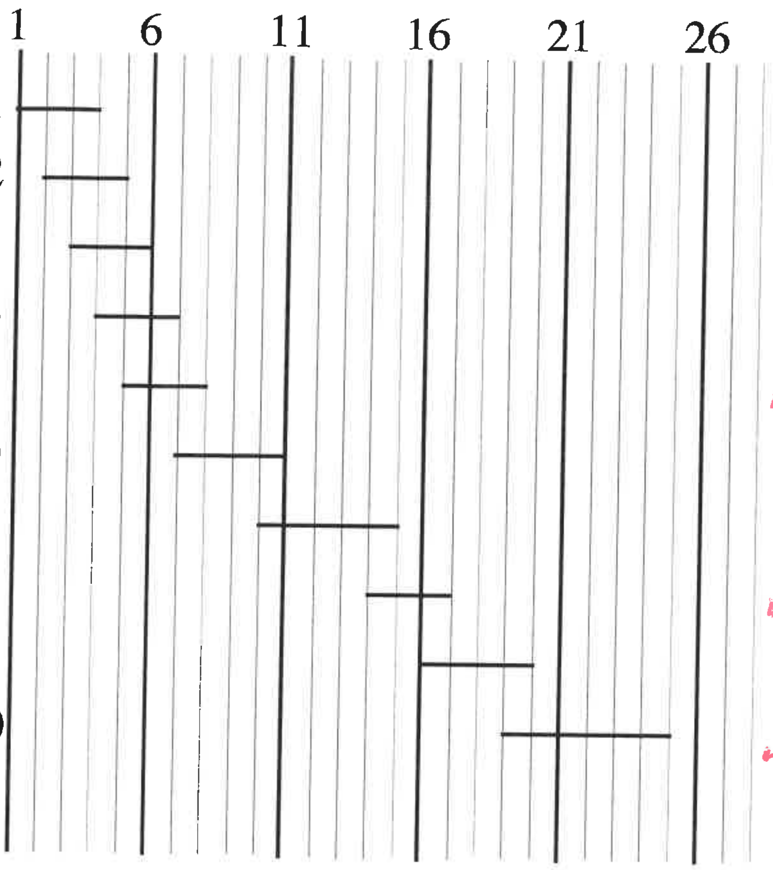
1. Give the tree corresponding to the following instance of optimal matrix multiplication. 5 points

```
6
5 4 3 2 3 4 7
```

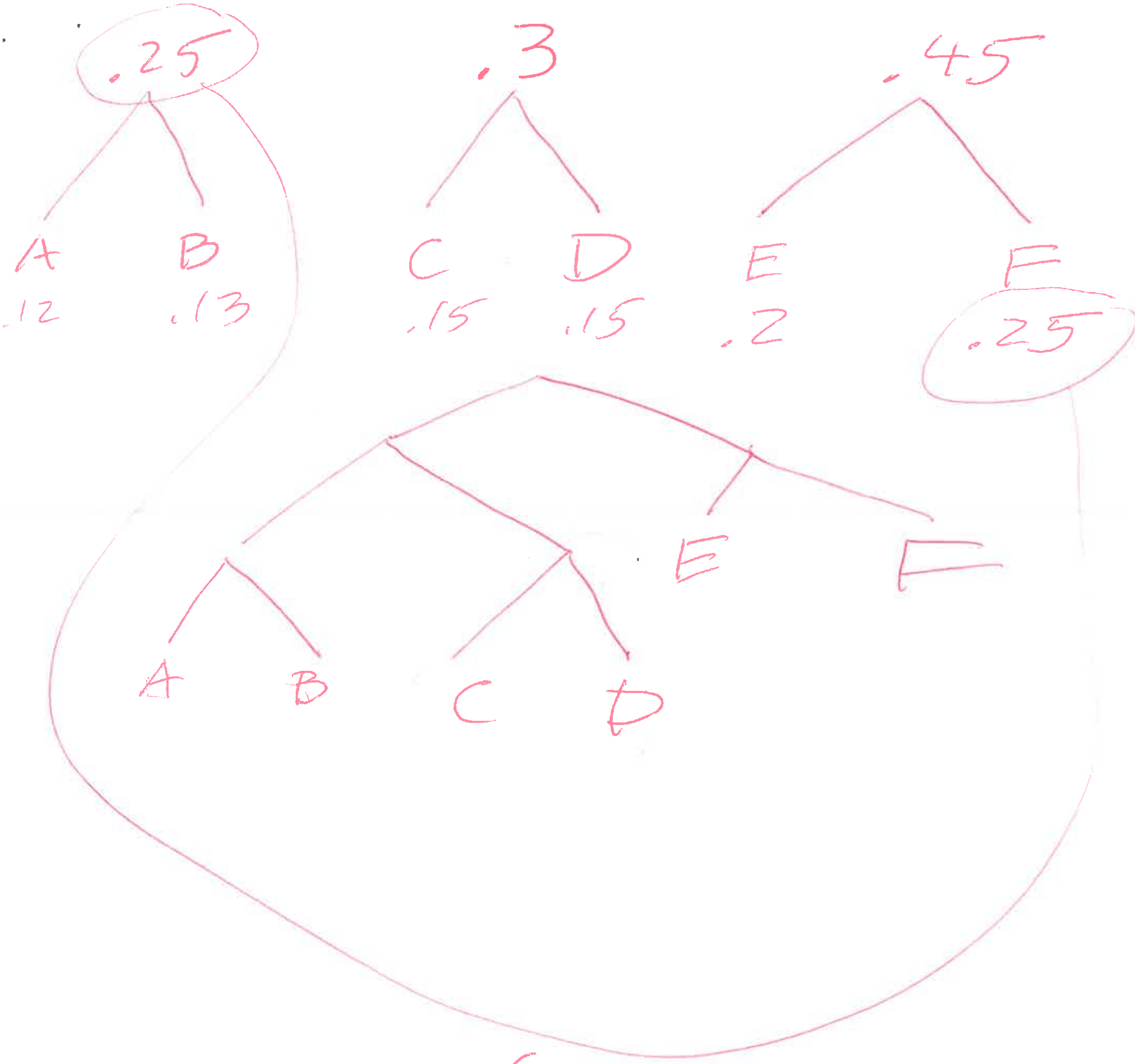
	1		2		3		4		5		6	
1	0	0	60	1	64	1	94	3	128	3	214	3
2	-------		0	0	24	2	48	3	80	3	160	3
3	-------		-------		0	0	18	3	48	3	122	3
4	-------		-------		-------		0	0	24	4	80	5
5	-------		-------		-------		-------		0	0	84	5
6	-------		-------		-------		-------		-------		0	0



2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



	v	p	m
1	4	0	4
2	5	0	5
3	6	0	6
4	3	1	7
5	5	2	10
6	1	4	10
7	3	5	13
8	4	6	14
9	2	7	15
10	5	8	19



Can swap

3. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

A	0.12	3	.36
B	0.13	3	.39
C	0.15	3	.45
D	0.15	3	.45
E	0.2	2	.40
F	0.25	2	.5

The answer is not unique

2.55 bits/symbol expected

4. Complete the C function below that is intended to verify (in linear time) that a maxHeap stored in an `int` array named `heap` has its `n` priorities stored correctly. If the maxHeap is correct, then `return 1`. Otherwise, `return 0`. `n` will not be negative. 10 points

(Details of input/output, allocation, error checking, comments and style **are unnecessary**. Calls to other functions are also unnecessary.)

```
int verify(int n, int heap[])
{
int i;

for (i=2; i<=n; i++)
    if (heap[i] > heap[i/2])
        return 0;

return 1;

}
```

5. $T(n)$ is in $O(n^2 \log n)$

$$T(k) \le ck^2 \log_{\boxed{2}} k \quad \text{for } k<n$$

$$T\left(\frac{n}{2}\right) \le c\left(\frac{n}{2}\right)^2 \log_2 \frac{n}{2}$$

$$= c\frac{n^2}{4}(\log_2 n - 1)$$

$$= c\frac{n^2}{4}\log_2 n - \frac{cn^2}{4}$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$\le 4\left[c\frac{n^2}{4}\log_2 n - \frac{cn^2}{4}\right] + n^2$$

$$= cn^2\log_2 n - cn^2 + n^2$$

$$\le cn^2 \log_2 n \quad \text{when } c \ge 1$$

5. Use the substitution method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $O\left(n^2 \log n\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^2 \log n\right)$.) 10 points

6. $T(n)$ is in $\Theta(n^2 \log n)$

$= 4T(\frac{n}{2}) + n^2$

$T(n) \Rightarrow n^2$

4

$T(\frac{n}{2}) \Rightarrow \frac{n^2}{4} \cdot 4 \Rightarrow n^2$

4

$T(\frac{n}{4})$ ″

″

⋮

$T(1)$ $\#\text{leaves} = 4^{\log_2 n}$

$= n^{\log_2 4}$

$= n^2$

$1 + \log_2 n$

$n^2(1 + \log_2 n) = \Theta(n^2 \log_2 n)$

6. Use the recursion-tree method to show that $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ is in $\Theta\left(n^2 \log n\right)$. 10 points