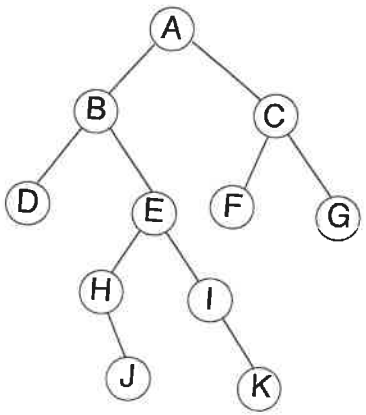


Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. The maximum number of rotations while inserting a key into a red-black tree is:
B A. 1 B. 2 C. 3 D. the black-height

2. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 8? (Write the node's letter on the line.)



A

3. Memoization is associated with which technique?
D A. bottom-up dynamic programming B. PARTITION
C. greedy methods D. top-down dynamic programming

4. In the example of recycling the elements of a list in $O(1)$ time, which situation holds?
C A. Both lists are circular
B. Both lists are not circular
C. The list to be recycled is circular, the garbage list is not
D. The garbage list is circular, the list to be recycled is not

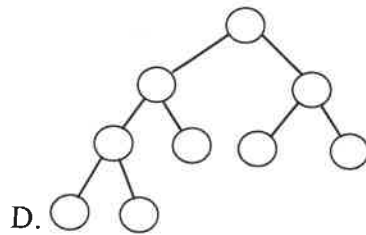
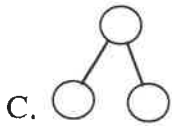
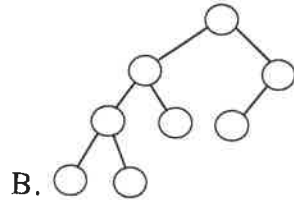
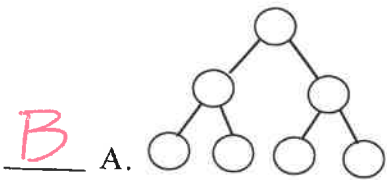
5. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with n nodes in ascending order is:
D A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

6. Suppose that only numbers in $1 \dots 100$ appear as keys in a binary search tree. While searching for 50, which of the following sequences of keys could not be examined?

C

- A. 10, 30, 70, 60, 50
- B. 100, 20, 80, 30, 50
- C. 10, 40, 70, 30, 50
- D. 1, 100, 20, 70, 50

7. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



8. Which phase of counting sort clears the count table?

- A A. first B. second C. third D. fourth

9. Why is it common for a circular queue implementation to waste one table element?

- B A. To make sure the queue always has at least one element in use
 B. To avoid confusing an empty queue with a full queue
 C. To have a place to store the tail and head values
 D. To perform some loops faster

10. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?

- C A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
 B. The height of the tree is $\Omega(n \log n)$.
 C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
 D. There will be $n!$ leaves.

11. Suppose a node x in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete x ?

- A A. Find the successor of x B. Inorder traversal
 C. Rotate x so it becomes a leaf D. Splice the parent of x to either child of x

12. If POP is implemented as `return stack[--SP]`, then the test for an empty stack is implemented as:

- C A. `return stack[SP++]` B. `return SP==(-1)` C. `return SP==0` D. `stack[SP++] = X`

13. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

- A A. The head points to the first element and the tail points to the last element.
 B. The tail points to the first element and the head points to the last element.
 C. Like a circular queue, the maximum number of items is determined at initialization.
 D. One node is always wasted.

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
  if (x==null)
    return;
  traverse(x.left);
  traverse(x.right);
  // process x here
}
```

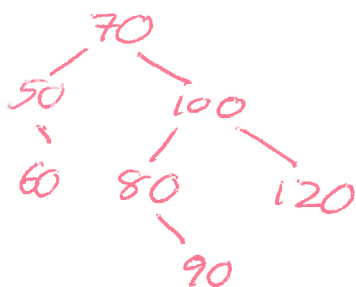
- B A. inorder B. postorder C. preorder D. search for key x

15. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:

- B A. to assure that the final solution is free of duplicate values
 B. to determine the longest possible increasing subsequence terminated by a particular input value
 C. to search a table that will contain only the LIS elements at termination
 D. to sort the original input

Long Answer

1. Give the unbalanced binary search tree that results when the keys 70, 50, 100, 80, 60, 90, 120 are inserted, *in the given order*, into an initially empty tree. (5 points)



i	S
0	0
1	1
2	2
3	3
4	4
5	8
i	C

-1	0	0
-3	1	1
	2	2
	3	2
	4	3
	5	3
	6	3
	7	4
	8	4
	9	4
	10	4
	11	5
	12	5

Solution

i	S
5	8
3	3
1	1

2. Use the dynamic programming solution for subset sums to determine a subset that sums to 12.
(10 points)

i	0	1	2	3	4	5
S_i	0	1	2	3	4	8

AB	8		2		6		1		9		0		7		3		4		5
A	8	B	2		6		1		9		0		7		3		4		5
	2	A	8	B	6		1		9		0		7		3		4		5
	2	A	8		6	B	1		9		0		7		3		4		5
	2		1	A	6		8	B	9		0		7		3		4		5
	2		1	A	6		8		9	B	0		7		3		4		5
	2		1		0	A	8		9		6	B	7		3		4		5
	2		1		0	A	8		9		6		7	B	3		4		5
	2		1		0		3	A	9		6		7		8	B	4		5
	2		1		0		3		4	A	6		7		8		9	B	5
	2		1		0		3		4	<	5	>	7		8		9		6

3. Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

8 2 6 1 9 0 7 3 4 5

LCS is 1010, length==4

		0	1	1	0	1	0
	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1
0	0	1	1	1	2	2	2
0	0	1	1	1	2	2	3
1	0	1	2	2	2	3	3
0	0	1	2	2	3	3	4
1	0	1	2	3	3	4	4

4. Complete the following example of the efficient dynamic programming technique for finding a longest common subsequence. Be sure to provide the backtrace for your LCS using arrows in the matrix. (10 points)

```

1 0 0 1 0 1
0 1 1 0 1 0

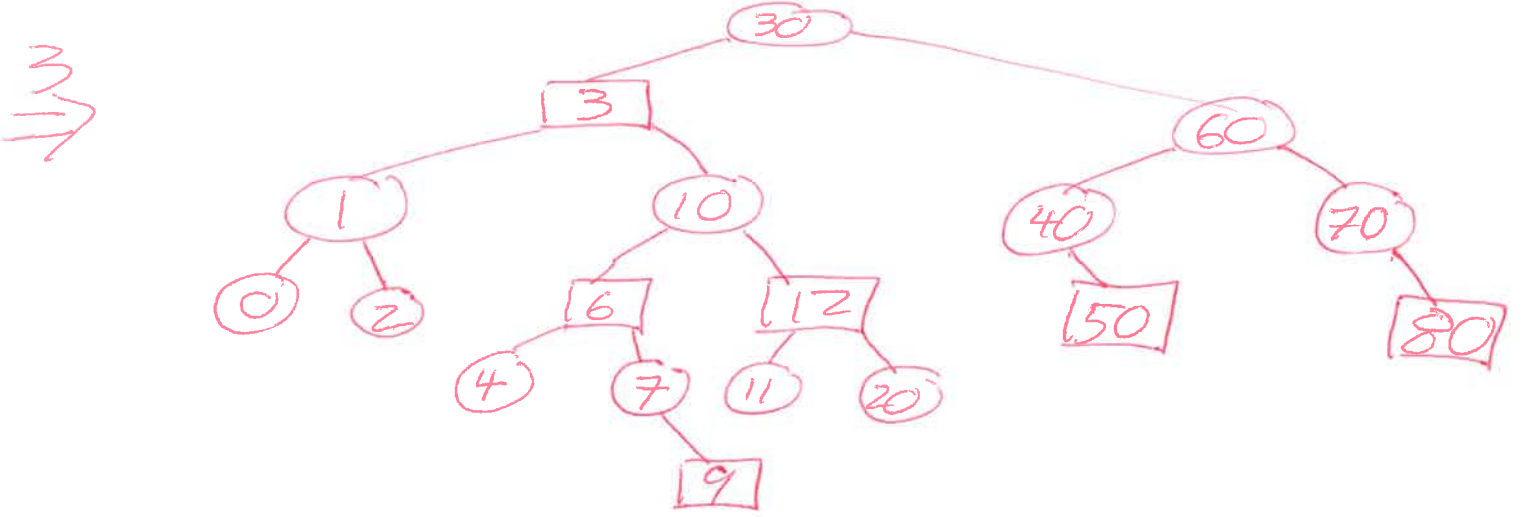
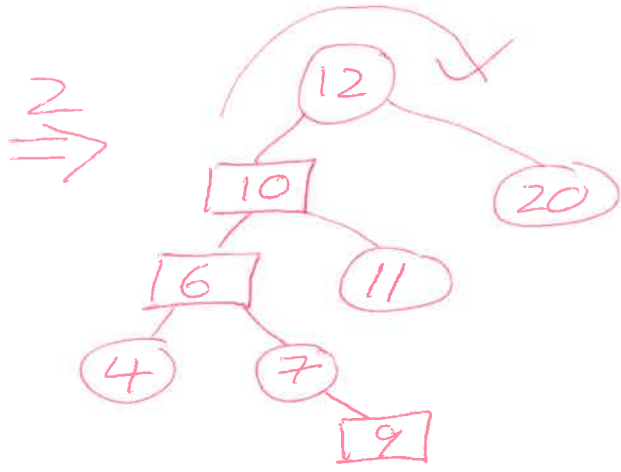
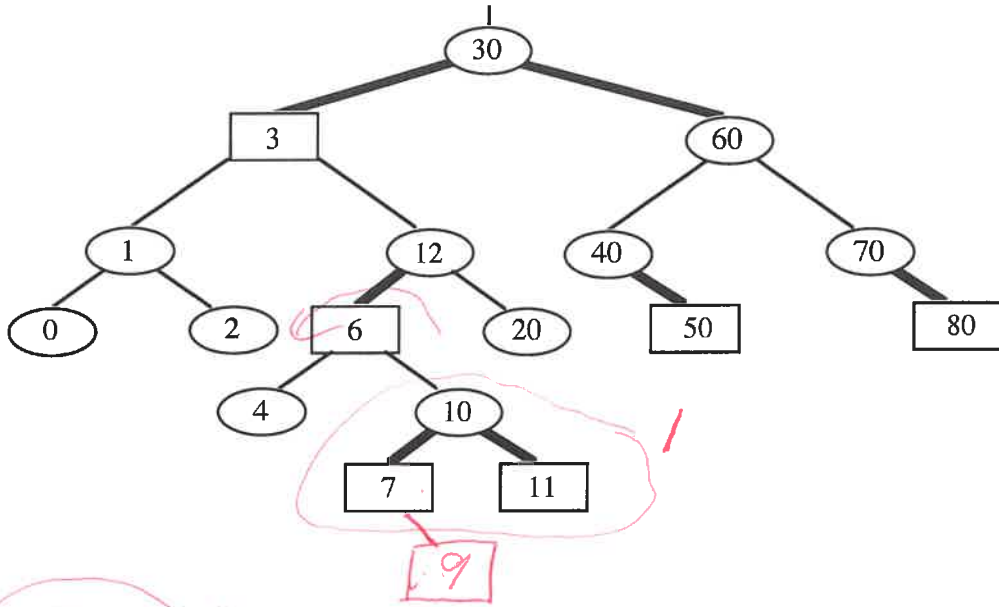
```

```

      0 1 1 0 1 0
    0 0 0 0 0 0 0
1 0 0 1 1 1 1 1
0 0 1 1 1 2 2 2
0 0
1 0
0 0
1 0

```

5. Insert 9 into the given red-black tree. Be sure to indicate the cases that you used (10 points)



6. A billion integers in the range $0 \dots 2^{32} - 1$ will be sorted by LSD radix sort. How much faster is this done using radix $0 \dots 2^4 - 1$ rather than $0 \dots 2^2 - 1$? Show your work. (10 points)

$0 \dots 2^4 - 1$	$0 \dots 2^2 - 1$
$d = 8$	$d = 16$
$k = 16$	$k = 4$
$n = 1B$	$n = 1B$
$\Theta(d(k+n))$	
$8(16 + 1B)$	$16(4 + 1B)$

↑ 2 times faster