

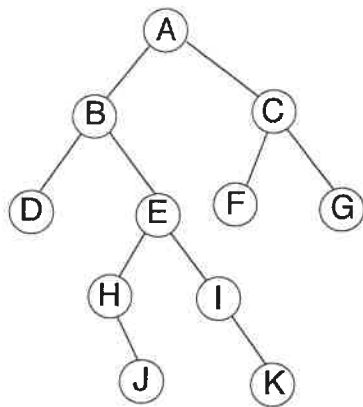
Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The minimum number of rotations while inserting a key into a red-black tree is:

- A A. 0 B. 1 C. 2 D. 3

2. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 5? (Write the node's letter on the line.)



E

3. Memoization is associated with which technique?

- D A. bottom-up dynamic programming B. top-down QUICKSORT
 C. bottom-up MERGESORT D. top-down dynamic programming

4. Circular linked lists are occasionally useful because

- A A. some operations may be done in constant time.
 B. they are an alternative to binary search trees.
 C. they are useful for implementing circular queues.
 D. they avoid mallocs.

5. Given a pointer to a node, the worst-case time to delete the node from an unsorted, doubly-linked list with n nodes is:

- A A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

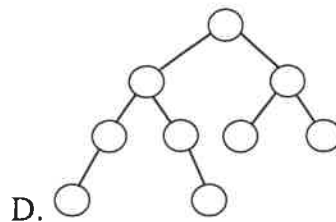
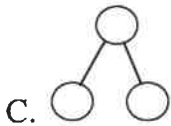
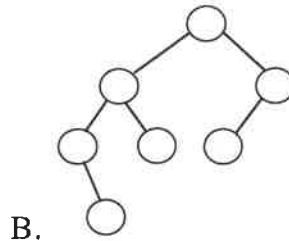
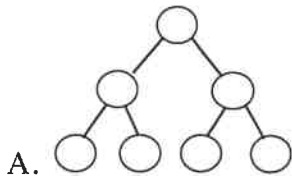
6. Suppose that only numbers in $1 \dots 1000$ appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?

D

- A. 200, 700, 600, 300, 400, 500
 B. 300, 400, 900, 800, 500
 C. 700, 200, 600, 550, 500
 D. 100, 1000, 200, 800, 300, 900, 500

7. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?

B



8. Which phase of counting sort clears the count table?

B

- A. second B. first C. fourth D. third

9. How will a circular queue implementation test for an empty queue?

B

- A. return count==0 B. return tail==head
 C. return head==0 D. return tail==0

10. Which of the following will not be true regarding the decision tree for QUICKSORT for sorting n input values?

B

- A. There will be $n!$ leaves.
 B. Every path from the root to a leaf will have $O(n \log n)$ decisions.
 C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
 D. The height of the tree is $\Omega(n \log n)$.

11. Suppose a node x in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete x ?

D

- A. Find the predecessor of x B. Inorder traversal
 C. Rotate x so it becomes a leaf D. Find the successor of x

12. If POP is implemented as return stack[SP--], then the test for an empty stack is implemented as:

B A. return stack[SP++] B. return SP==(-1) C. return SP==0 D. return stack.empty

13. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

A A. The head points to the first element and the tail points to the last element.
 B. One node is always wasted.
 C. The tail points to the first element and the head points to the last element.
 D. Like a circular queue, the maximum number of items is determined at initialization.

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
  if (x==null)
    return;
  traverse(x.left);
  // process x here
  traverse(x.right);
}
```

A A. inorder B. postorder C. preorder D. search for key x

15. Suppose a binary search tree includes each subtree's size at the subtree's root. How should the rank for the key stored at the tree's root be computed?

B A. Add 1 to the subtree size stored at the root.
 B. Add 1 to the subtree size for the subtree to the left of the root.
 C. Subtract 1 from the subtree size for the subtree to the right of the root.
 D. Count nodes while doing an inorder traversal.

16. Assuming the input has been sorted, Huffman coding may use:

A A. queues B. dynamic programming C. linked lists D. stacks

17. In the example of recycling the elements of a list in $O(1)$ time, which element becomes the first element of the garbage list?

B A. The first element of the circular list
 B. The second element of the circular list
 C. The last element of the circular list
 D. The second element of the original garbage list

18. What is minimized in the dynamic programming solution to the subset sum problem?

C A. The number of input values used to sum to each $C(i)$ B. S_j
 C. The index stored for each $C(i)$ D. m

19. The worst-case number of comparisons for finding the k th largest of n keys using PARTITION is in which asymptotic set?

- D A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

20. Given a pointer to a node, the worst-case time to insert the node into an unsorted, doubly-linked list with n nodes is:

- A A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

Long Answer

1. Use dynamic programming to solve the following instance of the *monotonically* longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. (10 points, no points for solving by inspection)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	5	10	15	20	25	7	10	15	22	25	5	10	22	26	27
C	1	2	3	4	5	2	3	4	5	6	2	4	6	7	8
j	0	1	2	3	4	1	6	7	8	9	1	7	9	13	14

1: 5/1

2: ~~10/2~~ ~~7/6~~ 5/11

3: ~~15/3~~ 10/7

4: ~~20/4~~ ~~15/8~~ 10/12

5: ~~25/5~~ 22/9

6: ~~25/10~~ 22/13

7: 26/14

8: 27/15

i	S
0	0
1	1
2	2
3	3
4	4
5	8
i	C

0	0
1	1
2	2
3	2
4	3
5	3
6	3
7	4
8	4
9	4
10	4
11	5
12	5
13	5

Solution

i	S
5	8
3	3
2	2

2. Use the dynamic programming solution for subset sums to determine a subset that sums to 13. (10 points, no points for solving by inspection)

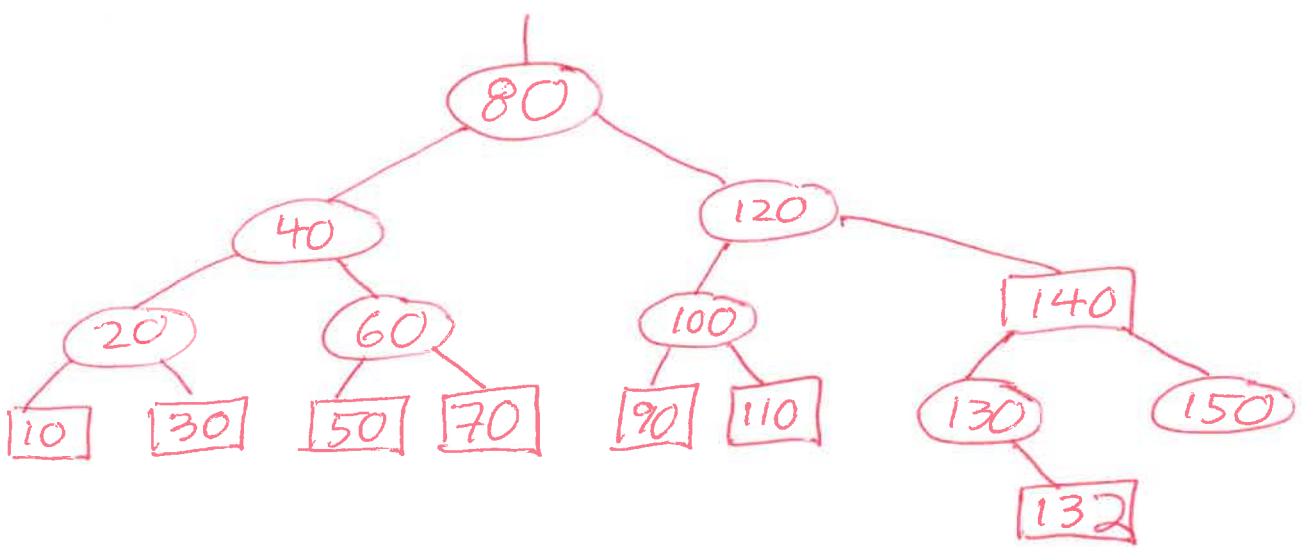
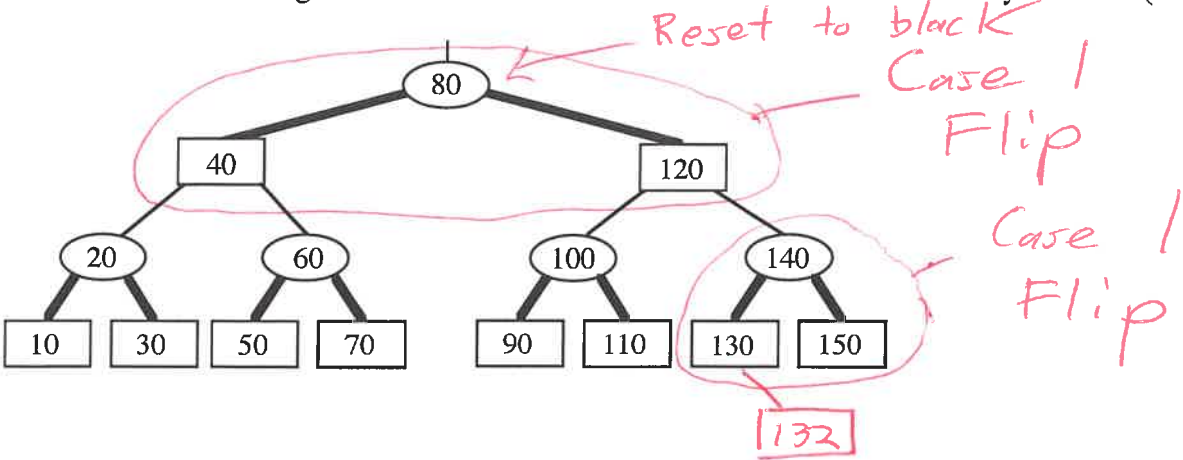
i	0	1	2	3	4	5
S_i	0	1	2	3	4	8

AB 9 0 7 3 8 2 5 1 4 6
 A 9 B 0 7 3 8 2 5 1 4 6
 0 A 9 B 7 3 8 2 5 1 4 6
 0 A 9 7 B 3 8 2 5 1 4 6
 0 3 A 7 9 B 8 2 5 1 4 6
 0 3 A 7 9 8 B 2 5 1 4 6
 0 3 2 A 9 8 7 B 5 1 4 6
 0 3 2 5 A 8 7 9 B 1 4 6
 0 3 2 5 1 A 7 9 8 B 4 6
 0 3 2 5 1 4 A 9 8 7 B 6
 0 3 2 5 1 4 < 6 > 8 7 9

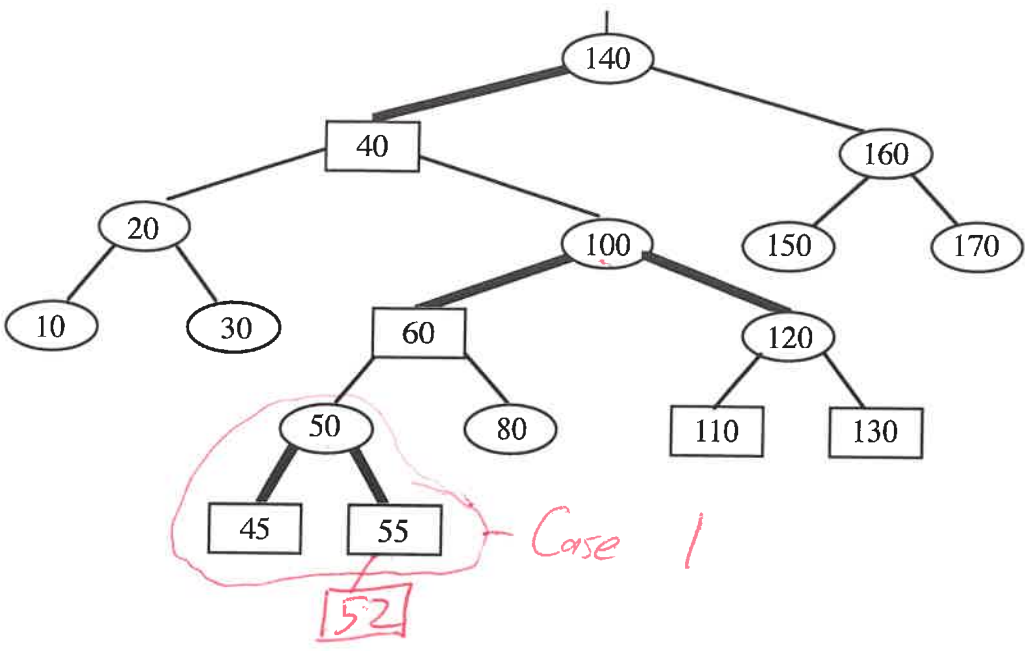
3. Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

9 0 7 3 8 2 5 1 4 6

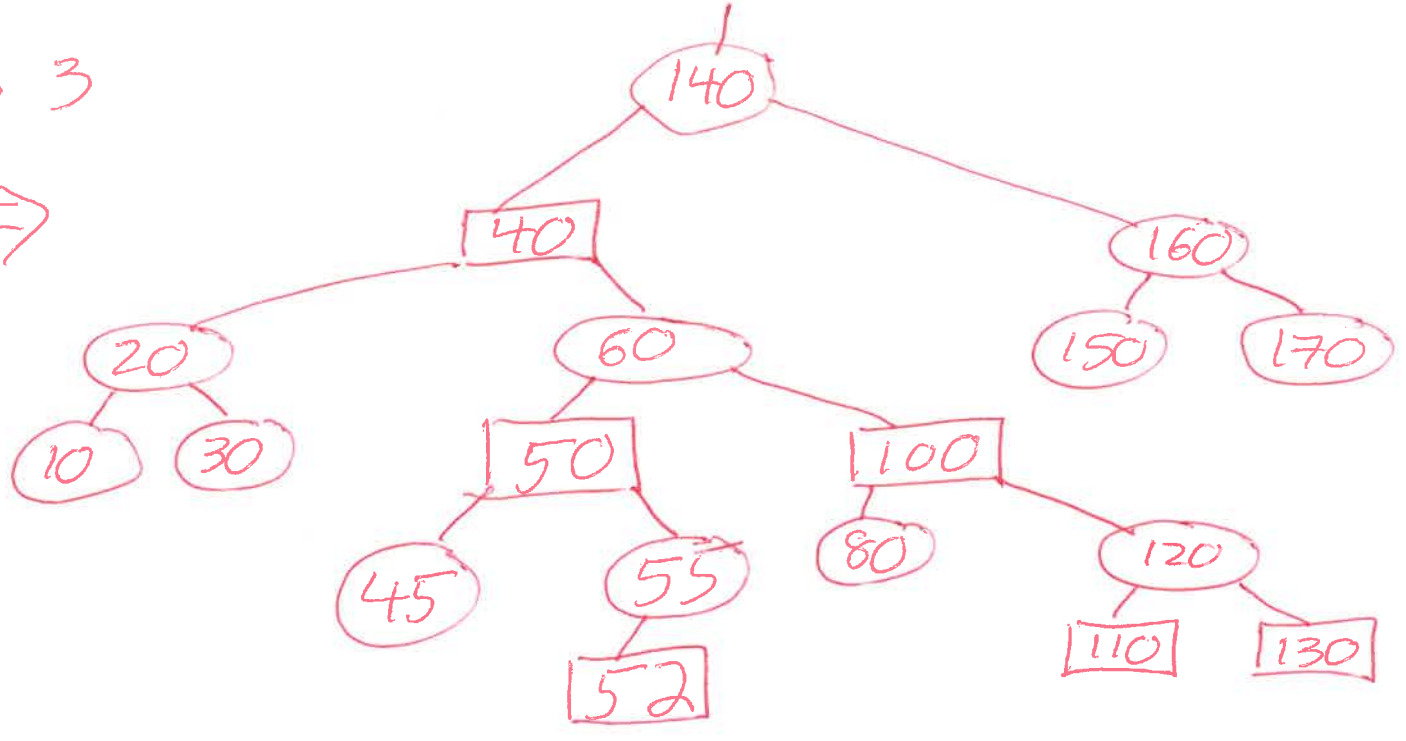
4. Insert 132 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



5. Insert 52 into the given red-black tree. Be sure to indicate the cases that you used. 10 points.



Case 3



6. A billion integers in the range $0 \dots 999,999$ are to be sorted by LSD radix sort. How much faster will this be done if radix $0 \dots 999$ is used rather than decimal ($0 \dots 9$) radix? Show your work. (10 points)

$$n = 1B$$

$$\text{range size} = 10^6$$

$0 \dots 999$	$0 \dots 9$
$k = 1000 = 10^3$	$k = 10$
$d = 2$	$d = 6$
$\Theta(d(n+k))$	$\Theta(6(1B+10))$
$\Theta(2(1B+1000))$	



~~Twice~~ as fast

Three times